TASI Lectures on Flavor Physics

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Lecture II

In this lecture we will develop further ideas for understanding the flavor puzzle. I will describe how Grand Unification, combined with flavor symmetries helps understand many of the features and indicate possible tests of this idea. Subsequent lectures will address radiative mass generation mechanism as an attractive way of explaining the mass hierarchy, and the strong CP problem and its possible resolutions.

§9. Grand Unification and the flavor puzzle

Grand Unification is an ambitious program that attempts to unify the strong, weak and electromagnetic interactions. It is strongly suggested by the unification of gauge couplings that happens in the minimal supersymmetric standard model (MSSM). Besides its aesthetic appeal, in practical terms, Grand Unified Theories (GUTs) reduce the number of parameters. For example, the three gauge couplings of the SM are unified into one at a very high energy scale $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV. The apparent differences in the strengths of the various forces is attributed to the spontaneous breakdown of the GUT symmetry to the MSSM and the resulting renormalization flow of the gauge couplings. SUSY GUTs are perhaps the best motivated extensions of the SM. They explain the quantization of electric charge, as well as the quantum numbers of quarks and leptons. They provide ideal settings for understanding the flavor puzzle, which will be the focus of this discussion.

The simplest GUT model is based on $SU(5)$. I will assume low energy supersymmetry, motivated by the gauge coupling unification and a solution to the hierarchy problem. For an understanding of quark–lepton masses and mixings SUSY is not crucial, but within the context of SUSY there will be many interesting flavor violating processes. In $SU(5)$, the fifteen components of one family of quarks and leptons are organized into two multiplets: A $\mathbf{10}$–plet and a $\overline{\mathbf{5}}$–plet. The $\overline{\mathbf{5}}$ is of course the anti–fundamental representation of $SU(5)$, while the $\mathbf{10}$ is the anti-symmetric second rank tensor. It is very nontrivial that this assignment of fermions under $SU(5)$ is anomaly free. The $\overline{\mathbf{5}}$–plet contains $\{d^c,\ L\}$ fields, while the $\mathbf{10}$–plet contains the $\{Q,\ u^c,\ e^c\}$ fields. There is no ν^c field in the simplest version of $SU(5)$, but it can be added as a gauge singlet, as in the SM.

The symmetry breaking sector consists of two types of Higgs fields: And adjoint $\mathbf{24}_H$–plet which acquires vacuum expectation value and breaks $SU(5)$ down to the SM gauge symmetry, and a $\{\mathbf{5}_H + \overline{\mathbf{5}}_H\}$ pair which contain the H_u and H_d fields of MSSM. The Yukawa couplings of fermions only involve the latter fields and read as

$$W_{\rm Yuk} = (Y_u)_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{5}_H + (Y_d)_{ij}\mathbf{10}_i\overline{\mathbf{5}}_j\overline{\mathbf{5}}_H~. \quad (1)$$

Here (i,j) are family indices, and $SU(5)$ contraction is implicit. Y_u is now a symmetric matrix in generation space, while Y_d is a general non-hermitian matrix. Note that there are only two

Yukawa coupling matrices, unlike the three matrices we had in the SM. The reason for this reduction of parameters is the higher symmetry and the unification of quarks with leptons. When Eq. (1) is expanded, one would obtain a relation between the down quark and the charged lepton mass matrices:

$$M_d = M_\ell^T \ . \tag{2}$$

This leads to the asymptotic (valid at the GUT scale) relations for the mass eigenvalues

$$m_b^0 = m_\tau^0, \ m_s^0 = m_\mu^0, \ m_d^0 = m_e^0 \tag{3}$$

where the superscript ^{0} is used to indicate that the relation holds at the GUT scale.

In order to test the validity of the prediction of minimal SUSY $SU(5)$, we have to extrapolate the masses from GUT scale to low energy scale where the masses are measured. This is done by the renormalization group equations. The evolution of the b quark and τ lepton masses is shown in Fig. 1 for two differen values of $\tan\beta = (1.7, \ 50)$. Here we have extrapolated the observed masses from low scale to the GUT scale. It is remarkable that unification of masses occurs in this simple context. The main effect on the evolution comes from QCD enhancement of b quark mass as it evolves from high energy to low energy scale, which is absent for the τ lepton.

