TASI Lectures on Flavor Physics

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Lecture I

§1. Overview

This set of lectures will focus on the flavor sector of the Standard Model (SM). As you know, most of the free parameters of the SM reside in this sector. In case you have not thought of it lately, let me remind you of the counting of parameters. Not including neutrino masses, there are 19 parameters in the SM. Five of these are flavor universal – the three gauge couplings, one Higgs quartic coupling λ, and one Higgs mass-squared μ^2, while the remaining fourteen are flavor parameters. Six quark masses, three charged lepton masses, four quark mixing parameters (includes one CP violating phase) make up thirteen, while the strong CP violating parameter $\overline{\theta}$, which is intimately related to the quark masses, is the fourteenth flavor parameter. If we include small neutrino masses and neutrino mixings, as suggested by experiments, an additional nine parameters will have to be introduced (three neutrino masses, three neutrino mixing angles and three CP violating phases, in the case of Majorana neutrinos).

While there is abundant information on the numerical values these parameters take, a fundamental understanding of the parameters is currently lacking. Why are there so many free parameters? Why do these parameters exhibit a strong hierarchical structure spanning some six orders of magnitude? Are the mixing parameters related to the mass ratios? Why is $\overline{\theta} < 10^{-9}$? What is the origin of CP violation? The lack of understanding of such issues is often referred to as the "flavor puzzle". Various solutions to this puzzle have been proposed, inevitably leading to beyond the Standard Model, for within the SM these parameters can only be accommodated, and not explained. Forthcoming experiments, especially at the LHC, have the potential to confirm or refute many, but not all, of these proposed non–standard scenarios. The Higgs boson is waiting to be discovered at the LHC. Its production and decay rates can be significantly modified relative to the SM expectations in some of the flavor–extensions of the SM. I will describe explicit models in this category. Very little is known about the top quark properties currently. LHC will serve as a top factory where modifications in the top sector arising from flavor–extensions can be studied. We have learned a lot about the B meson system from the B factories lately, but there are still many open issues which will be probed at the LHC. These include precise determinations of the CP violating parameters, rare processes allowed in the SM but not yet observed, and new physics processes in B decays that require modification of the SM structure.

In Lecture I, we will take a tour of the flavor parameters of the SM and review how these are measured and interpreted. Various ideas attempting to understand aspects of the flavor puzzle will then be introduced. Experimental consequences of these ideas will be outlined. In Lecture II we will pursue such ideas further and seek their tests. Lecture III will be devoted to the status and expectations of the Standard Model quark mixings and CP violation. Lecture IV will deal with beyond the SM scenarios for the flavor sector and their experimental manifestations at the LHC.

§2. Flavor structure of the Standard Model

Fermion masses arise in the SM via Yukawa interactions given by

$$\mathcal{L}_{\rm Yukawa} = Q^T Y_u u^c H - Q^T Y_d d^c \tilde{H} - L^T Y_\ell e^c \tilde{H} + h.c. \tag{1}$$

Here I have used the standard notation for quark (Q, u^c, d^c) and lepton (L, e^c) fields. $(Q,\ L)$ are $SU(2)_L$ doublets as is the Higgs field H, and its cousin $\tilde{H} = i\tau_2 H^*$, while the $(u^c,\ d^c,\ e^c)$ fields are $SU(2)_L$ singlets. All fermion fields are left–handed, a charge conjugation matrix C is understood to be sandwiched between all of the fermion bilinears. Contraction of the color indices is not displayed, but should be obvious. $Y_{u,d,\ell}$ are the Yukawa couplings matrices spanning generation space which are complex and non–Hermitian. $SU(2)_L$ contraction between the fermion doublet and Higgs doublet involves the matrix $i\tau_2$. Explicitly, we have for family labeled by index i

$$Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix};\ L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix};\ H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix};\tilde{H} = \begin{pmatrix} \overline{H^0} \\ -H^- \end{pmatrix} \tag{2}$$

so that Eq. (1) expands to

$$\mathcal{L}_{\rm Yukawa} = (Y_u)_{ij}[u_i u_j^c H^0 - d_i u_j^c H^+] + (Y_d)_{ij}[u_i d_j^c H^- + d_i d_j^c \overline{H^0}] + (Y_\ell)_{ij}[\nu_i e_j^c H^- + e_i e_j^c \overline{H^0}] + h.c. \tag{3}$$

The neutral component of H acquires a vacuum expectation value (VEV) $\langle H^0 \rangle = v$, spontaneously breaking the electroweak symmetry ($v \simeq 174$ GeV). The Higgs field can then be parametrized as $H^0 = (\frac{h}{\sqrt{2}} + v)$ where h is a real physical field (the Higgs boson). In the unitary gauge $H^\pm$, which are eaten up by the $W^\pm$ gauge bosons and the phase of H^0, which is eaten up by the Z^0 gauge boson, do not appear.

The VEV of H^0 generates the following fermion mass matrices:

$$M_u = Y_u v, \qquad M_d = Y_d v, \qquad M_\ell = Y_\ell v\ . \tag{4}$$

The Yukawa coupling matrices contained in $(Y_u)_{ij}/\sqrt{2}(uu^c h)$, etc in each of the up, down and charged lepton sector becomes proportional to the corresponding mass matrix. Once the mass matrices are brought to diagonal forms, the Yukawa coupling matrices will also be brought to diagonal forms. There is thus no tree–level flavor changing current mediated by the neutral Higgs boson in the Standard Model. As we will see, this feature is generally lost as we extend the SM to address the flavor issue.

