Physics in extra dimensions: lecture #1

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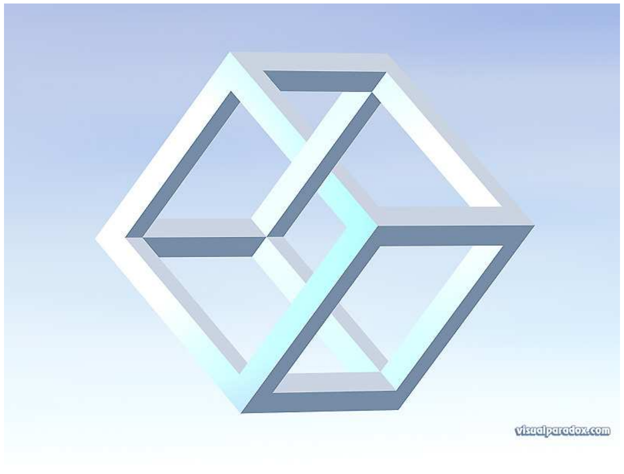
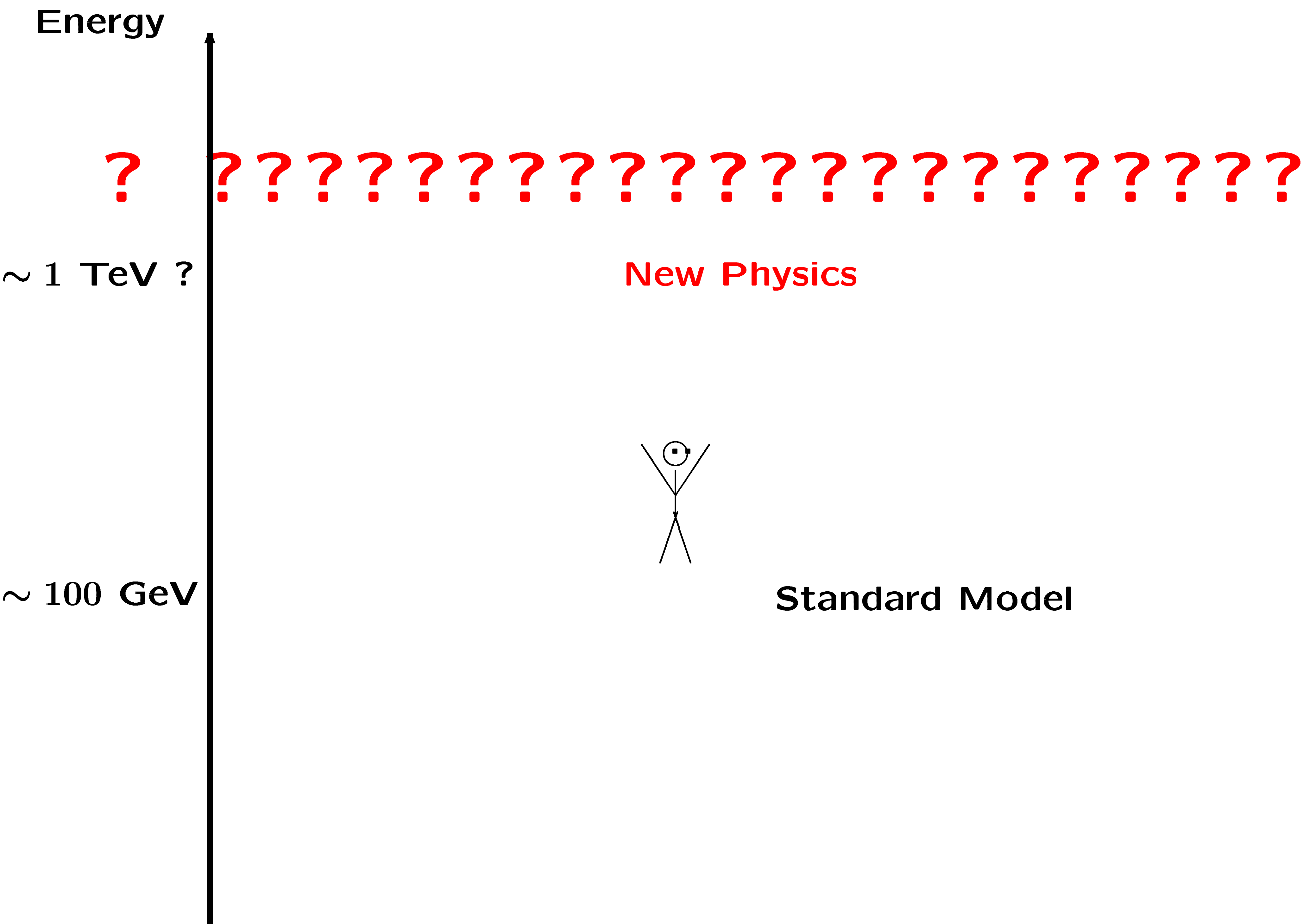
Lecture 1: **Field theory in compact dimensions.**
Bosons in the bulk and their collider signatures.

