Physics in extra dimensions: lecture #4

Bogdan Dobrescu (*Fermilab*)

Lecture 1: **Field theory in compact dimensions.**
Gauge bosons in the bulk and their collider signatures.

Lecture 2: **One universal extra dimension.**
Discrete symmetries and cascade decays at colliders.

Lecture 3: **Two universal extra dimensions.**

Lecture 4: **Particles in a warped extra dimension.**
Strongly-coupled physics at the TeV scale

Interactions of spinless adjoints with 0-mode fermions – Addendum to Lecture 3

$SU(3)_c$ spinless adjoints $G_H^{(j,k)}$ transform as (8,1,0) under $SU(3)_c \times SU(2)_W \times U(1)_Y$

No renormalizable interactions with SM fermions, because:

$\overline{Q}_L u_R G_H^{(1,1)}$ is not $SU(2)_W$ invariant,

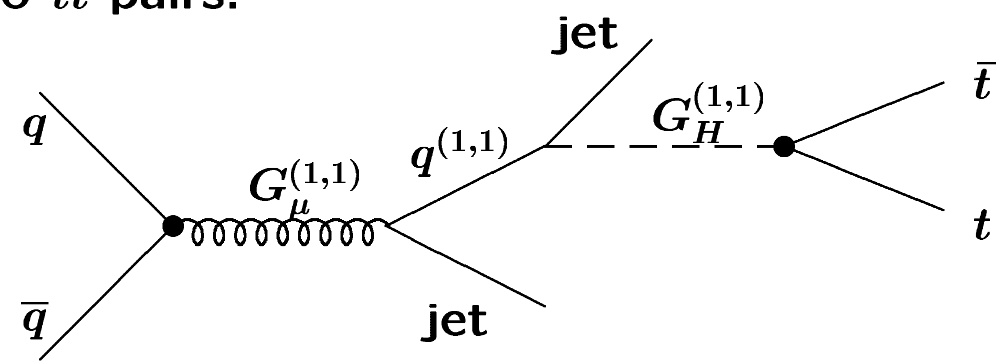
$\overline{Q}_L Q_L G_H^{(1,1)}$ is not invariant under 4D Lorentz transformations.

There are however dimension-5 operators, such as $\mathcal{L} = \frac{1}{M}\,\overline{Q}_L \gamma^\mu T^a Q_L\, \partial_\mu G_H^{(1,1)a}$

Integrate by parts:
$$\mathcal{L} = -\frac{1}{M}\, G_H^{(1,1)a} \left[\left(\partial_\mu \overline{Q}_L\right) \gamma^\mu T^a Q_L + \overline{Q}_L \gamma^\mu T^a \left(\partial_\mu Q_L\right) \right]$$

Use Dirac equation:
$$\mathcal{L} = \sum_q i \frac{m_q}{M}\, G_H^{(1,1)a} \overline{q} T^a \gamma_5 q$$

$\Rightarrow$ $G_H^{(1,1)}$ decays predominantly to $t\bar{t}$ pairs:



Homework: write down the interactions of $W_H^{(1,1)}$ with the SM fermions.

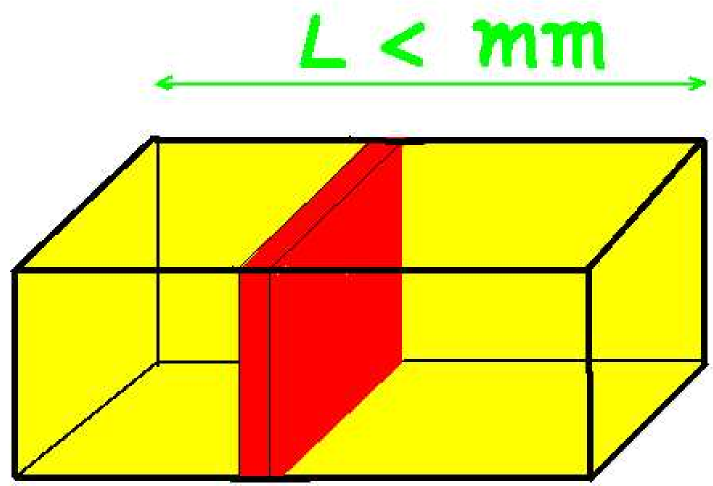
Graviton only in flat extra dimensions

Gravitational interactions measured at distances $\gtrsim 10^{-3}$cm:

$$F_N = \frac{m_1 m_2}{M_{\rm Planck}^2 r^2}$$

We may live on a wall in extra dimensions!

(Arkani-Hamed, Dimopoulos, Dvali, 1998)



Newton's law in extra dimensions: $$F_N = \frac{m_1 m_2}{(M_s r)^{2+n}}$$

Scale of quantum gravity may be as low as $\sim$ 5 TeV: $$M_s = \left(\frac{M_{\rm Planck}^2}{L^n}\right)^{1/(2+n)}$$

A warped extra dimension

L. Randall, R. Sundrum, hep-ph/9905221

4D flat spacetime of coordinates x^μ and one dimension of coordinate z.