We make unitary rotations on the quark fields in family space. Unitarity of these rotation will ensure that the quark kinetic terms remain canonical. Specifically, we define mass eigenstates $(u^0, u^{c0}, d^0, d^{c0})$ via

$$\begin{aligned} u &= V_u\ u^0, \quad u^c = V_{u^c}\ u^{c0}, \\ d &= V_d\ d^0, \quad d^c V_{d^c}\ d^{c0} \end{aligned} \tag{5}$$

and we choose the unitary matrices such that

$$V_u^T(Y_u v)V_{u^c} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}, \quad V_d^T(Y_d v)V_{d^c} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}. \tag{6}$$

We have assumed here that the number of families is three, but the procedure applies to any number of families. Bi-unitary transformations such as the ones in Eq. (6) can diagonalize non–Hermitian matrices. The same transformations should be applied to all interactions of the quarks. As already noted, this transformation will bring the Yukawa interactions of quarks with the Higgs boson into diagonal form. The couplings of the Z^0 boson and the photon to quarks will have the original diagonal form even after this rotation. For example, $(\overline{u}\gamma_\mu I u)Z^\mu$ where I is the identity matrix acting on family space will transform to $(\overline{u^0}\gamma_\mu(V_u^\dagger I V_u)u^0)Z^\mu$, which is identical to $(\overline{u^0}\gamma_\mu I u^0)Z^\mu$. Similarly, $(\overline{u^c}\gamma_\mu I u^c)Z^\mu$ will transform to $(\overline{u^{c0}}\gamma_\mu I u^{c0})Z^\mu$. We see that there is no tree level flavor changing neutral current (FCNC) mediated by the Z^0 boson and the photon in the SM.

Most significantly, the transformations of Eq. (6) will bring the charged current quark interaction, which originally is of the form $g/\sqrt{2}(\overline{u}\gamma_\mu d)W^{+\mu} + h.c.$, into the form

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}[\overline{u^0}\gamma_\mu V d^0]\ W^{\mu+} + h.c. \tag{7}$$

where

$$V = V_u^\dagger V_d \tag{8}$$

is the quark mixing matrix, or the Cabibbo–Kobayashi–Maskawa (CKM) matrix. In the SM, all the flavor violation is contained in V. Being product of unitary matrices, V is itself unitary. This feature has thus far withstood experimental scrutiny, with further scrutiny expected from LHC experiments.

Note that the right–handed rotation matrices V_{u^c} and V_{d^c} have completely disappeared, a result of the purely left–handed nature of charged weak current.

We can repeat this process in the leptonic sector. We define, in analogy with Eq. (5),

$$\nu = V_\nu\ \nu^0, \quad e = V_e\ e^0, \quad e^c = V_{e^c}\ e^{c0}\ . \tag{9}$$

We choose Y_e and Y_{e^c} such that

$$Y_e^T(Y_\ell v)Y_{e^c} = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} . \tag{10}$$

Note that there is no right–handed neutrino in the SM. If the Yukawa Lagrangian is as given in Eq. (1), there is no neutrino mass. In that case one can choose $V_\nu = V_e$, so that the charged current weak interactions will remain flavor diagonal. However, it is now established that neutrinos have small masses. Additional terms must be added to Eq. (1) in order to accommodate them. The simplest possibility is to add a non–renormalizable term

$$\mathcal{L}_{\nu \text{ mass}} = \frac{(L^T Y_\nu L)HH}{M_*} \tag{11}$$

where the $SU(2)_L$ contraction between the H fields is in the triplet channel and Y_ν is a complex symmetric matrix in generation space. Here M_* is a mass scale much above the weak interaction scale. Eq. (11) can arise by integrating out some heavy fields with mass of order M_*. The most celebrated realization of this is the seesaw mechanism, where M_* corresponds to the mass of the right–handed neutrino. The neutrino masses are suppressed, compared to the charged fermion masses, because of the inverse dependence on the heavy scale M_*. Right–handed neutrinos, if they exist, are complete singlets of the SM gauge symmetry, and can possess bare SM invariant mass terms, unlike any other fermion of the SM. This is an elegant explanation of why the neutrinos are much lighter than other fermions, relying only on symmetry principles and dimensional analysis. Eq. (11) leads to a light neutrino mass matrix given by

$$M_\nu = Y_\nu \frac{v^2}{M_*} . \tag{12}$$

Now we choose V_ν so that

$$V_\nu^T Y_\nu \frac{v^2}{M_*} V_\nu = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} , \tag{13}$$

with $m_{1,2,3}$ being the tiny masses of the three light neutrinos. The leptonic charge current interaction now becomes

$$\mathcal{L}_{cc}^\ell = \frac{g}{\sqrt{2}}[\overline{e^0}\gamma_\mu U \nu^0]\ W^{-\mu} + h.c. \tag{14}$$

where

$$U = V_e^\dagger V_\nu \tag{15}$$

is the leptonic mixing matrix, or the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix. As V, U is also unitary. Neutrino oscillations observed in experiments are attributed to the off–diagonal entries of the matrix U. We assumed here that the neutrino mass generation mechanism violated total lepton number by two units. While this is very attractive, it should be mentioned that neutrinos could acquire masses very much like the quarks do. That would require the right–handed ν^c states to be part of the low energy theory. M_ν will then be similar to M_ℓ of Eq. (10). Neutrino oscillation phenomenology will be identical to the case of L–violating neutrino masses.

The fermionic states (e_i^0) are simply the physical electron, the muon, and the tau lepton states. Similarly, the quark fields with a superscript ^{0} are the mass eigenstates. It is conventional to drop these superscripts, which we shall do from now on.

§3. Lepton masses

Conceptually charged lepton masses are the easiest to explain. Leptons are propagating states, and their masses are simply the poles in the propagators. Experimental information on charged lepton masses is rather accurate:

$$\begin{aligned} m_e &= 0.510998902 \pm 0.000000021 \text{ MeV} \\ m_\mu &= 105.658357 \pm 0.000005 \text{ MeV} \\ m_\tau &= 1777.03^{+0.30}_{-0.26} \text{ MeV} . \end{aligned} \tag{16}$$

The direct kinematic limits on the three neutrino masses are:

$$m_{\nu_e} \leq 3 \text{ eV}, \; m_{\nu_\mu} \leq 0.19 \text{ MeV}, \; m_{\nu_\tau} \leq 18.2 \text{ MeV} . \tag{17}$$

Neutrino oscillation experiments have provided much more accurate determinations of the squared mass differences $\Delta m_{ij}^2 = m_i^2 - m_j^2$. Solar and atmospheric neutrino oscillation experiments suggest the following allowed values (with 2σ error quoted):

$$\begin{aligned} \Delta m_{21}^2 &= (7.3 - 8.1) \times 10^{-5} \text{ eV}^2 \\ \Delta m_{31}^2 &= \pm(2.1 - 2.7) \times 10^{-3} \text{ eV}^2 . \end{aligned} \tag{18}$$

While this still leaves some room for the absolute masses, when combined with the direct limit on $m_{\nu_e} \leq 3$ eV, the options are limited. Current data allows for two possible ordering of the mass hierarchies: (i) normal hierarchy where $m_1 \leq m_2 \ll m_3$, and inverted hierarchy where $m_1 \simeq m_2 \gg m_3$.

