TASI Lectures on Flavor Physics

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Lecture III

§10. Radiative fermion mass generation

The hierarchical structure of the quark and lepton masses and the quark mixing angles can be elegantly understood by the radiative mass generation mechanism. This is an alternative to the Froggatt–Nielsen mechanism. Here the idea is that only the heaviest fermion(s) acquires a tree level mass. The next heaviest fermions acquire masses as one loop radiative corrections, which are suppressed by a a typical loop factor $\sim 1/(16\pi^2) \sim 10^{-2}$ relative to the heaviest fermion. The lightest fermions acquire masses as two loop radiative corrections, which are then a factor $\sim [1/(16\pi^2)]^2 \sim 10^{-4}$ suppressed relative to the heaviest fermion. Thus even without putting in small Yukawa couplings one understands the mass spectrum of the fermions.

There is another appeal to this idea. If the electron mass is radiatively generated from the muon mass, then there must be no counter–term needed in the Lagrangian to absorb infinity associated with the electron mass. In other words, electron mass is "calculable", in terms of other parameters of the model. This idea was originally suggested by 'tHooft in his classic paper on the renormalizability of non–Abelian gauge theories. This also implies that there must be some symmetry reason for the light fermions not to have tree level masses, otherwise the idea cannot be implemented successfully.

There is a resurgence of interest in this idea as the LHC turns on, since new particles with specific properties which may be seen at the LHC are predicted. There exist very nice models of this type by Mohapatra and collaborators from the late 80's. Recently Dobrescu and Fox has written a nice paper on the subject, which I recommend to you. As in past examples, I will try to convey the main idea, with the understanding that implementation can vary considerably. I will discuss an implementation I worked out with Mohapatra based on the permutation symmetry.

Let us focus on the quark sector of the SM first. We wish to have a scenario where only the top quark and the bottom quark have tree level masses. In the same limit, there should be no CKM mixing induced. This can be realized if one has the following "democratic" mass matrices for up and down quarks.

$$M_{u,d} = \frac{m_{t,b}}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} . \tag{1}$$

Of course, these matrices have rank 1, implying that only the top and the bottom get mass. The unitary matrix that diagonalize $M_{u,d}$ are the same, so there is no CKM mixing induced at this stage.

How do we obtain democratic mass matrices of Eq. (1)? It turns out that the symmetry of these matrices is $S_{3L} \times S_{3R}$, where S_3 is the group of permutation of three objects. The

Lagrangian that would generate Eq. (1) for M_u is of the form

$$\mathcal{L}_{\text{Yukawa}} = h_u(\overline{Q}_{1L} + \overline{Q}_{2L} + \overline{Q}_{3L})\tilde{H}(u_{1R} + u_{2R} + u_{3R}) \tag{2}$$

which is manifestly symmetric under separate permutations of the left and the right handed quark fields. So it is tempting to start with this symmetry group $S_{3L} \times S_{3R}$, but it is not necessary to have the S_{3R} group, since right-handed rotations are unphysical in the SM. So consider the following Lagrangian which only has the S_{3L} symmetry.

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= (\overline{Q}_{1L} + \overline{Q}_{2L} + \overline{Q}_{3L})\tilde{H}(h_1^u u_{1R} + h_2^u u_{2R} + h_3^u u_{3R}) \\ &+ (\overline{Q}_{1L} + \overline{Q}_{2L} + \overline{Q}_{3L})H(h_1^d d_{1R} + h_2^d d_{2R} + h_3^d d_{3R})\end{aligned} \tag{3}$$

By right–handed rotations on u_R and d_R fields, we can bring Eq. (3) into the form of Eq. (2). Two combinations of the Q_{iL} and (u_{iR}, d_{iR}) fields orthogonal to Eq. (3) will be massless.