Line element:

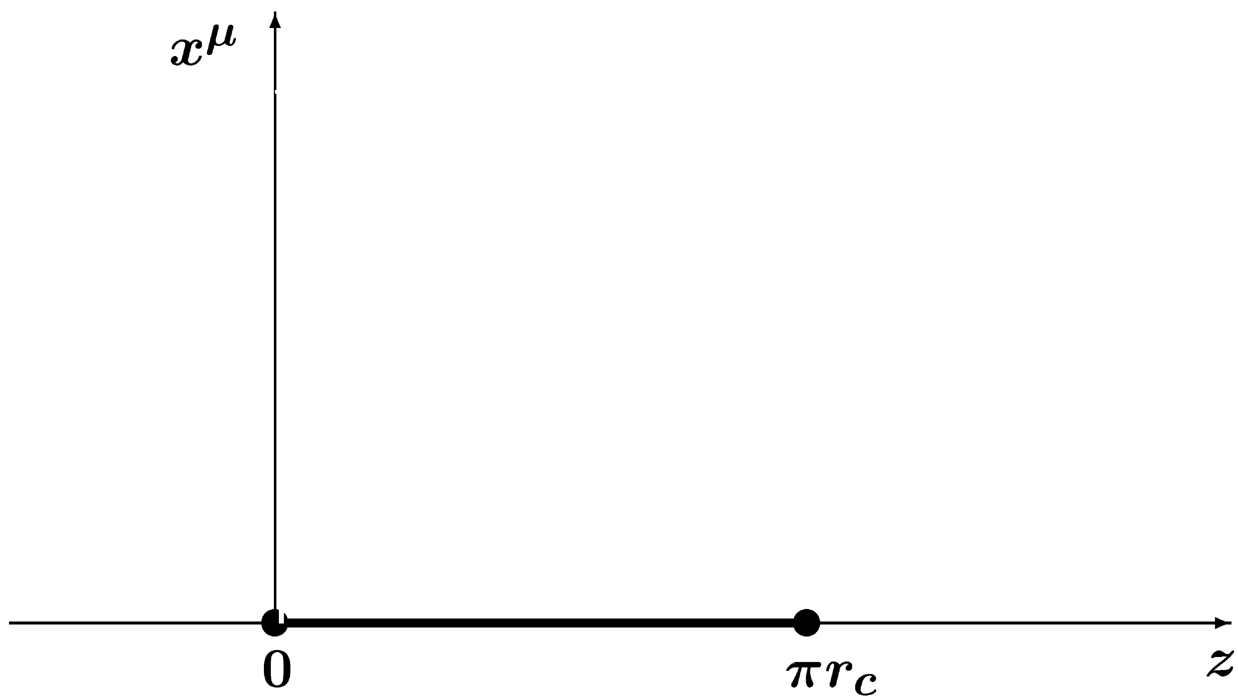
$$ds^2 = e^{-2kz}\,\eta_{\mu\nu}dx^\mu dx^\nu - dz^2 \quad , \quad \eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Anti-de-Sitter space along the 5th dimension.

k is the AdS curvature (has dimensions of mass).

The unit of length depends on the position along z!

A slice of Anti-de-Sitter space: space exists only for $0 \leq z \leq \pi r_c$.