§4. Leptonic mixing matrix

The PMNS matrix, being unitary, has N^2 independent components for N families of leptons. Out of these, $N(N-1)/2$ are Euler angles, while the remaining $N(N+1)/2$ are phases. Many of these phases can be absorbed into the fermionic fields and removed. If one writes $U = Q\hat{U}P$, where P and Q are diagonal phase matrices, then by redefining the phases of e fields Q, which has N phases, can be removed. P has only $N-1$ non–removable phases. For $N=3$, $P = diag.(e^{i\alpha},\ e^{i\beta},\ 1)$. α, β are called the Majorana phases. (If the neutrino masses are of the Dirac type, these phases can also be removed by redefining the ν^c fields.) $\hat{U}$ will then have $N(N+1)/2 - (2N-1) = \frac{1}{2}(N-1)(N-2)$ phases. For $N=3$, there is a single "Dirac" phase in U.

In general, the PMNS matrix can be written as

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} . \tag{19}$$

To enforce the unitarity relations it is convenient to adopt specific parametrizations. The Euler angles, as you know, can be parametrized in many different ways. Furthermore, the Dirac phase can be chosen to appear in different ways (by field redefinitions). The "standard parametrization" that is now widely used has $U_{PMNS} = U.P$ where

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} . \tag{20}$$

Here $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$.

Our current understanding of these mixing angles arising from neutrino oscillations can be summarized as follows (2 σ error bars quoted):

$$\begin{aligned} \sin^2\theta_{12} &= 0.28 - 0.37 \\ \sin^2\theta_{23} &= 0.38 - 0.63 \\ \sin^2\theta_{13} &\leq 0.033 . \end{aligned} \tag{21}$$

Here θ_{12} limit arises from solar neutrino data, θ_{23} from atmospheric neutrinos, and θ_{13} from reactor neutrino data.

It is intriguing that the current understanding of leptonic mixing can be parametrized by the unitary matrix

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P . \tag{22}$$

This mixing is known as tri-bimaximal mixing. As we will see, such a geometric structure is far from being similar to the quark mixing matrix. Note that currently θ_{13} is allowed to be zero, in which case the Dirac phase δ becomes irrelevant. We also have no information on the Majorana phases, which can only be tested in neutrinoless double beta decay experiments.

§5. Quark masses

Unlike the leptons, quarks are not propagating particles. So their masses have to be inferred indirectly from properties of hadrons. There are various techniques to do this. Let me illustrate this for the light quark masses by the method of chiral perturbation theory.

Consider the QCD Lagrangian at low energy scales. Electroweak symmetry has already been broken, and heavy quarks (t, b, c) have decoupled. The Lagrangian takes the form

$$\mathcal{L} = \sum_{k=1}^{N_F} \overline{q}_k (i\not{D} - m_k) q_k - \frac{1}{4} G_{\mu\nu} G^{\mu nu} \tag{23}$$

where $G_{\mu\nu}$ is the gluon field strength and $\not{D}$ is the covariant derivative. m_k is the mass of the kth quark and q_k the quark field. This Lagrangian has a chiral symmetry in the limit where the quark masses vanish. The symmetry is $SU(3)_L \times SU(3)_R \times U(1)_V$, with the axial $U(1)_A$ explicitly broken by anomalies. The $U(1)_V$ is baryon number, which remains unbroken even after QCD dynamics. QCD dynamics breaks the $SU(3)_L \times SU(3)_R$ symmetry down to $SU(3)_V$. In the limit of vanishing quark masses, there must be 8 Goldstone bosons corresponding to this symmetry breaking. These Goldstone bosons are identified as the pseudoscalar mesons, which are however, not exactly massless. The quark masses break the chiral symmetry explicitly and thus generate masses for the mesons.

Chiral perturbation theory is a systematic expansion in p/Λ_χ, where p is the particle momentum and $\Lambda_\chi \sim 1$ GeV is the chiral symmetry breaking scale. Since the masses of the light quarks $(u,\ d,\ s)$ are smaller than Λ_χ, we can treat them to be small and apply chiral expansion. The explicit breaking of chiral symmetry occurs via the mass term

$$M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} . \tag{24}$$

M can be thought of as a field which broke the chiral symmetry spontaneously. M can be split as $M = M_1 + M_8$, where M_1 is a singlet under $SU(3)_V$, while M_8 transforms as an octet:

$$\begin{aligned} M_1 &= \frac{(m_u + m_d + m_s)}{3} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \\ M_8 &= \frac{(m_u - m_d)}{2} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} + \frac{(m_u - m_d - 2m_s)}{6} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \end{aligned} \tag{25}$$

The octet of mesons can be written down as a (normalized) matrix

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\sqrt{\frac{2}{3}}\eta^0 \end{pmatrix} . \tag{26}$$

The lowest order invariants involving Φ bilinear and M are

$$A \ \mathrm{Tr}(\Phi^2)M_1 + B \ \mathrm{Tr}(\Phi^2 M_8) \ . \tag{27}$$

A and B are arbitrary coefficients here. Eq. (27) can be readily expanded, which will give relations for the masses of mesons. Now, in the limit of $m_u = 0, m_d = 0, m_s \neq 0$, the $SU(2)_L \times SU(2)_R$ chiral symmetry remains unbroken, and so the pion fields should be massless. Working out the mass terms, and demanding that the pion mass vanishes in this limit, gives a relation $A = 2B$. Using this relation we can write down the pseudoscalar meson masses. In doing so, let us also recall that electromagnetic interactions will split the masses of the neutral and charged members. To lowest order, this splitting will be universal. Then we have

$$\begin{aligned} m_{\pi^0}^2 &= B(m_u + m_d) \\ m_{\pi^\pm}^2 &= B(m_u + m_d) + \Delta_{\mathrm{em}} \\ m_{K^0}^2 &= m_{\overline{K^0}}^2 = B(m_d + m_s) \\ m_{K^\pm}^2 &= B(m_u + m_s) + \Delta_{\mathrm{em}} \\ m_\eta^2 &= \frac{1}{3}B(m_u + m_d + 4m_s) \end{aligned} \tag{28}$$

Here small $\pi^0 - \eta^0$ mixing has been neglected, which vanishes in the limit $m_u - m_d$ vanishes.