These massless Q_{iL} modes actually form the **2** dimensional representations of S_3. It is convenient to directly go to the irreducible representations of S_3. They are a true singlet **1**, an odd singlet **1′** and a doublet $\mathbf{2} = (x_1, x_2)$. The product of two **1′** gives a **1**, while the product of two **2** gives $\mathbf{1} + \mathbf{1}' + \mathbf{2}$. The Clebsch–Gordon coefficients for this product are

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \times \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \mathbf{1} : (x_1y_1 + x_2y_2); \quad \mathbf{1}' : (x_1y_2 - x_2y_1); \quad \mathbf{2} : \begin{pmatrix} x_1y_2 + x_2y_1 \\ x_1y_1 - x_2y_2 \end{pmatrix} \tag{4}$$

Now, consider the following assignment of quarks and scalars under S_3:

$$\begin{aligned}\begin{pmatrix} Q_{1L} \\ Q_{2L} \end{pmatrix} &: \mathbf{2}; \quad Q_{3L} : \mathbf{1}; \quad u_{iR} : \mathbf{1} \\ H &: \mathbf{1}, \quad \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} : \mathbf{2}, \quad \omega_3 : \mathbf{1}\end{aligned} \tag{5}$$

Here the gauge structure is simply that of SM with H being the SM Higgs doublet. In order to radiatively generate light fermion masses, new ingredients are needed. The simplest possibility is to introduce scalar fields which have Yukawa couplings connecting the heavy (3rd generation) and the light fermions. We have assumed existence of $\omega_i(3, 1, -1/3)$ fields, which can have such Yukawa couplings, without inducing direct mass terms for the light fermions. Note that these ω_i fields are colored and charged, so they do not acquire vacuum expectation values.

The most general Yukawa couplings allowed in this SM $\times S_3$ model is given by

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= h_t\overline{Q}_{3L}t_R\tilde{H} + h_b\overline{Q}_{3L}b_RH + h_1(Q_{1L}^TCQ_{3L}\omega_1 + Q_{2L}^TCQ_{3L}\omega_2) \\ &+ h_2(Q_{1L}^TCQ_{1L} + Q_{2L}^TCQ_{2L})\omega_3 + h_3Q_{3L}^TCQ_{3L}\omega_3 \\ &+ h_4\{Q_{1L}^TCQ_{2L} + Q_{2L}^TCQ_{1L})\omega_1 + (Q_{1L}^TCQ_{1L} - Q_{2L}^TCQ_{2L})\omega_2\} + h.c.\end{aligned} \tag{6}$$

Here we have redefined the combination of u_R that couples to Q_{3L} as simply t_R (and similarly for b_R).

Clearly, from Eq. (6), only the top and bottom quarks acquire tree–level masses. There is no tree–level CKM mixing angle. So by symmetry reason, we have achieved the first stage of the program. Now, if S_3 is unbroken, none of the light fermions will acquire masses, even though they have Yukawa couplings via the ω_i fields. We can break S_3 spontaneously, or by soft bilinear terms in the Higgs potential:

$$V = \sum_{i,j=1}^{3} \mu_{ij}^2 \omega_i^* \omega_j + h.c. \tag{7}$$

With these soft breaking terms, light fermion masses will be induced. In Fig. 1 we have the one-loop and the two–loop mass generation diagrams.

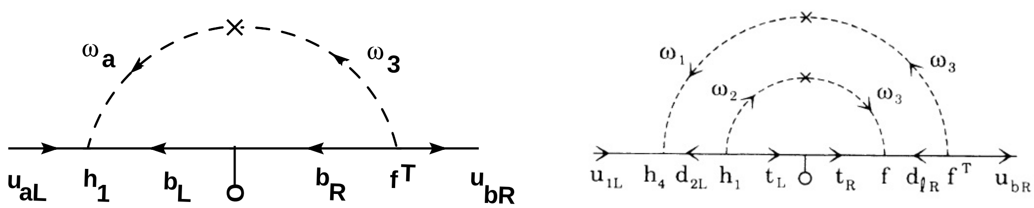

Figure 1: One–loop diagram inducing charm quark mass (left) and two–loop diagram inducing up quark mass (right).