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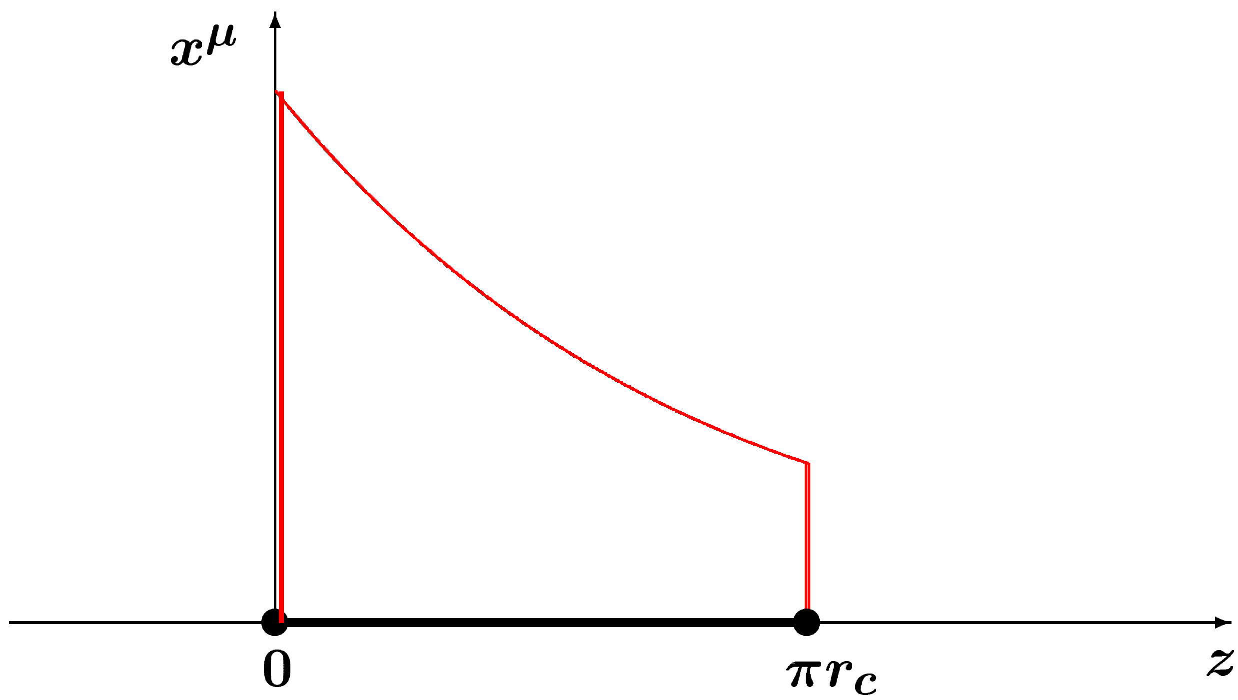
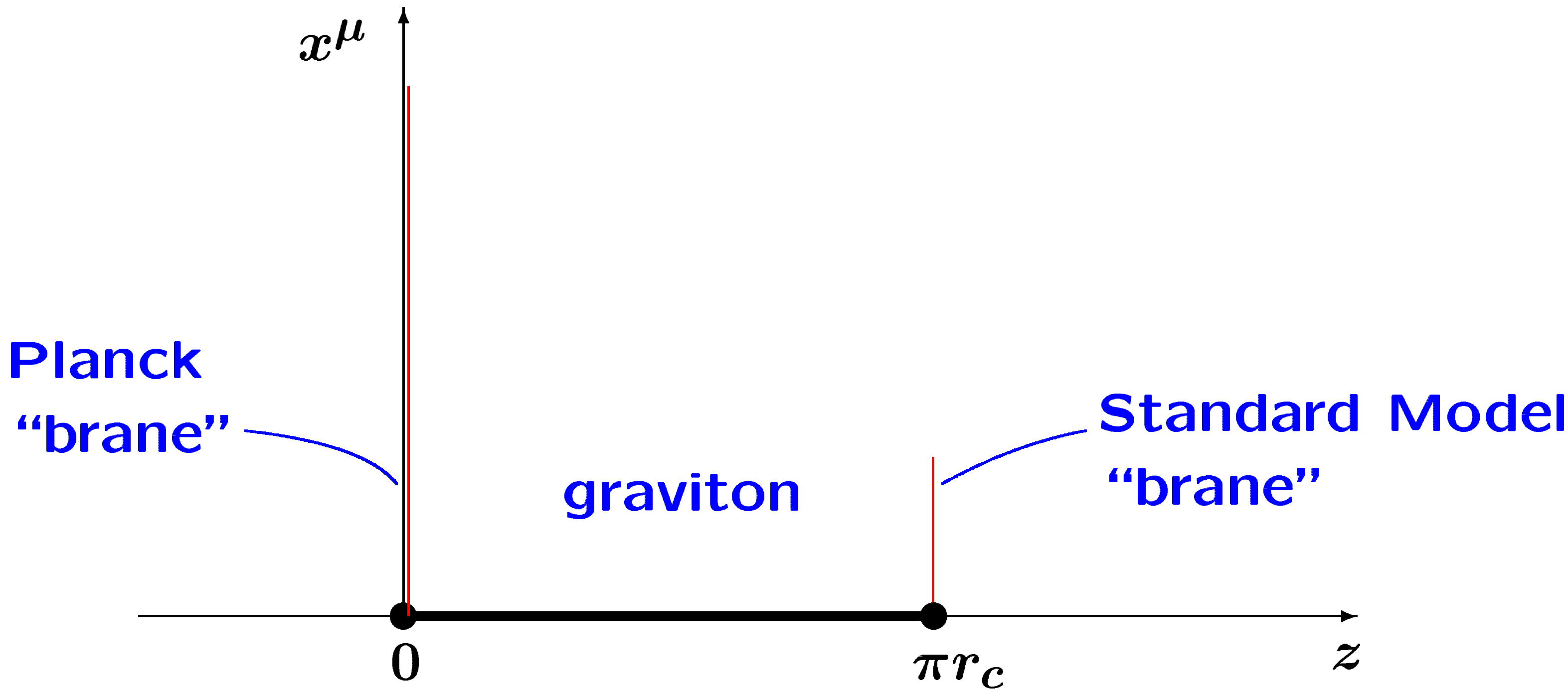


Boundary conditions at $z = 0$ and $z = \pi r_c$ must be specified for each field propagating in the bulk.

RS1 model:



Scales: $kr_c \approx 10$, $k \lesssim M_{\text{Planck}} \approx 10^{19}$ **GeV**

Fluctuations of the metric:

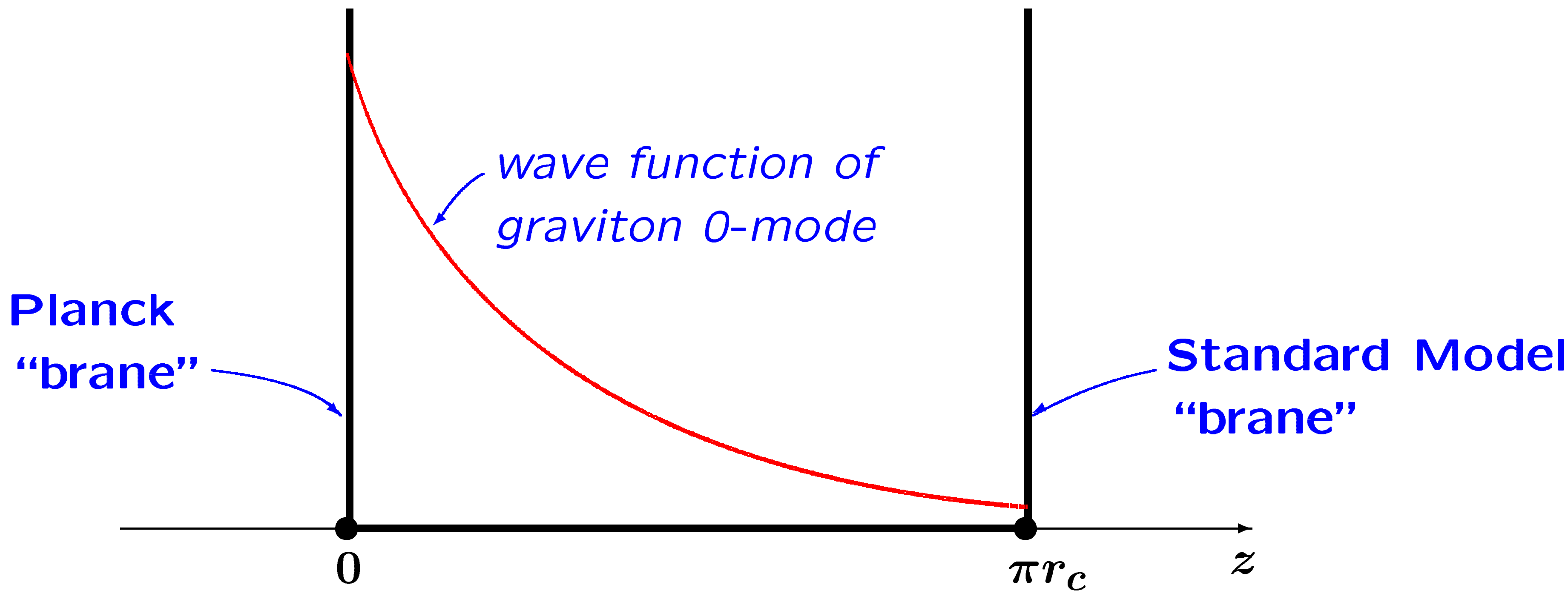
$$ds^2 = e^{-2kz} \left[\eta_{\mu\nu} + h_{\mu\nu}(x)\right] dx^\mu dx^\nu - dz^2$$

$h_{\mu\nu}$ **is the graviton zero-mode, responsible for the long-range gravitational interactions.**

Strength of gravitational force: $G_N \sim 1/M^2_{\text{Planck}}$

Fundamental scale on the Standard Model brane:

$$M_{\text{Planck}} e^{-\pi k r_c} \sim O(1) \text{ TeV}$$



Interaction of the graviton 0-mode (the massless 4D spin-2 field) with Standard Model particles is suppressed by its exponentially small wave function at the SM brane.

Hierarchy between the Planck and electroweak scales is explained!

Comparison between various solutions to the hierarchy problem:

1. Technicolor

Exponential hierarchy between $M_{\rm Planck}$ *and the scale where the technicolor gauge interaction becomes strong.*

Problem: fit to the electroweak data? (some solutions exist)

2. Dynamically-broken supersymmetry

Susy breaking scale is exponentially suppressed compared to $M_{\rm Planck}$ *due to gauge dynamics.*

Problem: μ **term (the Higgsino mass) is supersymmetric.**

Why $\mu \sim v$**? (some solutions exist)**

3. RS1

$1/M_{\rm Planck}$ *is exponentially suppressed compared to* $1/v$.