Eliminating B and Δ_{em} from Eq. (28) we obtain two relations for quark mass ratios:

$$\begin{aligned} \frac{m_u}{m_d} &= \frac{2m_{\pi^0}^2 - m_{\pi^+}^2 + m_{K^+}^2 - m_{K^0}^2}{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^+}^2} = 0.56 \\ \frac{m_s}{m_d} &= \frac{m_{K^0}^2 + m_{K^+}^2 - m_{\pi^+}^2}{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^+}^2} = 20.1 \end{aligned} \tag{29}$$

This is the lowest order chiral perturbation theory result for these mass ratios. Note that the absolute masses cannot be determined in this way. QCD sum rules and lattice calculations are needed for this.

For heavy quarks (c and b), one can invoke heavy quark effective theory (HQET). When the mass of the quark is heavier than the typical momentum of the partons $\Lambda \sim m_p/3 = 330$ MeV, one can make another type of expansion. In analogy with atomic physics, where different isotopes exhibit similar chemical behavior, the behavior of charm hadrons and bottom hadrons will be similar. In fact, there will be an $SU(2)$ symmetry relating the two, to lowest order in HQET expansion. One consequence is that the mass splitting between the vector and scalar mesons in the b and c sector should be the same. This leads to a relations $M_{B^*} - M_B = \Lambda^2/m_b$ and $M_{D^*} - M_D = \Lambda^2/m_c$, leading to the prediction

$$\frac{M_{B^*} - M_B}{M_{D^*} - M_D} = \frac{m_c}{m_b} \tag{30}$$

in good agreement with experiments.

There is general agreement with these types of evaluations and lattice QCD calculations, which are perhaps more reliable. Listed below are the masses of quarks obtained from lattice QCD:

$$\begin{aligned} \frac{1}{2}(\overline{m}_u + \overline{m}_d)(2 \text{ GeV}) &= 3.8 \pm 0.8 \text{ MeV} \\ \overline{m}_s(2 \text{ GeV} &= 95 \pm 20 \text{ MeV} \\ \overline{m}_c(m_c) &= 1.30 \pm 0.20 \text{ GeV} \\ \overline{m}_b(m_b) &= 4.20 \pm 0.20 \text{ GeV} \end{aligned} \tag{31}$$

§6. Quark mixing

The unitary matrix V of Eq. (8) appears in a variety of processes. A lot of information has been gained on the matrix elements. The general matrix can be written as

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{32}$$

The standard parametrization of V is as in Eq. (20), but now understood to be for the quark sector. V has a single un-removable phase for three families of quarks and leptons. This allows for the violation of CP symmetry in the quark sector. Unlike in the leptonic sector, the quark mixing angles turn out to be small. This enables one to make a perturbative expansion of the mixing matrix a la Wolfenstein. The small parameter is taken to be $\lambda = V_{us}$ in terms of which one has

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 + \frac{1}{2}A\lambda^4\left(1 - 2(\rho + i\eta)\right) & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^5). \tag{33}$$

Here we have the exact correspondence with Eq. (20) as given by

$$s_{12} \equiv \lambda, \qquad s_{23} \equiv A\lambda^2, \qquad s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta) \ . \tag{34}$$

Matrix elements of V are determined usually via semileptonic decays of quarks. In Fig. 1 we have displayed the dominant processes enabling determination of these elements. Fig. 1 (a) is the diagram for nuclear beta decay, from which $|V_{ud}|$ has been extracted rather accurately:

$$|V_{ud}| = 0.97377 \pm 0.00027 \; . \tag{35}$$

Fig. 1 (b) shows semileptonic K decay, which yields the Cabibbo angle V_{us} to be

$$|V_{us}| = 0.2257 \pm 0.0021 \; . \tag{36}$$

V_{cd} is determined from $D \to K\ell\nu$ and $D \to \pi\ell\nu$ with assistance from lattice QCD for the computation of the relevant form factors. Both V_{cd} and V_{cs} have rather large errors:

$$\begin{aligned} |V_{cd}| &= 0.230 \pm 0.011 \\ |V_{cs}| &= 0.957 \pm 0.010 \; . \end{aligned} \tag{37}$$

V_{cb} is determined from both inclusive and exclusive decays of B hadrons into charm, yielding a value

$$|V_{cb}| = (41.6 \pm 0.6) \times 10^{-3} \; . \tag{38}$$

V_{ub} s determined from charmless B decays and gives

$$|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3} \; . \tag{39}$$

Elements V_{td} and V_{ts} cannot be currently determined, for a lack of top quark events, but can be inferred from B meson mixings where these elements appear through the box diagram. The result is

$$\begin{aligned} |V_{td}| &= (7.4 \pm 0.8) \times 10^{-3} \\ \frac{|V_{td}|}{|V_{ts}|} &= 0.208 \pm 0.008 \; . \end{aligned} \tag{40}$$

Fig. 1 (f) depicts the decay of top quark into $W + b$. It can also decay into $W + q$ where q is d, s, b. By taking the ratio of branching ratios $R = B(t \to Wb)/\sum_q B(t \to Wq)$, CDF and D0 have arrived at a limit on $|V_{td}| > 0.79$.