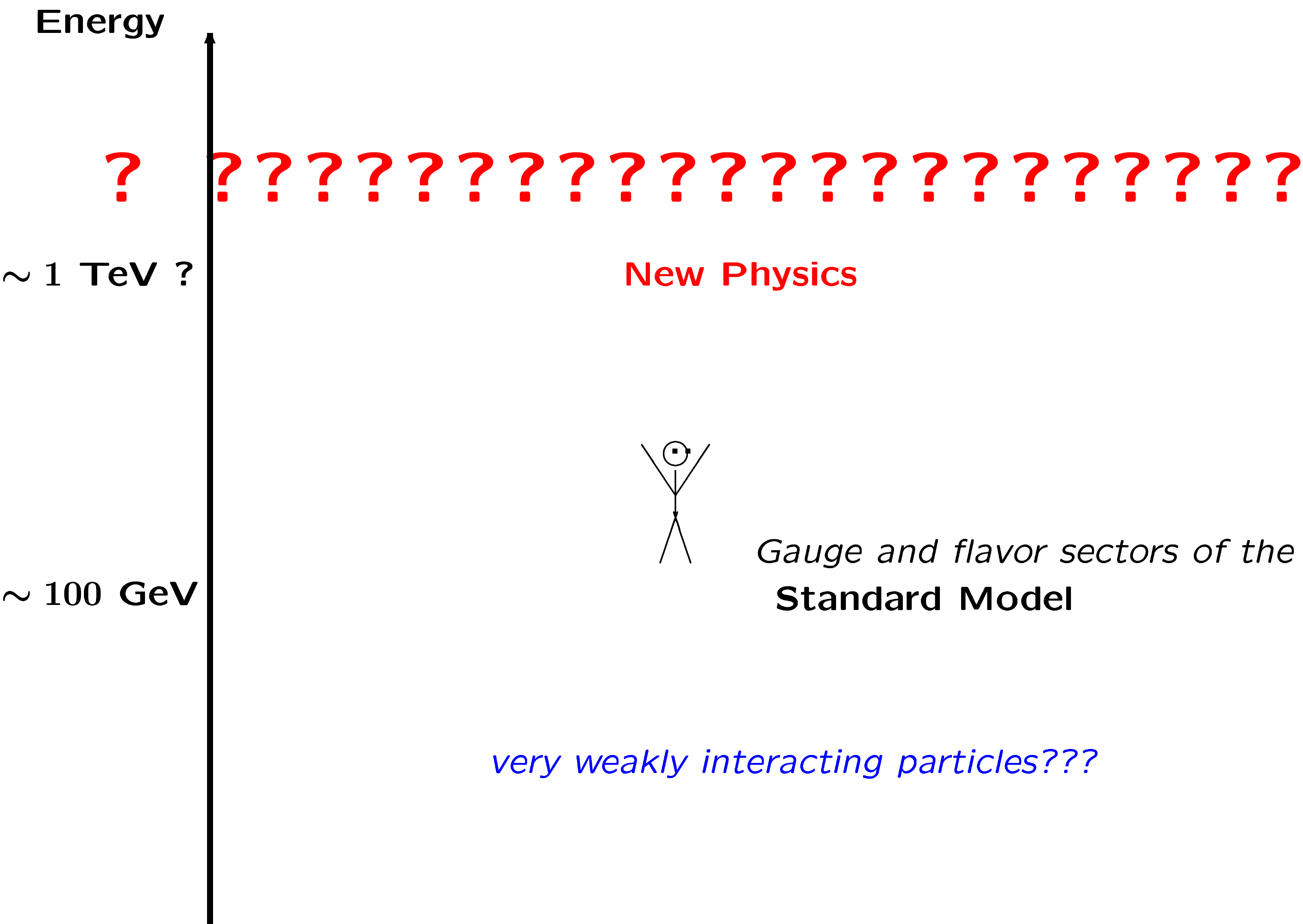
Lecture 2: **One universal extra dimension.**
Discrete symmetries and cascade decays at colliders.