Energy

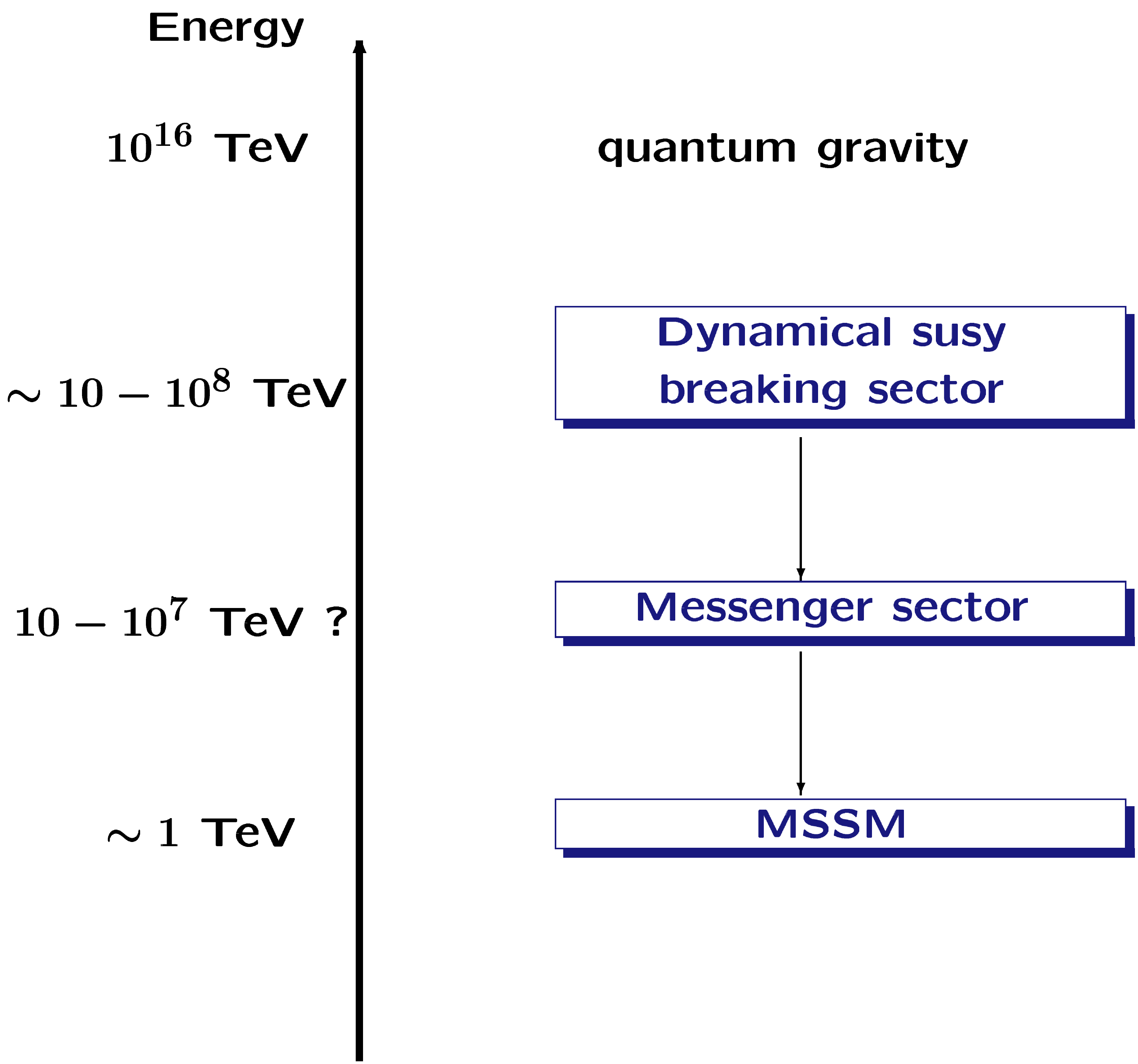
10^{16} **TeV** — **quantum gravity**

Technicolor gauge coupling: $g_{\rm TC} \sim O(1)$

logarithmic running of $g_{\rm TC}$
(increases at lower scales,
just as in QCD)

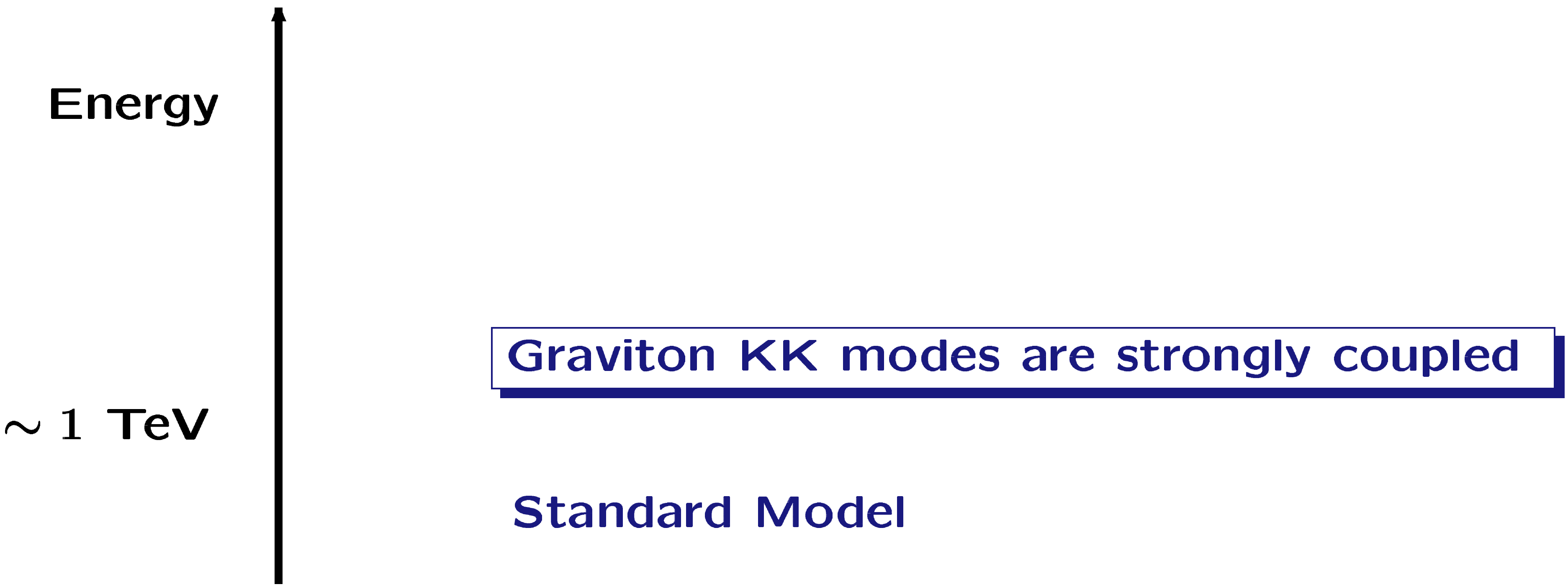
~ 1 **TeV**

$g_{\rm TC} \sim O(4\pi) \Rightarrow$ **Technifermions condense**
$\Rightarrow$ **electroweak symmetry is broken**



Energy
10^{16} TeV
quantum gravity
Dynamical susy
breaking sector
$\sim 10 - 10^8$ TeV
Messenger sector
$10 - 10^7$ TeV ?
MSSM
~ 1 TeV

Scales of RS1 model (measured on the SM brane):