The one–loop diagram of Fig. 1 only generates charm quark mass, and not up quark mass. This can be understood as follows. At tree–level, among the down quarks, only b has a mass. There is a single linear combination of up quarks which couples to the b quark via the ω_i fields. It is this combination that picks up mass at one–loop. The orthogonal combination remains massless at this order. Now, the two–loop diagram connects up quarks to both b and s quarks. The inner loop of the two–loop diagram is the one–loop diagram that generates the s quark mass. As a result, u quark will acquire a mass proportional to the s quark mass at two–loop.

Including the one–loop diagram, the mass matrix for the (c, t) sector has the form

$$M_u^{1-\text{loop}} = \begin{pmatrix} \epsilon & a\epsilon \\ 0 & m_t^0 \end{pmatrix} \tag{8}$$

where a is of order one and the small parameter ϵ is found to be

$$\epsilon \simeq \left(\frac{h_1 f}{8\pi^2}\right) m_b \left(\frac{\mu_{a3}^2}{M_\omega^2}\right) \log\left(\frac{M_\omega^2}{m_b^2}\right) \tag{9}$$

With the Yukawa couplings being order one, we can explain why the charm is much lighter than the top. The mixing angle V_{cb} is of order m_s/m_b, in agreement with observations. The two–loop diagrams which induce the up and down quark masses also induce the mixings of the first family. There is a natural hierarchy of mixing angles where $|V_{us}| \gg |V_{cb}| \gg |V_{ub}|$.

Note that we cannot constrain the masses of the ω fields from this process, since by taking mass of the ω fields and the soft breaking μ^2 term to large values, the charm mass will remain unchanged.

However, in the supersymmetric version of the radiative mass generation mechanism, the new scalars should remain light, to about 1 TeV, since the superpotential is un-renormalized. That is to say that in a SUSY context, in the exact SUSY limit, all the loop diagrams will vanish. Once SUSY breaking terms are turned on, these diagrams will no longer add to zero. Thus, there is a prediction in this scenario. In addition to SUSY particles, LHC should discover these ω_i particles and their superpartners.

It is straightforward to extend the S_3 model to the leptonic sector. Consider the following assignment of leptons and ω_ℓ fields under S_3, where ω_ℓ are $(3^*, 1, -1/3)$ scalar fields. (These are not the conjugates of the ω_i fields from the quark sector, or else, there will be proton decay mediated by these scalars. We assume separate baryon number conservation, so the proton is stable.)

$$\begin{aligned} \begin{pmatrix} \psi_{1L} \\ \psi_{2L} \end{pmatrix} &: \mathbf{2},\ \ \psi_{3L} : 1;\ e_{iR} : \mathbf{1}' \\ \omega_\ell &: \mathbf{1},\ \omega'_\ell : \mathbf{1}' \end{aligned} \tag{10}$$

The general Yukawa coupling of leptons is given by

$$\begin{aligned} \mathcal{L}'_{\text{Yukawa}} &= h'_1 Q^T_{3L} C \psi_{3L} \omega_\ell + h'_2 (Q^T_{1L} C \psi_{1L} + Q^T_{2L} C \psi_{2L}) \omega_\ell + h'_3 (Q^T_{1L} C \psi_{2L} - Q^T_{2L} C \psi_{1L}) \omega'_\ell \\ &+ f'_{ab} u^T_{aR} C e_{bR} \omega'_\ell + h.c \end{aligned} \tag{11}$$

Note that all leptons are massless at the tree level. The one–loop diagram shown in Fig. 2 will induce the τ lepton mass, and is proportional to the top quark mass with a loop suppression. Only τ acquires a one–loop mass. The muon mass arises from the two–loop diagram of Fig. 2. The electron remains massless at this order, and acquires a mass only via a three–loop diagram.