Lecture 3: **6D Standard Model.**

Lecture 4: **Particles in a warped extra dimension.**
Strong coupling at the TeV scale.




Energy
? ??????????????????????
~ 1 TeV ?
New Physics
~ 100 GeV
Standard Model


Energy
? ??????????????????????
∼ 1 TeV ?
New Physics
Gauge and flavor sectors of the
Standard Model
∼ 100 GeV
very weakly interacting particles???

"New physics" at the TeV scale could change the basic hypotheses of the Standard Model:

local quantum field theory
in 3 spatial + 1 time dimensions,
invariant under $SO(3,1)$ Lorentz transformations.

... "terra incognita" ... "uncharted waters" ...

Evidence that we live in 3 spatial dimensions:

- **it is obvious!** *(end of story?!?)*
- **Gauss law, in 3 + n spatial dimensions:** $V(r) \sim 1/r^{n+1}$
We observe $n = 0$ **for gravity and electromagnetism.**
- **Standard Model agrees with the data.**
- **there are no renormalizable field theories in more dimensions**

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Counter-arguments:

- **what's obvious may be due to preconception**
(*e.g.*, quantum mechanics is not obvious)
- **Gauss law may change at short distance**
- **Standard Model has not been tested below 10^{-16} cm**
- **gravitational interactions are non-renormalizable in $D = 3 + 1$**

Bosons in compact spatial dimensions

4D flat spacetime $\perp$ one dimension of size $L = \pi R$:

A scalar field in the bulk, $\phi(x^\alpha)$, $\alpha = 0, 1, ..., 4$:

$$\mathcal{L}_{5D} = (\partial^\mu \phi)^\dagger \partial_\mu \phi - \left(\partial^4 \phi\right)^\dagger \partial_4 \phi - m_0^2 \phi^\dagger \phi \ , \qquad \mu = 0, 1, 2, 3$$

$\Rightarrow$ Equation of motion: $\left(\partial^\mu \partial_\mu - \partial^4 \partial_4\right) \phi = m_0^2 \phi$

m_0 *is the 5D mass of* ϕ.

Neumann boundary conditions for "even" fields:

$$\frac{\partial}{\partial x^4}\phi(x^\mu, 0) = \frac{\partial}{\partial x^4}\phi(x^\mu, \pi R) = 0$$

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$$\phi(x^\mu, x^4) = \frac{1}{\sqrt{\pi R}}\left[\, \phi^{(0)}(x^\mu) + \sqrt{2}\sum_{j\geq 1}\phi^{(j)}(x^\mu)\cos\left(\frac{jx^4}{R}\right)\,\right]$$

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Kaluza-Klein decomposition

Zero-mode (wave function is constant along x^4)

Kaluza-Klein modes: particles of definite momentum along x^4

4D point of view: a tower of massive particles:

$$\mathcal{L}_{4D} = \int_0^{\pi R} dx^4\, \mathcal{L}_{5D} \qquad \Rightarrow \qquad m_j^2 = m_0^2 + \frac{j^2}{R^2}$$

Dirichlet boundary conditions for "odd" fields:

$$\phi(x,0) = \phi(x,\pi R) = 0$$

KK decomposition:

$$\phi(x^\mu, x^4) = \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{j\geq 1} \phi^{(j)}(x^\mu) \sin\left(\frac{j x^4}{R}\right)$$

There is no zero-mode.