Graviton KK modes

Kaluza-Klein decomposition for the graviton field

$$h_{\mu\nu}(x,z) = \frac{1}{\sqrt{r_c}} \sum_{j\geq 0} h^{(j)}_{\mu\nu}(x)\chi_j(z)$$

$\rightarrow$ A tower of spin-2 resonances

Graviton KK functions (solutions to the Einstein equations in the bulk):

$$\chi_j(z) = \frac{e^{2kz}}{N_j}\left[J_2\left(\frac{m_j}{k}e^{kz}\right) + \alpha_j Y_2\left(\frac{m_j}{k}e^{kz}\right)\right]$$

J_2, Y_2 are Bessel functions; N_j, α_j are normalization constants

m_j is the mass of the jth KK mode of spin-2:

$$m_1 = 3.8\bar{k},\ m_2 = 7.0\bar{k},\ m_3 = 10.2\bar{k},\ \ldots$$

$\bar{k} \equiv k e^{-kr_c\pi} \sim O(1)$ TeV

Gauge fields in a warped extra dimension

Kaluza-Klein decomposition for the gauge fields

$$A_\mu(x^\nu, z) = \frac{1}{\sqrt{2\pi r_c}} \left[A_\mu^{(0)}(x^\nu) + \sum_{j\geq 1} A_\mu^{(j)}(x^\nu) f_j(z) \right]$$

0-mode has a flat profile (unlike the graviton).

KK functions:

$$f_j(z) = \frac{e^{kz}}{N_j} \left[J_1\left(\frac{m_j}{k} e^{kz}\right) - \frac{J_0(m_j/k)}{Y_0(m_j/k)} Y_1\left(\frac{m_j}{k} e^{kz}\right) \right]$$

J, Y are Bessel functions; N_j is a normalization constant

m_j is the mass of the jth KK mode of spin-1:

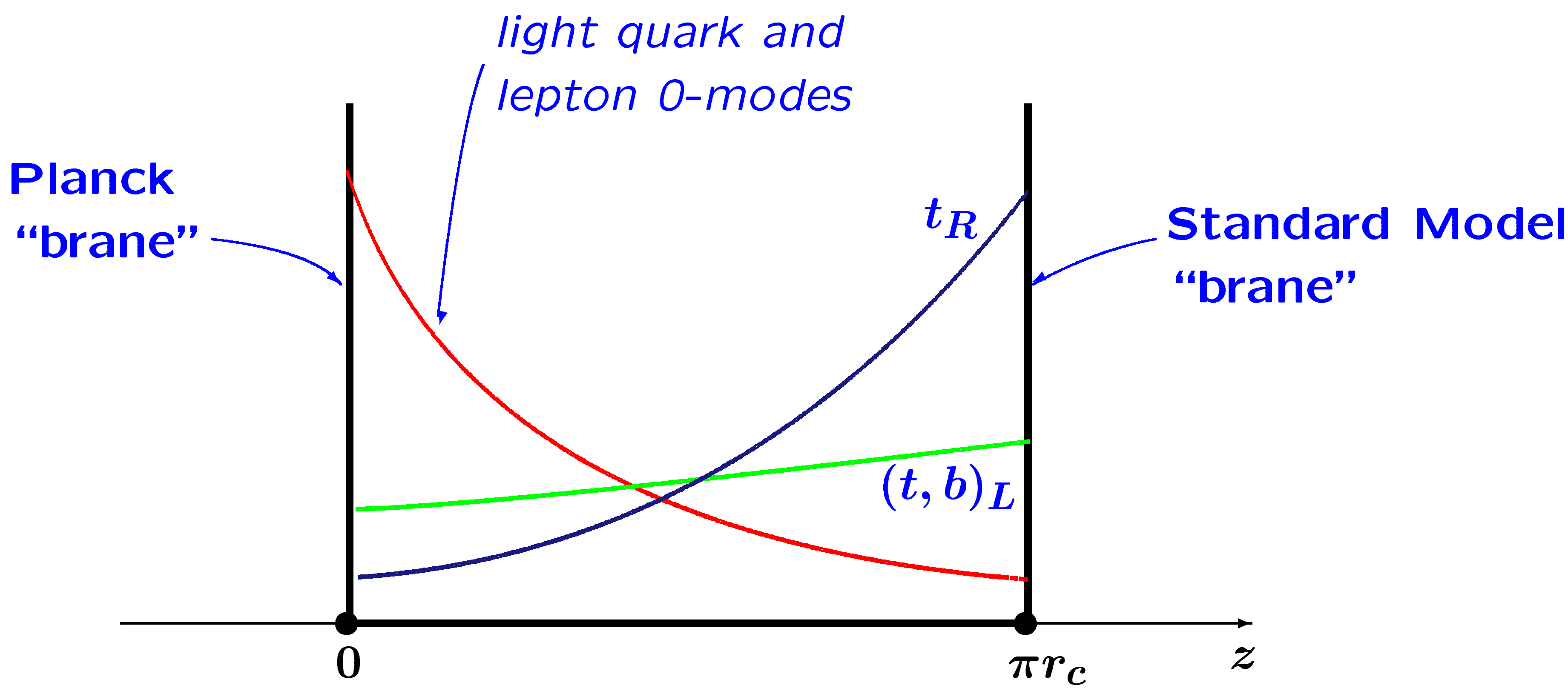
$$m_1 = 2.5\bar{k},\ m_2 = 5.6\bar{k},\ m_3 = 8.7\bar{k},\ \ldots$$

Fermions in a warped extra dimension

S. Chang et al, hep-ph/9912498

Dirac equation in a warped dimension ⇒ fermion zero-modes have a non-trivial profile, peaked on one of the branes.