The global fits to the Wolfenstein parameters (which includes information from CP violating observables) can be summarized as follows:

$$\lambda = 0.2272 \pm 0.0010, \;\; A = 0.818^{+0.007}_{-0.017}, \;\; \rho = 0.221^{+0.064}_{-0.028}, \;\; \eta = 0.340^{+0.017}_{-0.045} \tag{41}$$

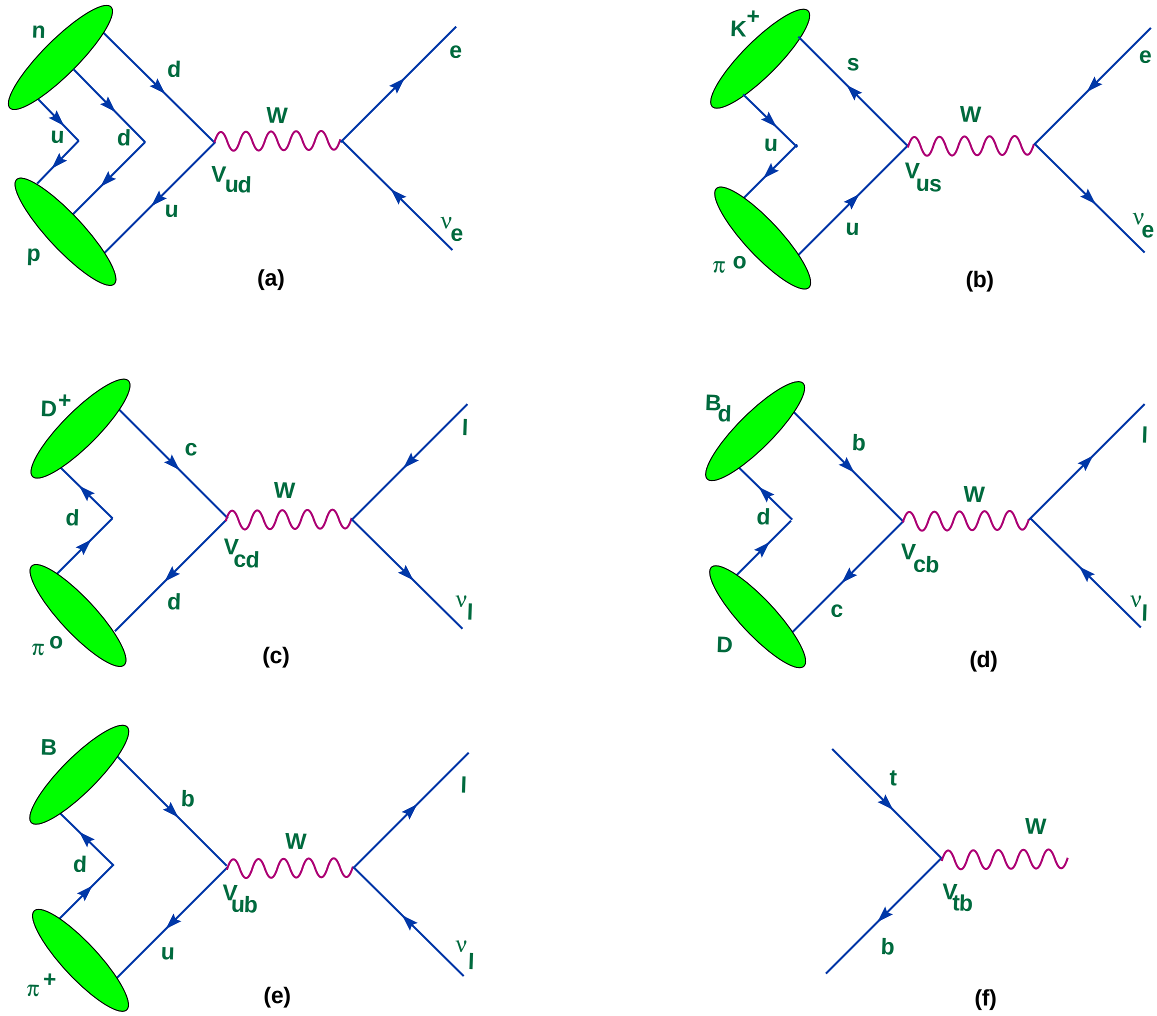

Figure 1: Processes determining V_{ij}.

§7. Relating quark mixings and masses with flavor symmetry

In the quark sector we have seen that the mass ratios such as m_d/m_s, m_u/m_c, etc are strongly hierarchical, while the mixing angles, such as V_{us} are also hierarchical, although the hierarchy here is not as strong. Can the quark mixing angle be computed in terms of the quark mass ratios? Clearly such attempts have to go beyond the SM. Here I give a simple two–family example which assumes a flavor $U(1)$ symmetry that distinguishes the two families.

Consider the mass matrices for $(u,\ c)$ and $(d,\ s)$ quarks given by

$$M_u = \begin{pmatrix} 0 & A_u \\ A_u^* & B_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & A_d \\ A_d^* & B_d \end{pmatrix}, \ . \tag{42}$$

The crucial features of these matrices are (i) the zeros in the (1,1) entries, and (ii) their hermiticity. Neither of these features can be realized within the SM. The zero entries can be enforced by a flavor $U(1)$ symmetry, the hermitian nature can be obtained if the gauge sector is left–right symmetric. Before constructing such a model, let us examine the consequences of Eq. (42). Matrices in Eq. (42) have factorizable phases. That is, $M_u = P_u \hat{M}_u P_u^*$, where $\hat{M}_u$ has the same form as M_u but with all entries real, and where $P_u = diag(e^{\imath\alpha_u},\ 1)$ is a diagonal phase matrix. A similar factorization applies to M_d with a phase matrix $P_d = diag(e^{\imath\alpha_d},\ 1)$. We can absorb these phase matrices into the quark fields, but since $\alpha_u \neq \alpha_d$, the matrix $P_u^* P_d = diag.(e^{\imath\phi},\ 1)$ will appear in the charged current matrix ($\phi = \alpha_d - \alpha_u$). The matrices $\hat{M}_u$ and $\hat{M}_d$, which have all real entries, can be diagonalized readily yielding for the mixing angles θ_u and θ_d

$$
\begin{aligned}
\tan^2\theta_u &= \frac{m_u}{m_c} \\
\tan^2\theta_d &= \frac{m_d}{m_s}
\end{aligned}
\tag{43}
$$

This yields a prediction for the Cabibbo angle

$$
|\sin\theta_C| \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi}\sqrt{\frac{m_u}{m_c}} \right| \ . \tag{44}
$$

This formula works rather well, especially since even without the second term, the Cabibbo angle is correctly reproduced. The phase ϕ is a parameter, however, its effect is rather restricted. For example, since $\sqrt{m_d/m_s} \simeq 0.22$ and $\sqrt{m_u/m_c} \simeq 0.07$, $|\sin\theta_C|$ must lie between 0.15 and 0.29, independent of the value of ϕ.