The lightest KK mode is $\phi^{(1)}$, of mass $\sqrt{1/R^2 + m_0^2}$

Homework: *Check that the normalization condition for KK functions requires the factor of $\sqrt{2}$.*
Why $j < 0$ is not allowed?

Gauge bosons in 5D:

$A_\mu(x^\nu, x^4)$, $\mu, \nu = 0, 1, 2, 3$, and

$A_4(x^\nu, x^4)$ – polarization along the extra dimension.

From the point of view of the 4D theory:

$A_4(x^\nu, x^4)$ is a tower of spinless KK modes.

Gauge invariance requires A_μ to have a zero-mode:

$$\partial_4 A_\mu(x^\nu, 0) = \partial_4 A_\mu(x^\nu, \pi R) = 0$$

$$A_\mu(x^\nu, x^4) = \frac{1}{\sqrt{\pi R}} \left[A_\mu^{(0)}(x^\nu) + \sqrt{2} \sum_{j \geq 1} A_\mu^{(j)}(x^\nu) \cos\left(\frac{j x^4}{R}\right) \right]$$

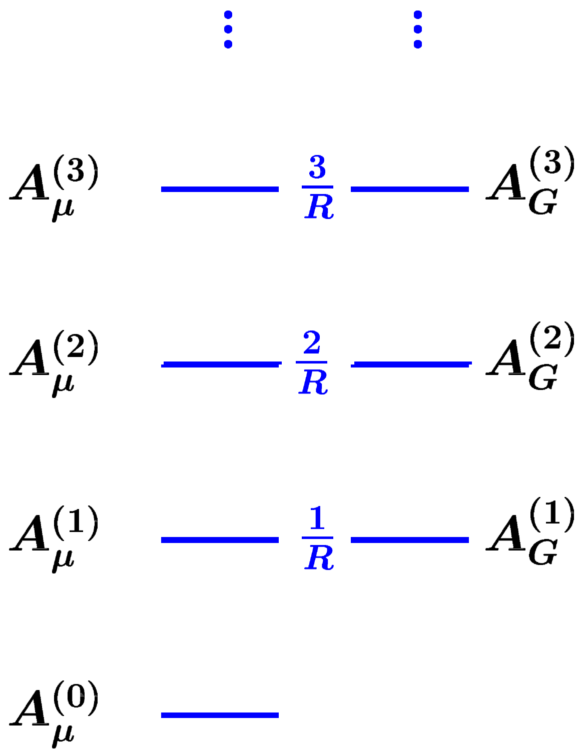
Dirichlet B.C : $\quad A_4(x^\nu, 0) = A_4(x^\nu, \pi R) = 0$

KK decomposition : $\quad A_4(x^\nu, x^4) = \sqrt{\frac{2}{L}} \sum_{j \geq 1} A_G^{(j)}(x) \sin\left(\frac{j x^4}{R}\right)$

$\rightarrow A_4(x^\nu, x^4)$ **does not have a 0-mode!** *(Odd field)*

Kaluza-Klein spectrum of gauge bosons

$A_G^{(j)}(x^\nu)$ becomes the longitudinal degree of freedom of the spin-1 KK mode $A_\mu^{(j)}(x^\nu)$.



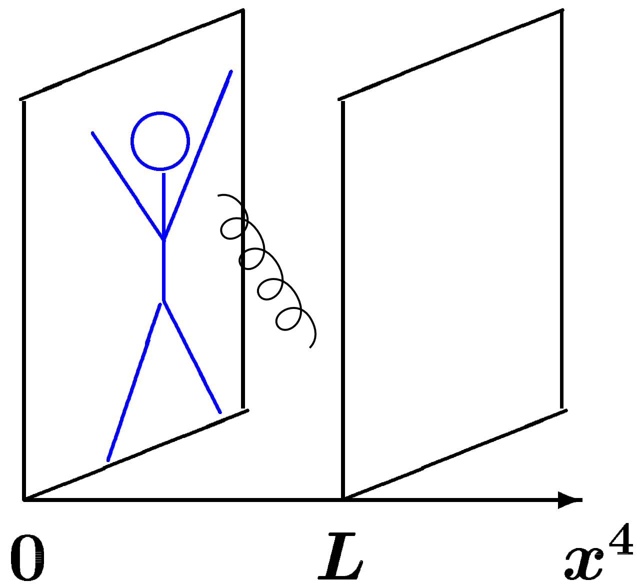
Extra dimensions may be classified according to:

- **number (1, 2, ... , 13?)**
- **type of compactification (*i.e.* boundary conditions)**
- **metric (flat, warped, ...)**
- **which fields propagate in the bulk (graviton, top quark, ...)**
- **existence of localized operators, stabilization mechanism, ...**

Types of extra dimensions:

- graviton only propagates in $n \geq 2$ flat extra dimensions (ADD)
- bosons only propagate in some flat extra dimensions (DDG)
- bosons and some fermions propagate in flat extra dimensions
- all particles propagate in some flat extra dimensions (UED)
- graviton only propagates in a warped extra dimension (RS)
- all particles propagate in a warped extra dimension
- ...

Assume that only bosons propagate in a flat extra dimension, and that the fermions are localized at $x^4 = 0$.



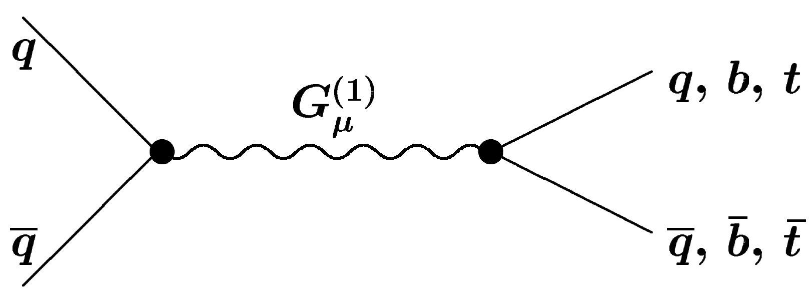
Interactions of the KK gluons with quarks:

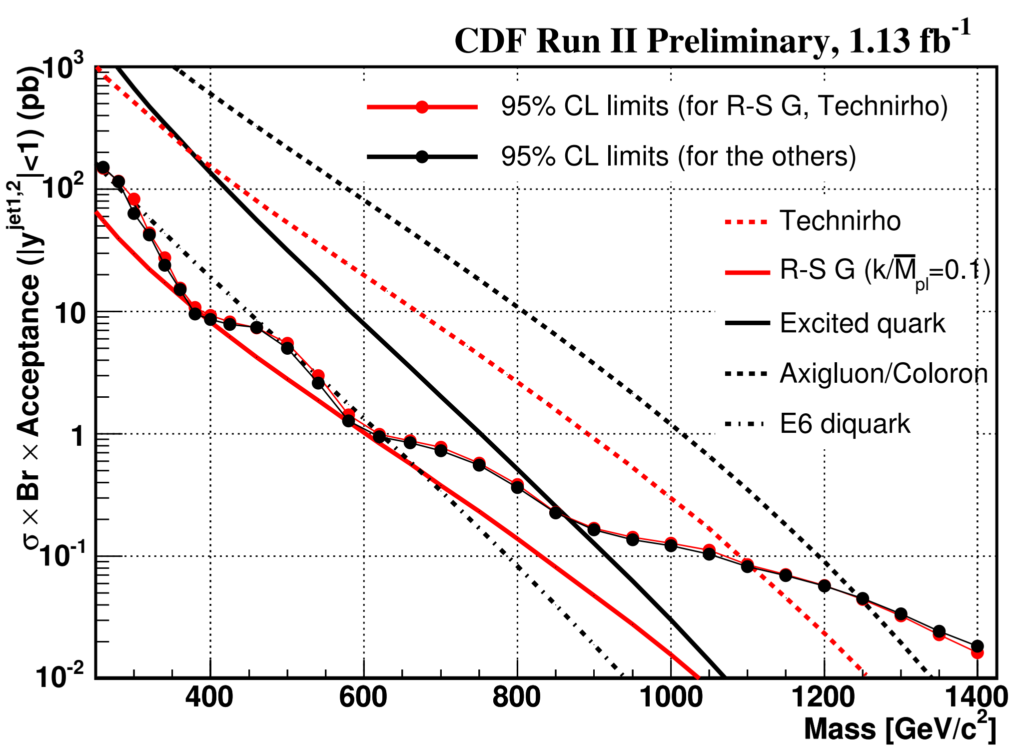
$$\mathcal{L}_{4D} = \int_0^L dx^4 \, g_5 G^a_\mu(x^\nu, x^4) \left[\delta\left(x^4\right) \bar{q}(x^\nu)\gamma^\mu T^a q(x^\nu)\right]$$