§11. The strong CP problem and its resolutions

The Lagrangian for QCD admits a term

$$\mathcal{L}_{QCD} = \frac{\theta g^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} \tag{12}$$

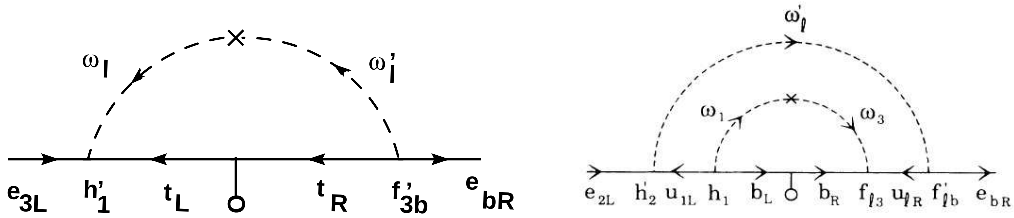

Figure 2: One loop diagram inducing τ lepton mass (left) and two–loop diagram inducing the muon mass (right).

where $\tilde{G}^{\mu\nu}$ is the dual field strength for the gluon. While this term is a total divergence, the surface term in the action does not vanish owing to finite energy "instanton" configurations. This θ term violates CP and P symmetries. But in strong interactions there is no evidence for CP violation. The most stringent constraint on θ arises from the limit on neuron electric dipole moment $d_n \leq 10^{-26}$ e-cm. In the presence of the θ term, we expect neutron to have an EDM estimated to be

$$d_n \simeq \left[10^{-16} \times \overline{\theta}\right] \text{ e} - \text{cm} . \tag{13}$$

Here $\overline{\theta} = \theta + Arg.Det(M_q)$ is the physical observable, since chiral rotations that make quark masses real induce an anomaly term of the same form as $G\tilde{G}$. This yields a constraint:

$$\overline{\theta} \leq 10^{-10} . \tag{14}$$

Now we are faced with a puzzle. Why is it that a parameter of the Lagrangian so small? This is the strong CP problem.

There are various proposed solutions to the problem. At some point in time it was thought that the up quark mass may be zero. If true, that will solve the strong CP problem, since by chiral rotation one can absorb $\overline{\theta}$. But now we know that this is not an acceptable solution.

The most widely studied solution of the problem is the Peccei–Quinn mechanism, which yields a light pseudo–Goldstoneboson, the axion. Here the parameter $\overline{\theta}$ is promoted to a dynamical filed. Because of a QCD anomaly, there is a term induced in the Lagrangian of the form $\mu^4 \cos(\overline{\theta})$. If $\overline{\theta}$ is a field, minimizing the potential with respect to the filed will yield the solution $\overline{\theta} = 0$. Implementing this idea requires an anomalous $U(1)$ symmetry which is spontaneously broken by a Higgs VEV, and explicitly broken by the QCD anomaly.

There is another class of solution to the strong CP problem. One can assume Parity or CP symmetry to set $\theta = 0$. If the fermion mass matrices have real determinant, then $\overline{\theta}$ can be zero at the tree level. Loop induced $\overline{\theta}$ needs to be small, but this is not difficult to obtain.

Let me illustrate this idea with the left–right symmetric model which has Parity invariance. The Yukawa couplings are hermitian in this set-up. To make the mass matrices also hermitian, we must ensure that the VEVs of scalars are real. This is easily done in the SUSY version, which is what I will describe.

Parity invariance implies that the QCD Lagrangian parameter $\theta = 0$, the gluino mass is real and the quark mass matrices $M_{u,d}$ are hermitian at tree level. Therefore $\bar{\theta} = 0$ at tree level. Since parity is broken at a high scale (denoted as v_R), a nonzero value of $\bar{\theta}$ will be induced at the weak scale through renormalization group extrapolation below v_R. We shall estimate this induced $\bar{\theta}$ in the constrained MSSM scenario where the squark masses are degenerate at the unification scale with the trilinear A matrices and the corresponding Yukawa coupling matrices being proportional.