Now to a possible derivation of Eq. (42). Since SM interactions do not conserve Parity, it is useful to extend the gauge sector to the left-right symmetric group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, wherein Parity invariance can be imposed. The (2,3) and (3,2) elements of $M_{u,d}$ being complex conjugates of each other will then result. The left–handed and the right–handed quarks transform as $Q_{iL}(3,2,1,1/3) + Q_{iR}(3,1,2,1/3)$. Under discrete parity operation $Q_{iL} \leftrightarrow Q_{iR}$. This symmetry can be consistently imposed, as $W_L \leftrightarrow W_R$ in the gauge sector under Parity. The Higgs field that couples to quarks should be $\Phi(1,2,2,0)$, and under Parity $\Phi \to \Phi^\dagger$. In matrix form Q_{iL}, Q_{iR}, Φ read as

$$
Q_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L , \quad Q_{iR} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_R , \quad \Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \tag{45}
$$

so that the Yukawa Lagrangian

$$
\mathcal{L}_{\text{Yukawa}} = \overline{Q}_L \Phi Y Q_R + h.c. \tag{46}
$$

is invariant. Imposing Parity, we see that the Yukawa matrix Y must be hermitian, $Y = Y^\dagger$, which is the desired result.

To enforce zeros in the (1,1) entries of $M_{u,d}$, we can employ the following $U(1)$ flavor symmetry: $Q_{1L} : 2,\ Q_{1R} : -2.\ Q_{2L} : 1,\ Q_{2R} : -1.\ \Phi_1 : 2,\ \Phi_2 : 3$. Note that two Higgs bidoublet fields are needed. Φ_1 generates the (2,2) entries, while Φ_2 generates the (1,2) and (2,1) entries. There is no (1,1) entry generated, since there is no Higgs field with $U(1)$ charge of $+4$.

The $U(1)$ flavor symmetry and the left–right symmetry, which were crucial for the derivation, could show up as new particles at the LHC. In general, one would also expect multiple Higgs bosons.

Eq. (42) can be generalized for the case of three families, a la Fritzsch. The up and down quark mass matrices have hermitian nearest neighbor interaction form:

$$M_{u,d} = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}_{u,d} . \tag{47}$$

These matrices preserve the prediction for Cabibbo angle of Eq. (44). In addition, this model predicts the following relations:

$$|V_{cb}| \simeq \left| \sqrt{\frac{m_s}{m_b}} - e^{i\chi} \sqrt{\frac{m_c}{m_t}} \right|,\ \frac{|V_{ub}|}{|V_{cb}|} \simeq \sqrt{\frac{m_u}{m_c}},\ \frac{|V_{td}|}{|V_{ts}|} \simeq \sqrt{\frac{m_d}{m_s}} . \tag{48}$$

Here χ is an undetermined phase. The relation for $|V_{cb}|$ would predict a top quark mass of order $(40 - 60)$ GeV, which is now excluded.

There have been attempts to fix the problem of Fritzsch mass matrices by modifying its form slightly. If small (2,2) elements are allowed in $M_{u,d}$, the troublesome relation for V_{cb} will be removed, while all other predictions will stay. A different alternative is to make the (2,3) and (3,2) entries different, while maintaining the relations between (1,2) and (2,1) entries. This can be achieved by non–Abelian discrete symmetries.

§8. Frogatt–Nielsen mechanism for mass and mixing hierarchy

The hierarchy in the masses and mixings of quarks and leptons can be understood by assuming a flavor $U(1)$ symmetry under which the fermions are distinguished. In this approach developed by Frogatt and Nielsen, there is a "flavon" field S, which is a scalar, usually a SM singlet field, which acquires a VEV and breaks the $U(1)$ symmetry. This symmetry breaking is communicated to the fermions at differen order in a small parameter $\epsilon = \langle S \rangle / M_*$. Here M_* is the scale of flavor dynamics, and usually is associated with some heavy fermions which are integrated out. The nice feature of this approach is that the mass and mixing hierarchies will be explained as powers of the expansion parameter ϵ. The effective theory below M_* is rather simple, while the full theory will have many heavy fermions, called Frogatt–Nielsen fields.

Let me illustrate this idea with a two family example which is realistic when applied to the second and third families of quarks. Consider M_u and M_d for the $(c,\ t)$ and $(s,\ b)$ sectors given by

$$M_u = \begin{pmatrix} \epsilon^4 & \epsilon^2 \\ \epsilon^2 & 1 \end{pmatrix} v, \quad M_d = \begin{pmatrix} \epsilon^3 & \epsilon^3 \\ \epsilon & \epsilon \end{pmatrix} v \ . \tag{49}$$

Here $\epsilon \sim 0.2$ is a flavor symmetry breaking parameter. Every term has an order one coefficient which is not displayed. We obtain from Eq. (49) the following relations for quark masses and V_{cb}:

$$\frac{m_c}{m_t} \sim \epsilon^4,\ \frac{m_s}{m_b} \sim \epsilon^2,\ |V_{cb}| \sim \epsilon^2 \ . \tag{50}$$

All of these relations work well, for $\epsilon \sim 0.2$. Although precise predictions have not been made, qualitative understanding of the hierarchies has been made.