For each standard model fermion there is a 5D mass parameter which controls the bulk profile.



Standard Model in a warped extra dimension:

unlike UED, there is no KK parity because of the warping.

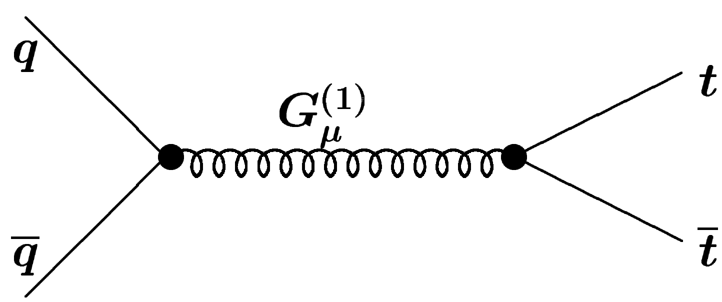
Severe constraints on the KK masses from electroweak fits: $m_1 > O(20)$ TeV

Csaki, J. Erlich and J. Terning, hep-ph/0203034; G. Burdman, hep-ph/0205329

Limits are lowered to $m_1 > O(3)$ TeV if the gauge group in the bulk is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

K. Agashe, et al hep-ph/0308036

Typical signature at the LHC:



The fermion KK modes may have masses below 1 TeV

(Carena, Ponton, Santiago, Wagner, hep-ph/0701055)

Conjecture: **SM in a warped extra dimension is dual to a 4D quasi-conformal strongly-coupled gauge theory**

(a deformation of the AdS/CFT conjecture)

Similar to "walking technicolor"?!

Fields in the 5D picture localized close to the SM brane are composite fields in the conformal theory:

→ Higgs doublet and t_R are composite fields

→ $(t, b)_L$ is partially composite (an admixture of a composite field and a fundamental field).

AdS/CFT interpretation of a warped extra dimension is yet another connection to 4D physics.

Recall from lecture #1:

5D theory = 4D theory with some heavy particles

$SU(3)_c$ **in extra dimensions** $\to$ **SM gluon** $+$ **heavy gluons**

4D theory describing the first N KK modes of the gluon:
$SU(3)_1 \times SU(3)_2 \times \cdots \times SU(3)_{N+1} \to SU(3)_c$ **gauge group, spontaneously broken by the VEVs of scalars transforming as** $(3, \bar{3}, 1, ..., 1)$, **...**

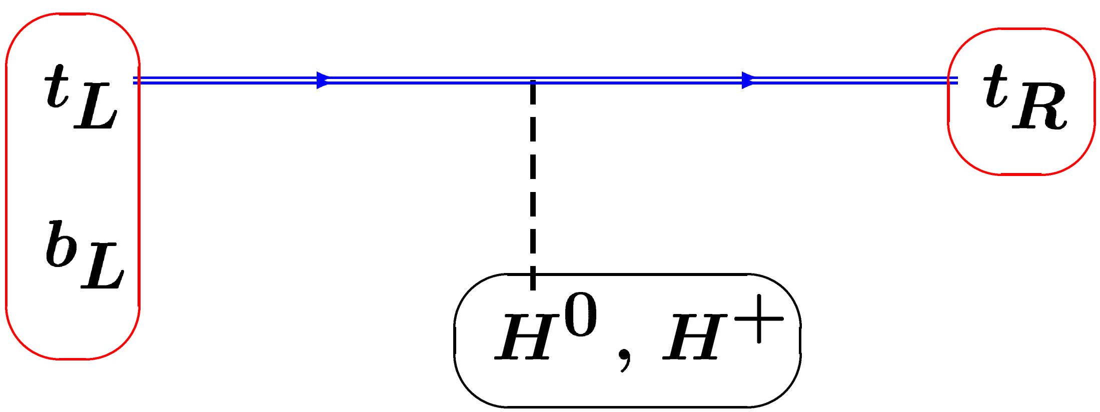
Composite Higgs models

The coupling of the top quark to the Higgs field changes with the distance (similar to vacuum polarization in electrodynamics).

$\lambda_t \, \bar{t}_R \langle H^0 \rangle t_L$, $\langle H^0 \rangle \approx 174$ GeV

