Physics in extra dimensions: lecture #4

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Lecture 1: Field theory in compact dimensions.  
Gauge bosons in the bulk and their collider signatures.

Lecture 2: One universal extra dimension.  
Discrete symmetries and cascade decays at colliders.

Lecture 3: Two universal extra dimensions.

Lecture 4: Particles in a warped extra dimension.  
Strongly-coupled physics at the TeV scale
$SU(3)_c$ spinless adjoints $G_H^{(j,k)}$ transform as $(8,1,0)$ under $SU(3)_c \times SU(2)_W \times U(1)_Y$

No renormalizable interactions with SM fermions, because:

$\overline{Q}_L u_R G_H^{(1,1)}$ is not $SU(2)_W$ invariant,

$\overline{Q}_L Q_L G_H^{(1,1)}$ is not invariant under 4D Lorentz transformations.

There are however dimension-5 operators, such as

$$\mathcal{L} = \frac{1}{M} \overline{Q}_L \gamma^\mu T^a Q_L \partial_\mu G_H^{(1,1)a}$$

Integrate by parts:

$$\mathcal{L} = -\frac{1}{M} G_H^{(1,1)a} \left[ (\partial_\mu \overline{Q}_L) \gamma^\mu T^a Q_L + \overline{Q}_L \gamma^\mu T^a (\partial_\mu Q_L) \right]$$

Use Dirac equation:

$$\mathcal{L} = \sum_q i \frac{m_q}{M} G_H^{(1,1)a} \overline{q} T^a \gamma_5 q$$

$\Rightarrow \ G_H^{(1,1)}$ decays predominantly to $t \bar{t}$ pairs:

**Homework:** write down the interactions of $W_H^{(1,1)}$ with the SM fermions.
Graviton only in flat extra dimensions

Gravitational interactions measured at distances \( \gtrsim 10^{-3}\text{cm} \):

\[
F_N = \frac{m_1 m_2}{M_{\text{Planck}}^2 r^2}
\]

We may live on a wall in extra dimensions!

(Arkani-Hamed, Dimopoulos, Dvali, 1998)

\[
L < \text{mm}
\]

Newton's law in extra dimensions:

\[
F_N = \frac{m_1 m_2}{(M_s r)^{2+n}}
\]

Scale of quantum gravity may be as low as \( \sim 5 \text{ TeV} \):

\[
M_s = \left( \frac{M_{\text{Planck}}^2}{L^n} \right)^{1/(2+n)}
\]
A warped extra dimension

L. Randall, R. Sundrum, hep-ph/9905221

4D flat spacetime of coordinates $x^\mu$ and one
dimension of coordinate $z$.

Line element:

$$ds^2 = e^{-2kz} \eta_{\mu\nu} dx^\mu dx^\nu - dz^2$$

$$\eta_{\mu\nu} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

Anti-de-Sitter space along the 5th dimension.

$k$ is the AdS curvature (has dimensions of mass).

The unit of length depends on the position along $z$!
A slice of Anti-de-Sitter space: space exists only for $0 \leq z \leq \pi r_c$. 
A slice of Anti-de-Sitter space: space exists only for $0 \leq z \leq \pi r_c$.

Boundary conditions at $z = 0$ and $z = \pi r_c$ must be specified for each field propagating in the bulk.
RS1 model:

$$x^\mu$$

Scales: \( kr_c \approx 10 \), \( k \lesssim M_{\text{Planck}} \approx 10^{19} \text{ GeV} \)
Fluctuations of the metric:

\[ ds^2 = e^{-2kz} \left[ \eta_{\mu\nu} + h_{\mu\nu}(x) \right] dx^\mu dx^\nu - dz^2 \]

\( h_{\mu\nu} \) is the graviton zero-mode, responsible for the long-range gravitational interactions.

*Strength of gravitational force:* \( G_N \sim 1/M_{\text{Planck}}^2 \)

*Fundamental scale on the Standard Model brane:*

\[ M_{\text{Planck}} e^{-\pi kr_c} \sim O(1) \text{ TeV} \]
Interaction of the graviton 0-mode (the massless 4D spin-2 field) with Standard Model particles is suppressed by its exponentially small wave function at the SM brane.