How do we arrive at Eq. (49)? We do it in two stages. First, let us look at the effective Yukawa couplings, which can be obtained from the Lagrangian

$$\begin{aligned} \mathcal{L}_{FN} \ &= \ \left[Q_3 u_3^c H_u + Q_2 u_3^c H_u S^2 + Q_3 u_2^c H_u S^2 + Q_2 u_2^c H_u S^4\right] \\ &+ \ \left[Q_3 d_3^c H_d S + Q_3 d_2^c H_d S + Q_2 d_2^c H_d S^3 + Q_2 d_3^c H_d S^3\right] + h.c. \end{aligned} \tag{51}$$

Here I assumed supersymmetry, so that there are two Higgs doublets $H_{u,d}$. It is not necessary to assume SUSY, in that case identify H_u as H of SM, and replace H_d by $\tilde{H}$. In Eq. (51) all couplings are taken to be order one. The symmetry of Eq. (51) is a $U(1)$ with the following charge assignment.

$$\{Q_3, u_3^c\} : 0; \quad \{Q_2, u_2^c\} : 2;\ \{d_2^c, d_3^c\} : 1;\ \{H_u, H_d\} : 0;\ S : -1 \ . \tag{52}$$

Now we wish to obtain Eq. (52) by integrating out certain Frogatt–Nielsen fields. This is depicted in Fig. 2 via a set of "spaghetti" diagrams. As you can see, there are a variety of fields denoted by $G_i, \overline{G}_i$ $(i = 1-4)$ for the up–quark mass generation. G_i have the same gauge quantum numbers as teh u^c quark of SM, while $\overline{G}_i$ have the conjugate quantum numbers. F_i have the quantum numbers of d^c quark, while $\overline{F}_i$ the conjugate quantum numbers.

You can readily read off the flavor $U(1)$ charges of the various F_i and G_i fields from the spaghetti diagrams.

Actually the flavor $U(1)$ that we used is anomalous. String theory generically gives an anomalous $U(1)_A$ where anomaly cancellation occurs by the Green–Schwartz mechanism. In this case, we can get rid of the complicated Frogatt–Nielsen fields, and simply write down

Field	$U(1)_A$ Charge	Charge notation
$Q_1,\, Q_2,\, Q_3$	4, 2, 0	q_i^Q
$L_1,\, L_2,\, L_3$	$1+s,\, s,\, s$	q_i^L
$u_1^c,\, u_2^c,\, u_3^c$	4, 2, 0	q_i^u
$d_1^c,\, d_2^c,\, d_3^c$	$1+p,\, p,\, p$	q_i^d
e_1^c, e_2^c, e_3^c	$4+p-s,\, 2+p-s,\, p-s$	q_i^e
$\nu_1^c,\, \nu_2^c,\, \nu_3^c$	1, 0, 0	q_i^ν
$H_u,\, H_d,\, S$	$0,\, 0,\, -1$	$(h, \bar{h}, q_s)$

Table 1: The flavor $U(1)_A$ charge assignment for the MSSM fields and the flavon field S.

higher dimensional operators suppressed by the string scale. A bonus in this approach is that the small expansion parameter ϵ can be computed in specific models, where it tends to come out close to 0.2.

An explicit and complete anomalous $U(1)$ model that fits well all quark and lepton masses and mixings is constructed below. Consider the quark and lepton mass matrices of the following form:

$$M_u \sim \langle H_u \rangle \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \qquad M_d \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix},$$
$$M_e \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon \\ \epsilon^4 & \epsilon^2 & 1 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \qquad M_{\nu_D} \sim \langle H_u \rangle \epsilon^s \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix},$$
$$M_{\nu^c} \sim M_R \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \quad \Rightarrow \quad M_\nu^{light} \sim \frac{\langle H_u \rangle^2}{M_R} \epsilon^{2s} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}. \tag{53}$$

These matrices can be obtained by the $U(1)$ charge assignment of Table 1. Note that this charge assignment is compatible with $SU(5)$ unification. Here p takes integer values 0, 1 or 2, corresponding to $\tan\beta$ taking large, medium or small values.

All qualitative features of quark and lepton masses and mixings are reproduced by these matrices. This includes small quark mixings and large neutrino mixings.

Can the SM Higgs field itself be the flavon field? Clearly, then new flavor dynamics must happen near the TeV scale. This is apparently possible with significant consequences for Higgs boson physics, as I shall now outline.

Consider an expansion in $H^\dagger H/M^2$, which is a SM singlet that can play the role of S. Immediately you may wonder how this is possible, since $H^\dagger H$ cannot carry any $U(1)$ quantum number. But think of SUSY at the TeV scale. $H_u H_d$ can carry $U(1)$ number, and when reduced to SM this expansion in terms of $H^\dagger H$ is consistent. Consider the following mass matrices in terms of the expansion parameter

$$\epsilon = \frac{v}{M} \ . \tag{54}$$

$$M_u = \begin{pmatrix} h^u_{11}\epsilon^6 & h^u_{12}\epsilon^4 & h^u_{13}\epsilon^4 \\ h^u_{21}\epsilon^4 & h^u_{22}\epsilon^2 & h^u_{23}\epsilon^2 \\ h^u_{31}\epsilon^4 & h^u_{32}\epsilon^2 & h^u_{33} \end{pmatrix} v, \qquad M_d = \begin{pmatrix} h^d_{11}\epsilon^6 & h^d_{12}\epsilon^6 & h^d_{13}\epsilon^6 \\ h^d_{21}\epsilon^6 & h^d_{22}\epsilon^4 & h^d_{23}\epsilon^4 \\ h^d_{31}\epsilon^6 & h^d_{32}\epsilon^4 & h^d_{33}\epsilon^2 \end{pmatrix} v \ . \tag{55}$$

These matrices give good fit to masses and mixings, as in the case of anomalous $U(1)$ model.