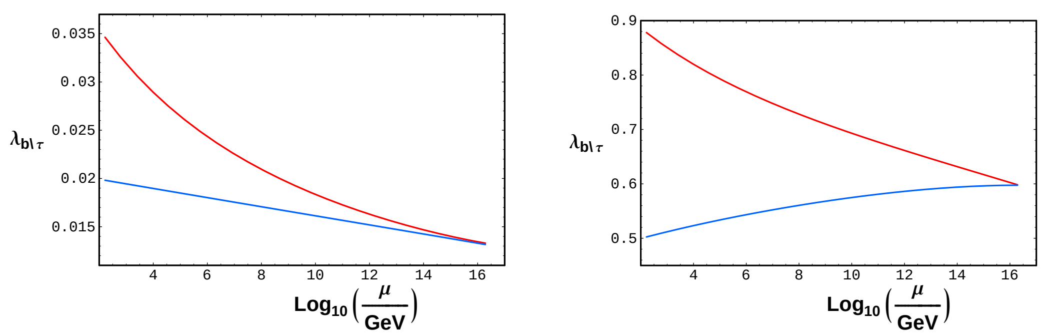

Figure 1: Evolution of the b–quark and τ–lepton masses in the MSSM for $\tan\beta = 1.7$ (left panel) and 50 (right panel). $m_b(m_b) = 4.65$ GeV has been used here.

Fig. 2 shows the predicted values of $m_b(m_b)$ using the asymptotic relation $m_b^0 = m_\tau^0$ as a function of m_t and the parameter $\tan\beta$. For low and large values of $\tan\beta$, there is always good solution for $m_b(m_b)$, while for intermediate values there is no acceptable solution. It should be mentioned that there are significant finite corrections to the b–quark mass from loops involving the gluino, which is not included in the RGE analysis. These graphs, while loop suppressed, are enhanced by a factor of $\tan\beta$, and thus can be as large 30-40% for $m_b(m_b)$. So even intermediate values of $\tan\beta$ are not totally excluded.

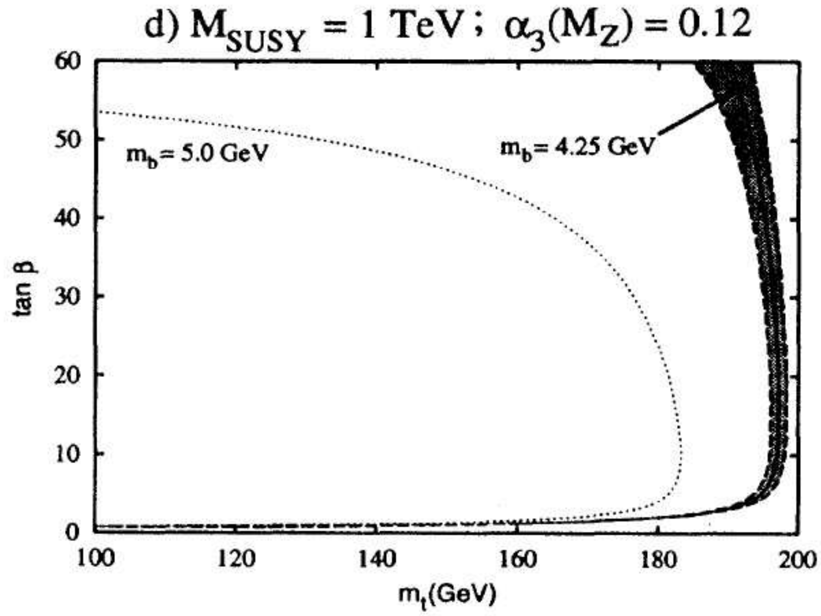

Figure 2: b quark mass prediction as a function of m_t in MSSM, assuming $m_b^0 = m_\tau^0$ at the GUT scale (from Barger, Berger, Ohmann).

The last two relations of Eq. (3) turn out to be not acceptable when compared to low energy values of the masses. One can see this without going through the RGE evolution. Eq. (3) implies $m_s^0/m_d^0 = m_\mu^0/m_e^0$. These mass ratios are RGE independent, so one can compare them directly with observations. We have seen that $m_s/m_d \simeq 20$, while $m_\mu/m_\tau \simeq 200$. So this relation is off by an order of magnitude.

There is an elegant way of fixing the light fermion masses in $SU(5)$. Consider modifying Eq. (3) to the following relations:

$$m_b^0 = m_\tau^0,\ m_s^0 = \frac{1}{3}m_\mu^0,\ m_d^0 = 3m_e^0\ . \tag{4}$$

These relations were proposed by Georgi and Jarlskog and are known as the GJ relations. The factors of 3 that appears in Eq. (4) have a simple group theoretic understanding in terms of $B-L$ under which lepton charges are 3 times that of quark charges. The RGE independent quantity from Eq. (4) gives us

$$\frac{m_s}{m_d} = \frac{1}{9}\frac{m_\mu}{m_e} \tag{5}$$

which is in excellent agreement with observations. There is one other prediction, which can be taken to be the value of $m_d(1\text{ GeV}) \simeq 8$ MeV, which is also is good agreement with data.