**Hierarchy between the Planck and electroweak scales is explained!**
Comparison between various solutions to the hierarchy problem:

1. Technicolor

*Exponential hierarchy between $M_{\text{Planck}}$ and the scale where the technicolor gauge interaction becomes strong.*

**Problem:** fit to the electroweak data? (some solutions exist)

2. Dynamically-broken supersymmetry

*Susy breaking scale is exponentially suppressed compared to $M_{\text{Planck}}$ due to gauge dynamics.*

**Problem:** $\mu$ term (the Higgsino mass) is supersymmetric.

    Why $\mu \sim v$? (some solutions exist)

3. RS1

$1/M_{\text{Planck}}$ is exponentially suppressed compared to $1/v$. 
Energy

$10^{16}$ TeV

quantum gravity

Technicolor gauge coupling: $g_{TC} \sim O(1)$

logarithmic running of $g_{TC}$
(increases at lower scales, just as in QCD)

$g_{TC} \sim O(4\pi) \Rightarrow$ Technifermions condense
$\Rightarrow$ electroweak symmetry is broken
Energy

$10^{16}$ TeV

quantum gravity

$\sim 10 - 10^8$ TeV

Dynamical susy breaking sector

$10 - 10^7$ TeV ?

Messenger sector

$\sim 1$ TeV

MSSM
Scales of RS1 model (measured on the SM brane):

- $\sim 1$ TeV
- Graviton KK modes are strongly coupled
- Standard Model
**Graviton KK modes**

Kaluza-Klein decomposition for the graviton field

\[ h_{\mu\nu}(x, z) = \frac{1}{\sqrt{r_c}} \sum_{j \geq 0} h_{\mu\nu}^{(j)}(x) \chi_j(z) \]

→ A tower of spin-2 resonances

**Graviton KK functions (solutions to the Einstein equations in the bulk):**

\[ \chi_j(z) = \frac{e^{2kz}}{N_j} \left[ J_2 \left( \frac{m_j}{k} e^{kz} \right) + \alpha_j Y_2 \left( \frac{m_j}{k} e^{kz} \right) \right] \]

\( J_2, Y_2 \) are Bessel functions; \( N_j, \alpha_j \) are normalization constants

\( m_j \) is the mass of the \( j \)-th KK mode of spin-2:

\[ m_1 = 3.8 \bar{k}, \ m_2 = 7.0 \bar{k}, \ m_3 = 10.2 \bar{k}, \ldots \]

\( \bar{k} \equiv k e^{-kr_c \pi} \sim O(1) \) TeV
Gauge fields in a warped extra dimension

Kaluza-Klein decomposition for the gauge fields

$$A_\mu(x', z) = \frac{1}{\sqrt{2\pi r_c}} \left[ A_\mu^{(0)}(x') + \sum_{j \geq 1} A_\mu^{(j)}(x') f_j(z) \right]$$

0-mode has a flat profile (unlike the graviton).

**KK functions:**

$$f_j(z) = \frac{e^{kz}}{N_j} \left[ J_1 \left( \frac{m_j e^{kz}}{k} \right) - \frac{J_0(m_j/k)}{Y_0(m_j/k)} Y_1 \left( \frac{m_j e^{kz}}{k} \right) \right]$$

$J, Y$ are Bessel functions; $N_j$ is a normalization constant.

$m_j$ is the mass of the $j$th KK mode of spin-1:

$$m_1 = 2.5k, \ m_2 = 5.6k, \ m_3 = 8.7k, \ldots$$
Fermions in a warped extra dimension

S. Chang et al, hep-ph/9912498

Dirac equation in a warped dimension ⇒ fermion zero-modes have a non-trivial profile, peaked on one of the branes.

For each standard model fermion there is a 5D mass parameter which controls the bulk profile.
Standard Model in a warped extra dimension:

unlike UED, there is no KK parity because of the warping.