The Yukawa couplings of the physical quark fields are no longer proportional to the mass matrices. We obtain for the Yukawa couplings,

$$Y_u = \begin{pmatrix} 7h^u_{11}\epsilon^6 & 5h^u_{12}\epsilon^4 & 5h^u_{13}\epsilon^4 \\ 5h^u_{21}\epsilon^4 & 3h^u_{22}\epsilon^2 & 3h^u_{23}\epsilon^2 \\ 5h^u_{31}\epsilon^4 & 3h^u_{32}\epsilon^2 & h^u_{33} \end{pmatrix}, \qquad Y_d = \begin{pmatrix} 7h^d_{11}\epsilon^6 & 7h^d_{12}\epsilon^6 & 7h^d_{13}\epsilon^6 \\ 7h^d_{21}\epsilon^6 & 5h^d_{22}\epsilon^4 & 5h^d_{23}\epsilon^4 \\ 7h^d_{31}\epsilon^6 & 5h^d_{32}\epsilon^4 & 3h^d_{33}\epsilon^2 \end{pmatrix}, \tag{56}$$

This leads to flavor changing Higgs processes, which however are within experimental limits. The branching ratios of the Higgs get modified, and is shown in Fig. 3.

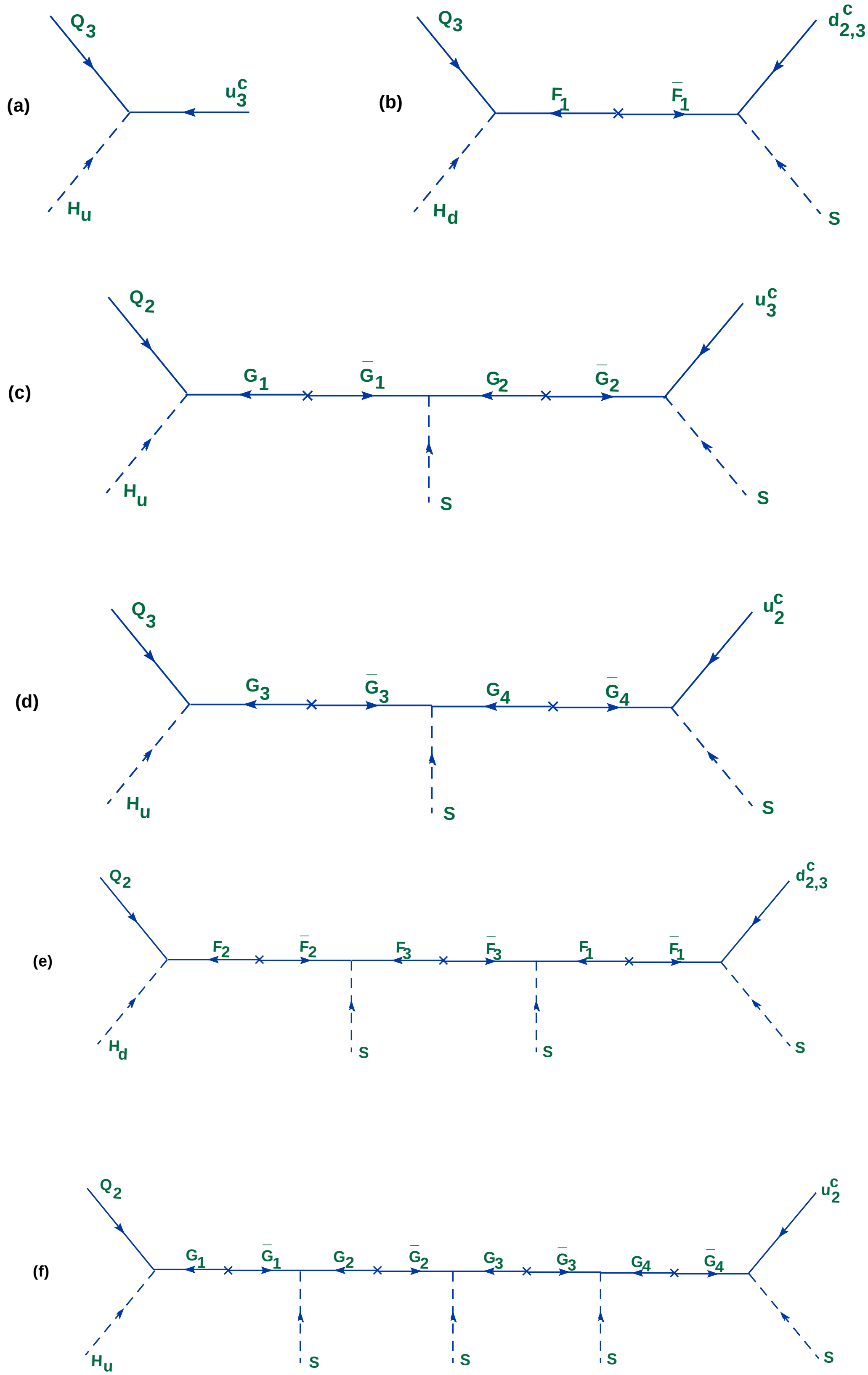

Figure 2: Frogatt–Nielsen fields generating Eq. (52).

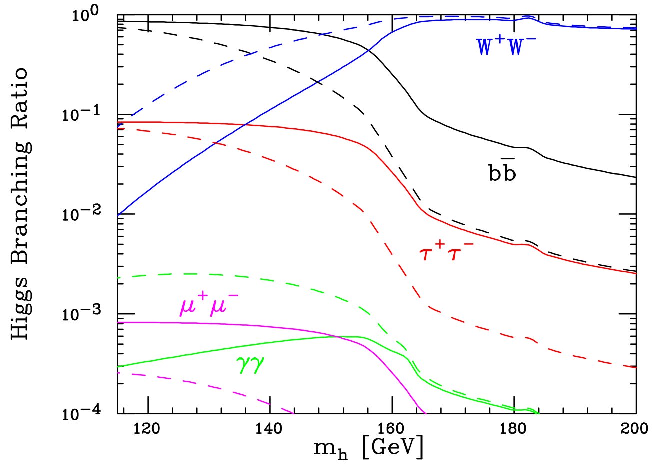

Figure 3: Higgs branching ratios with the SM Higgs as a flavon field.