How would one go about deriving the Georgi–Jarlskog mass relations? We invoke a flavor $U(1)$ symmetry as before. Consider the following mass matrices for up quarks, down quarks and charged leptons.

$$M_u = \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix}; \; M_d = \begin{pmatrix} 0 & de^{i\phi} & 0 \\ de^{-i\phi} & f & 0 \\ 0 & 0 & g \end{pmatrix}; \; M_\ell = \begin{pmatrix} 0 & d & 0 \\ d & -3f & 0 \\ 0 & 0 & g \end{pmatrix}. \tag{6}$$

The factor 3 in quark versus lepton is attributed to the $B - L$ quantum number, and the zeros are enforced by a flavor symmetry. All parameters are complex to begin with, but after field redefinitions, only a single complex phase survives. There are 7 parameters in all to fit the 13 observables (9 masses, 3 mixing angles and one CP phase), thereby resulting in six predictions. Three of these predictions are the b, s and d–quark masses. We write them at the low energy scale by incorporating factors denoted as η which are the RGE evolution factors.

$$m_b = \eta_{b/\tau}^{-1} m_\tau; \; \frac{m_d/m_s}{(1 - m_d/m_s)^2} = 9\frac{m_e/m_\mu}{(1 - m_e/m_\mu)^2}; \; (m_s - m_d) = \frac{1}{3}\eta_{s/\mu}^{-1}(m_\mu - m_e) \, . \tag{7}$$

The other three predictions are for the quark mixing angles and the CP phase J. J is the rephasing invariant CP violation parameter (Jarlskog invariant) which can be defined as

$$J = \mathrm{Im}(V_{us}V_{cb}V_{ub}^* V_{cs}^*) \tag{8}$$

and has a value of $J \simeq 2.8 \times 10^{-5}$. We have for the predictions

$$|V_{cb}| = \eta_{KM}^{-1}\eta_{u/t}^{1/2}\sqrt{\frac{m_c}{m_t}} \; ; \qquad \frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{m_u}{m_c}} \; ;$$

$$J = \eta_{KM}^{-2}\eta_{u/t}\sqrt{\frac{m_d}{m_s}}\sqrt{\frac{m_c}{m_t}}\sqrt{\frac{m_u}{m_t}}\left[1 - \frac{1}{4}\left(\sqrt{\frac{m_u}{m_c}}\sqrt{\frac{m_s}{m_d}} + \sqrt{\frac{m_c}{m_u}}\sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_c}{m_u}}\sqrt{\frac{m_s}{m_d}}|V_{us}|^2\right)^2\right]^{\frac{1}{2}} \tag{9}$$

Here the η's are renormalization factors for the various parameters in going from the low energy scale to the GUT scale. If the bottom–quark Yukawa coupling h_b is much smaller than the top Yukawa coupling h_t, (corresponding to $\tan\beta \lesssim 10$ or so) these RGE factors can be expressed analytically as

$$\begin{aligned} \eta_{KM} &= \eta_{d/b} = \left(1 - \frac{Y_t}{Y_f}\right)^{\frac{1}{12}}; \; \eta_{u/t} = \left(1 - \frac{Y_t}{Y_f}\right)^{\frac{1}{4}}; \eta_{s/\mu} = \left(\frac{\alpha_1}{\alpha_G}\right)^{-10/99}\left(\frac{\alpha_3}{\alpha_G}\right)^{-8/9}; \\ \eta_{b/\tau} &= \left(\frac{\alpha_1}{\alpha_G}\right)^{-10/99}\left(\frac{\alpha_3}{\alpha_G}\right)^{-8/9}\left(1 - \frac{Y_t}{Y_f}\right)^{-1/12} . \end{aligned} \tag{10}$$

Here α_G is the unified gauge coupling strength, $Y_t = h_t^2$ at the weak scale and Y_f is the fixed point value of Y_t. Y_t cannot strictly be equal to Y_f, since that would correspond to infinite Y_t at the GUT scale. If we demand that $Y_t \lesssim 4$ at GUT scale, Y_t/Y_f can at most be 0.98. The renormalization factors in Eq. (4) are all well–behaved even when Y_t differs from Y_f by only 2% due to the small exponents.