$$= g_s \left(G_\mu^{(0)a} + \sqrt{2} \sum_{j\geq 1} G_\mu^{(j)a}(x) \right) \bar{q}\gamma^\mu T^a q$$

KK gluon production (in the narrow width approximation):

$$\sigma\left(p\bar{p} \to G_\mu^{(1)} X\right) \approx \frac{16\pi^2\alpha_s}{9s} \sum_q \int_{M^2/s}^{1} \frac{dx}{x} \left[q(x)\, q\left(\frac{M^2}{xs}\right) + \overline{q}(x)\, \overline{q}\left(\frac{M^2}{xs}\right) \right]$$





Run II limit on dijet resonances:

$\longrightarrow \quad 1/R > 1.4$ TeV

5D theory = 4D theory with some heavy particles

$SU(3)_c$ **in extra dimensions → SM gluon + heavy gluons**

4D theory with the same spectrum must include a larger gauge symmetry.

$SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_c$ **spontaneously broken by the VEV of a scalar transforming as** $(3, \bar{3})$

Quarks transform as 3 of $SU(3)_1$

G^a_μ - massless gluon as in QCD, with $g_s = \frac{h_1 h_2}{\sqrt{h_1^2 + h_2^2}}$

G'^a_μ - massive gluon ("coloron") with couplings $g_s \frac{h_1}{h_2} G'^a_\mu \bar{q} \gamma^\mu T^a q$

KK gluon coupling recovered if the two $SU(3)$ gauge couplings satisfy $h_1/h_2 = \sqrt{2}$.

The 4D theory describing the first N KK modes of the gluon has a $SU(3)_1 \times SU(3)_2 \times \cdots \times SU(3)_{N+1} \to SU(3)_c$ gauge structure.

Assume that the Higgs doublet and electroweak gauge bosons propagate in one flat extra dimension, while the fermions are localized at one end of the interval.

Homework: Derive the couplings of the KK electroweak gauge bosons to quark and leptons.

LEP-II limits on four-fermion interactions imply $1/R > 6$ TeV.

Fermions in 5D:

Chiral boundary conditions:

$$\chi_L(x^\mu, 0) \;=\; \chi_L(x^\mu, \pi R) = 0$$
$$\frac{\partial}{\partial x^4}\chi_R(x^\mu, 0) \;=\; \frac{\partial}{\partial x^4}\chi_R(x^\mu, \pi R) = 0$$

Kaluza-Klein decomposition:

$$\chi = \frac{1}{\sqrt{\pi R}}\left\{\chi_R^0(x^\mu) + \sqrt{2}\sum_{j\geq 1}\left[\chi_R^j(x^\mu)\cos\left(\frac{\pi j x^4}{L}\right) + \chi_L^j(x^\mu)\sin\left(\frac{\pi j x^4}{L}\right)\right]\right\}$$

Something to read:

G. D. Kribs, TASI lectures on the "Phenomenology of extra dimensions", hep-ph/0605325.

E. H. Simmons, "Coloron phenomenology," Phys. Rev. D 55, 1678 (1997), hep-ph/9608269.

K. R. Dienes, E. Dudas and T. Gherghetta, "Grand unification at intermediate mass scales through extra dimensions," Nucl. Phys. B 537, 47 (1999), hep-ph/9806292.

...

Conclusions so far

• Extra spatial dimensions may exist if their size is small enough. The limits are model dependent.

• Any particle that propagates in $D \geq 5$ would appear in experiments as a tower of heavy 4-dimensional particles.

• If bosons propagate in extra dimensions while fermions are localized, then the Kaluza-Klein bosons may be singly produced, leading to s-channel resonances.

• Kaluza-Klein modes of the quarks and leptons are vectorlike fermions.