Severe constraints on the KK masses from electroweak fits: \( m_1 > O(20) \) TeV


Limits are lowered to \( m_1 > O(3) \) TeV if the gauge group in the bulk is \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \)


Typical signature at the LHC:

The fermion KK modes may have masses below 1 TeV

(Carena, Ponton, Santiago, Wagner, hep-ph/0701055)
Conjecture: SM in a warped extra dimension is dual to a 4D quasi-conformal strongly-coupled gauge theory
(a deformation of the AdS/CFT conjecture)

Similar to “walking technicolor”?! 

Fields in the 5D picture localized close to the SM brane are composite fields in the conformal theory:

→ Higgs doublet and \( t_R \) are composite fields
→ \( (t,b)_L \) is partially composite (an admixture of a composite field and a fundamental field).
AdS/CFT interpretation of a warped extra dimension is yet another connection to 4D physics.

Recall from lecture #1:

5D theory = 4D theory with some heavy particles

$SU(3)_c$ in extra dimensions $\rightarrow$ SM gluon $+$ heavy gluons

4D theory describing the first $N$ KK modes of the gluon:

$SU(3)_1 \times SU(3)_2 \times \cdots \times SU(3)_{N+1} \rightarrow SU(3)_c$ gauge group, spontaneously broken by the VEVs of scalars transforming as $(3, \bar{3}, 1, \ldots, 1, \ldots)$
Composite Higgs models

The coupling of the top quark to the Higgs field changes with the distance (similar to vacuum polarization in electrodynamics).

\[ \lambda_t \bar{t}_R \langle H^0 \rangle t_L , \quad \langle H^0 \rangle \approx 174 \text{ GeV} \]

In a world of only top and Higgs:

\[ \lambda_t (\mu) = \frac{\lambda_t (m_t)}{\sqrt{1 - \frac{9 \lambda_t^2 (m_t)}{64\pi^2} \ln \frac{\mu}{m_t}}} \]
Infrared Fixed Point for $\lambda_t$

(C.T. Hill, 1981, ...)

$$\frac{\lambda_t^2}{4\pi} \approx \log\left(\lambda_t^2/1\text{GeV}\right)$$

MSSM (hep-ph/9610296)
Top condensation $\Rightarrow$ Higgs boson is a $\bar{t}t$ bound state!
*(Bardeen, Hill, Lindner, 1990, ...)*

*Binding may be due to some strongly-interacting heavy gauge bosons*

New heavy quarks (vectorlike) could accelerate the $\lambda_t$ running: scale of Higgs compositeness may be as low as a few TeV.

*Explicit models: top seesaw, QCD in extra dimensions, ...*
Is there a Higgs boson?

EWSB by boundary conditions

Csaki, Grojean, Pilo, Terning: hep-ph/0308038

One warped ED, bulk $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group broken by boundary conditions.

AdS dual of a (walking) technicolor-like theory,
(the presence of the IR brane breaks electroweak symmetry)

→ lightest $W$, $Z$ and photon resonances around 1.2 TeV
→ no fundamental (or composite) Higgs boson
What to look for at colliders

- Vector-like fermions (KK modes)

- New gauge bosons
  (*e.g.*, unitarity restored by $Z'$, $W'$, ...)

- extended Higgs sectors
  (*e.g.*, radion-Higgs mixing, two composite $H$, ...)

$\Rightarrow$ many possibilities! Will the experiments be able to differentiate between models?
Conclusions

• A warped extra dimension provides a nice explanation for the hierarchy problem (related to walking technicolor).

• Physics at the TeV scale may be strongly coupled. Extra dimensional theories provide some examples, as well as some tools for analyzing strong dynamics.

• Any particle that propagates in $D \geq 5$ would appear in experiments as a tower of heavy 4D particles. There are purely 4D theories with similar spectra and interactions, which are interesting whether or not extra dimensions exist in nature!

• Phenomenological implications of extra dimensions are highly model dependent. Even closely related theories, such as 1 versus 2 universal extra dimensions, predict quite different signals.
Find out what theory describes physics at the TeV scale.

Energy

$\sim 1 \text{ TeV}$

$\sim 100 \text{ GeV}$

New Physics

Standard Model

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