For the light fermion masses there is QCD and QED running factors below m_t as well. These are obtained numerically using three loop QCD and one–loop QED β and γ functions. Corresponding to $\alpha_s(M_Z) = 0.12$ and $\alpha^{-1}(M_Z) = 127.9$, these factors are $(\eta_u, \eta_{d,s}, \eta_c, \eta_b, \eta_{e,\mu}, \eta_\tau) = (0.401, 0.404, 0.460, 0.646, 0.982, 0.984)$. Using $\alpha_s(M_Z) = 0.12, m_c(m_c) = 1.27\, GeV, m_u(1\, GeV) = 5.1\ MeV$, $|V_{us}| = 0.22$, $m_t^{\rm phys} = 174\ GeV$ and $Y_t/Y_f = 0.98$ as input values, we obtain

$$\begin{aligned} m_d(1\ GeV) &= 7.7\ MeV,\ m_s(1\ GeV) = 193\ MeV,\ m_b(m_b) = 4.26\ GeV \\ |V_{cb}| &= 0.050,\ |V_{ub}|/|V_{cb}| = 0.059,\ J = 2.96 \times 10^{-5}\ . \end{aligned} \tag{11}$$

These predictions of the GJ ansatz are generally in good agreement with experiments. V_{cb} is on the large side, but again there is finite threshold correction from the gaugino loops, which can fix it at the 20% level.

Now let us turn to an even more interesting class of GUTs, those based on the gauge symmetry $SO(10)$. All members of a family are unified into a **16** dimensional spinor representation of $SO(10)$. This requires the existence of right–handed neutrino ν^c, leading naturally to the seesaw mechanism and small neutrino masses. $SU(5)$ has the option of having neutrino mass, but in that context there is no compelling argument for it. The spinor of $SO(10)$ breaks down under $SU(5)$ as

$$\mathbf{16} = \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1} \tag{12}$$

where the **1** is the ν^c field. Symmetry breaking is accomplished by Higgs fields in the adjoint **45**, spinor $\mathbf{16} + \overline{\mathbf{16}}$ and vector **10** representations. The MSSM Higgs doublets $H_{u,d}$ are contained partially in the $\mathbf{10}_H$ but partially also in the $\mathbf{16}_H$.

Let me work out a specific flavor model based on $SO(10) \times U(1)$. Under the flavor $U(1)$, the charges of the three family fermions and the Higgs fields are taken as

$$\begin{array}{cccccccc} \mathbf{16}_3 & \mathbf{16}_2 & \mathbf{16}_1 & \mathbf{10}_H & \mathbf{16}_H & \overline{\mathbf{16}}_H & \mathbf{45}_H & \mathbf{S} \\ a & a+1 & a+2 & -2a & -a-1/2 & -a & 0 & -1 \end{array}\ . \tag{13}$$

With this symmetry, only the third family Yukawa coupling arises at the renormalizable level, other couplings are suppressed by inverse power of some mass scale. The lowest dimensional operators that can be written are:

$$\mathcal{L}_{\rm Yuk} = h_{33}\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H + [h_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H(S/M)$$

$$+ \quad a_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H(\mathbf{45}_H/M')(S/M)^p + g_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^q]$$
$$+ \quad \left[h_{22}\mathbf{16}_2\mathbf{16}_2\mathbf{10}_H(S/M)^2 + g_{22}\mathbf{16}_2\mathbf{16}_2\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^{q+1}\right] . \tag{14}$$

Here p, q are integers which can be taken to be 0 or 1. This leads to the following mass matrices:

$$M_u = \begin{bmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \zeta_{22}^u & \sigma+\epsilon \\ 0 & \sigma-\epsilon & 1 \end{bmatrix} \mathcal{M}_u^0; \qquad M_d = \begin{bmatrix} 0 & \eta'+\epsilon' & 0 \\ \eta'-\epsilon' & \zeta_{22}^d & \eta+\epsilon \\ 0 & \eta-\epsilon & 1 \end{bmatrix} \mathcal{M}_d^0$$
$$M_\nu^D = \begin{bmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & \zeta_{22}^u & \sigma-3\epsilon \\ 0 & \sigma+3\epsilon & 1 \end{bmatrix} \mathcal{M}_u^0; \quad M_l = \begin{bmatrix} 0 & \eta'-3\epsilon' & 0 \\ \eta'+3\epsilon' & \zeta_{22}^d & \eta-3\epsilon \\ 0 & \eta+3\epsilon & 1 \end{bmatrix} \mathcal{M}_d^0 \tag{15}$$