We first turn to the correction to $\bar{\theta}$ arising from the non–hermiticity of the Yukawa coupling matrices. The induced $\bar{\theta}$ will have the general structure given by

$$\delta\bar{\theta} = \mathrm{ImTr}[\Delta M_u M_u^{-1} + \Delta M_d M_d^{-1}] - 3\mathrm{Im}(\Delta M_{\tilde{g}} M_{\tilde{g}}^{-1}) \tag{15}$$

where $M_{u,d,\tilde{g}}$ denote the tree level contribution to the up–quark matrix, down–quark matrix and the gluino mass respectively, and $\Delta M_{u,d,\tilde{g}}$ are the loop corrections. To estimate the corrections from ΔM_u and ΔM_d, we note that the beta function for the evolution of Y_u below v_R is given by $\beta_{Y_u} = Y_u/(16\pi^2)(3Y_u^\dagger Y_u + Y_d^\dagger Y_d + G_u)$ with the corresponding one for Y_d obtained by the interchange $Y_u \leftrightarrow Y_d$ and $G_u \to G_d$. Here G_u is a family–independent contribution arising from gauge bosons and the $\mathrm{Tr}(Y_u^\dagger Y_u)$ term. The $3Y_u^\dagger Y_u$ term and the G_u term cannot induce non–hermiticity in Y_u, given that Y_u is hermitian at v_R. The interplay of Y_d with Y_u will however induce deviations from hermiticity. Repeated iteration of the solution with $Y_u \propto Y_u Y_d^\dagger Y_d$ and $Y_d \propto Y_d Y_u^\dagger Y_u$ in these equations will generate the following structure:

$$\delta\bar{\theta} \simeq \left(\frac{\ln(M_U/M_W)}{16\pi^2}\right)^4 \left[c_1 \mathrm{ImTr}\left(Y_u^2 Y_d^4 Y_u^4 Y_d^2\right) + c_2 \mathrm{ImTr}\left(Y_d^2 Y_u^4 Y_d^4 Y_u^2\right)\right] \,, \tag{16}$$

where M_U is the unification scale. Here c_1 and c_2 are order one coefficients which are not equal since the flavor independent parts G_u and G_d are not the same for the evolution of Y_u and Y_d (hypercharge gauge couplings and the tau lepton Yukawa couplings differentiate the two.) These contributions to $\delta\bar{\theta}$ are very high order in the Yukawa couplings since the trace of products of two hermitian matrices, having the form Im $\mathrm{Tr}(Y_u^n Y_d^m Y_u^p Y_d^q ..)$ contains an imaginary piece only at this order. To estimate the induced $\bar{\theta}$, we choose a basis where Y_u is diagonal, $Y_u = D$ and $Y_d = V D' V^\dagger$ where $D_u v_u = \mathrm{diag}(m_u,\ m_c,\ m_t)$, $D_d v_d = \mathrm{diag}(m_d,\ m_s,\ m_b)$ with V being the CKM matrix. The Trace of the first term in Eq. (7) is then $\mathrm{Im}(D_i^2 D_k^4 D_j'^4 D_l'^2 V_{ij} V_{kl} V_{il}^* V_{kj}^*)$. The leading contribution in this sum is $(m_t^4 m_c^2 m_b^4 m_s^2)/(v_u^6 v_d^6)\mathrm{Im}(V_{cb} V_{ts} V_{cs}^* V_{tb}^*)$. The second Trace in

Eq.(16) is identical, except that it has an opposite sign. Numerically then,

$$\delta\bar{\theta} \sim 3 \times 10^{-27} (\tan\beta)^6 (c_1 - c_2) \tag{17}$$

where we have used the running quark masses at m_t to be $(m_t, m_c, m_b, m_s) = (166, 0.6, 2.8, .063)$ GeV. Clearly, $\delta\bar{\theta}$ is very small, even for $\tan\beta = 50$ its value is 10^{-16}, much below the experimental limit of 10^{-10} from neutron EDM.