Excellent agreement with data is obtained by choosing the parameters to be

$$\begin{aligned} &\sigma \approx 0.110, \quad \eta \approx 0.151, \quad \epsilon \approx -0.095, \quad |\eta'| \approx 4.4 \times 10^{-3}, \\ &\epsilon' \approx 2 \times 10^{-4}, \quad \mathcal{M}_u^0 \approx m_t(M_X) \approx 100 \text{ GeV}, \quad \mathcal{M}_d^0 \approx m_\tau(M_X) \approx 1.1 \text{ GeV}. \end{aligned} \tag{16}$$

One obtains the following predictions.

$$\begin{aligned} &m_b(m_b) \approx 4.9 \text{ GeV}, \quad \sqrt{\Delta m_{23}^2} \approx m(\nu_3) \approx (1/24 \text{ eV})(1/2\text{-}2), \quad V_{cb} \approx 0.044, \\ &\sin^2 2\theta_{\nu_\mu\nu_\tau}^{\text{osc}} \approx 0.98 - 0.995, \ (\text{for } m(\nu_2)/m(\nu_3) \approx 1/10 - 1/5), \\ &|V_{us}| \approx 0.20, \quad \left|\frac{V_{ub}}{V_{cb}}\right| \approx 0.07, \quad m_d(1 \text{ GeV}) \approx 8 \text{ MeV} \end{aligned} \tag{17}$$

all of which are good agreement, to within 10% accuracy.

How do we go about testing such ideas? Since the GUT scale is below the Planck scale, even though the flavor symmetry is broken near the GUT scale, SUSY breaking parameters can remember flavor violating interaction due to their running above the GUT scale. The most significant flavor violation in this model arises due to the splitting of the third family sfermions from those of the first two families. This is seen by the solution to the RGE equations for these masses.

$$\Delta\hat{m}_{\tilde{b}_L}^2 = \Delta\hat{m}_{\tilde{b}_R}^2 = \Delta\hat{m}_{\tilde{\tau}_L}^2 = \Delta\hat{m}_{\tilde{\tau}_R}^2 \equiv \Delta \approx -(\frac{30m_o^2}{16\pi^2})h_t^2 \ ln(M^*/M_{GUT}) \ . \tag{18}$$

Here M^* is the fundamental scale where SUSY breaking messengers reside, with $M^* > M_{\text{GUT}}$. h_t is the top quark Yukawa coupling. Note that leptons also feel the effect of top Yukawa, because leptons and quarks are unified. m_0 is the universal SUSY breaking scalar mass parameter. One sees that, because of the GUT threshold, universality is not preserved in this type of models. In going from gauge basis to the mass eienbasis for the fermions, Eq. (18) would imply that there will be flavor changing scalar interactions. Because SUSY particles have masses of order TeV, these flavor violation can manifest in the SM sector via SUSY loops.

The most constraining FCNC process in the present model turns out to be $\mu \to e\gamma$. Predictions for this process are depicted in Fig. 3 as a function of slepton mass. Part of the parameter space is already ruled out, so there is a good chance that this process will be discovered at the MEG experiment at PSI.

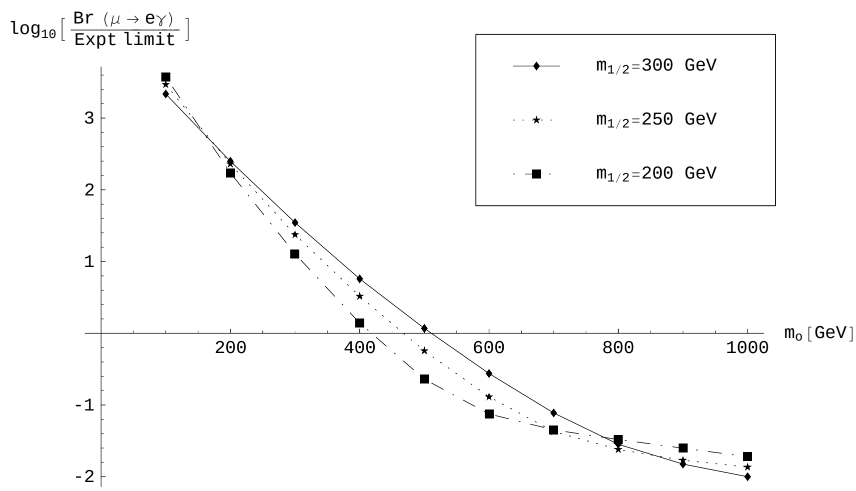

Figure 3: Prediction for the branching ratio for $\mu \to e\gamma$ in the SUSY $SO(10)$ model as a function of slepton mass. The horizontal line indicates current experimental limit.