Consider the finite one loop corrections to the quark mass matrices. A typical diagram involving the exchange of squarks and gluino is shown in Fig. 3, where the crosses on the $\tilde{Q}$ and $\tilde{Q}^c$ lines represent (LL) and (RR) mass insertions that will be induced in the process of RGE evolution. From this figure we can estimate the form for $\Delta M_u = \frac{2\alpha_s}{3\pi} m^2_{\tilde{Q}} A_u m^2_{\tilde{u^c}}$ where $\tilde{Q}$ is the squark doublet and $\tilde{u^c}$ is the right–handed singlet up squark. Without RGE effects, the trace of this term will be real, and will not contribute to $\bar{\theta}$. Looking at the RGE for $m^2_{\tilde{u^c}}$ upto two loop order, we see that for the case of proportionality of A_u and Y_u, $m^2_{\tilde{u^c}}$ gets corrections having the form $m_0^2 Y_u^2$ or $m_0^2 Y_u^4$ or $m_0^2 Y_u Y_d^2 Y_u$. Therefore in $\Delta M_u M_u^{-1}$, the M_u^{-1} always cancels and we are left with a product of matrices of the form $Y_u^n Y_d^m Y_u^p Y_d^q \cdot \cdots$. A similar comment applies when we look at the RGE corrections for $m^2_{\tilde{Q}}$ or A_u. If the product is hermitian, then its trace is real. So to get a nonvanishing contribution to theta, we have to find the lowest order product of Y_u^2 and Y_d^2 that is non–hermitian and we get

$$\delta\bar{\theta} = \frac{2\alpha_s}{3\pi} \left(\frac{\ln(M_U/M_W)}{16\pi^2} \right)^4 \left(k_1 \mathrm{ImTr}[Y_u^2 Y_d^4 Y_u^4 Y_d^2] + k_2 \mathrm{ImTr}[Y_d^2 Y_u^4 Y_d^4 Y_u^2] \right) \tag{18}$$

where $k_{1,2}$ are calculable constants. The numerical estimate of this contribution parallels that of the previous discussions, $\delta\bar{\theta} \sim (k_1 - k_2) \times 10^{-28} (\tan\beta)^6$. The contributions from the up–quark and down quark matrices tend to cancel, but since the $\tilde{d^c}$ and the $\tilde{u^c}$ squarks are not degenerate, $k_1 \neq k_2$ and the cancellation is incomplete.

In Fig. 3 we have displayed the one–loop contribution to the gluino mass arising from the quark mass matrix. Here again one encounters the imaginary trace of two hermitian matrices Y_u and Y_d, in the case of universality and proportionality of SUSY breaking parameters. Our estimate for $\delta\bar{\theta}$ is similar to that of the quark mass matrix of Eq. (18).

This exercise shows that the strong CP problem can be consistently resolved in the left–right symmetric model.

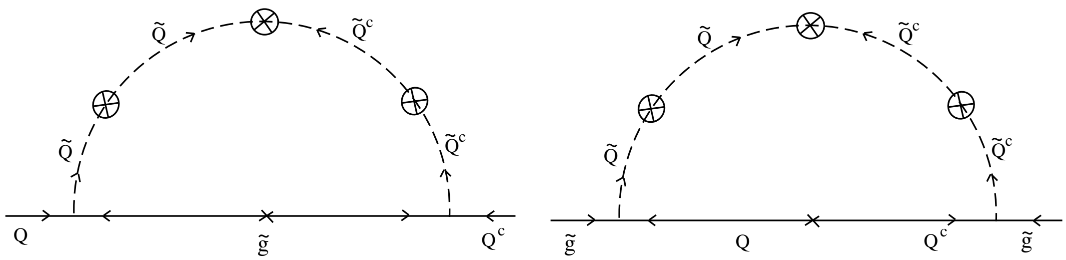

Figure 3: One loop diagram inducing complex correction to the quark mass (left) and to the gluino mass (right).