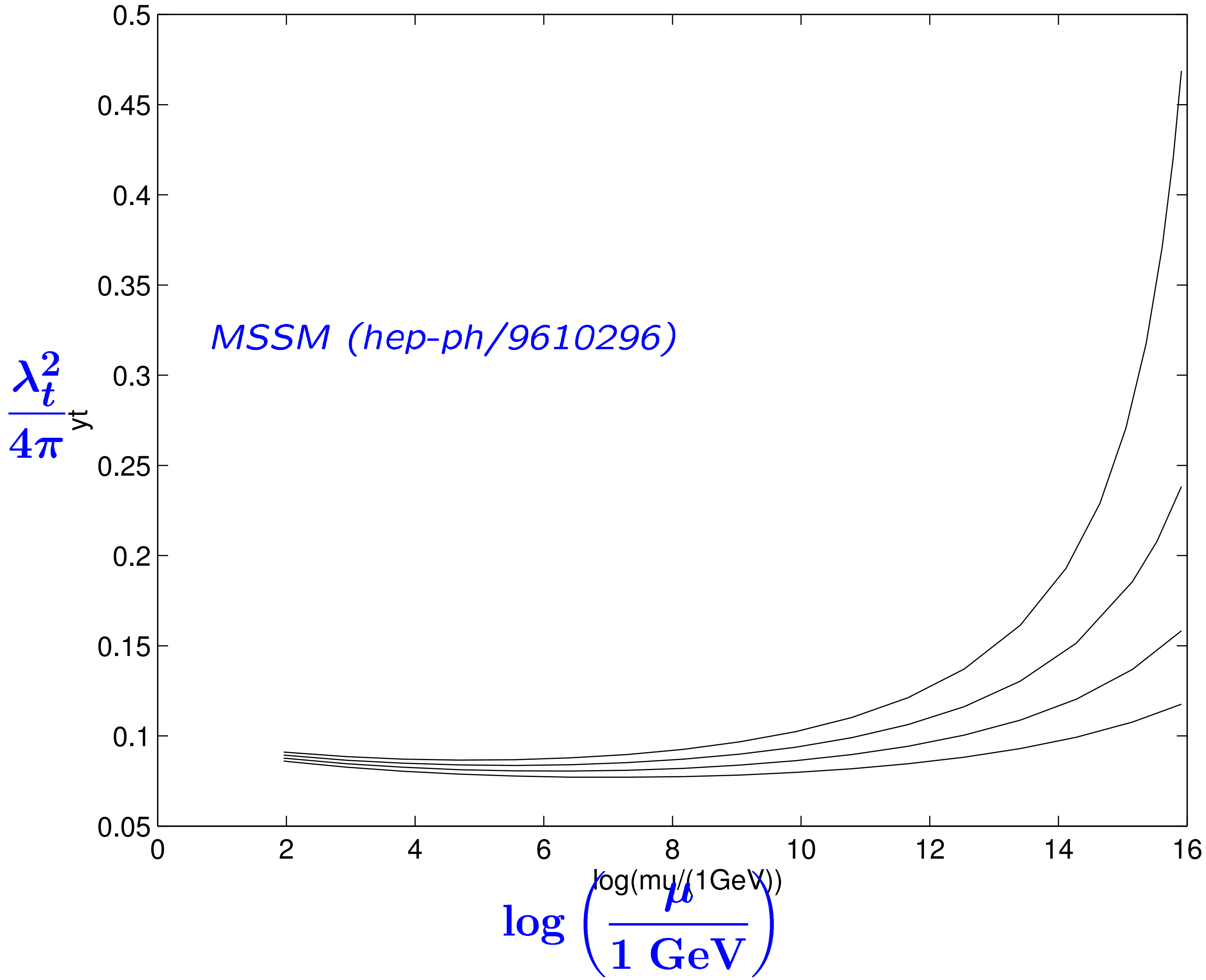
In a world of only top and Higgs:

$$\lambda_t\,(\mu) = \frac{\lambda_t(m_t)}{\sqrt{1 - \frac{9\lambda_t^2(m_t)}{64\pi^2}\ln\frac{\mu}{m_t}}}$$



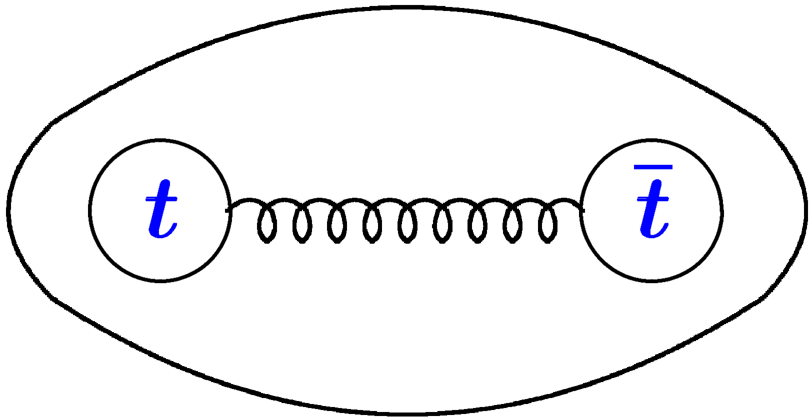
Infrared Fixed Point for λ_t

(C.T. Hill, 1981, ...)



Top condensation ⇒ Higgs boson is a $\bar{t}t$ bound state!
(Bardeen, Hill, Lindner, 1990, ...)

Binding may be due to some strongly-interacting heavy gauge bosons



New heavy quarks (vectorlike) could accelerate the λ_t running:
scale of Higgs compositeness may be as low as a few TeV.

Explicit models: top seesaw, QCD in extra dimensions, ...

Is there a Higgs boson?

EWSB by boundary conditions

Csaki, Grojean, Pilo, Terning: hep-ph/0308038

One warped ED, bulk $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group broken by boundary conditions.

AdS dual of a (walking) technicolor-like theory,
(the presence of the IR brane breaks electroweak symmetry)

$\rightarrow$ lightest W, Z and photon resonances around 1.2 TeV
$\rightarrow$ no fundamental (or composite) Higgs boson

What to look for at colliders

- **Vector-like fermions (KK modes)**

- **New gauge bosons**

(*e.g.*, unitarity restored by Z', W', ...)

- **extended Higgs sectors**

(*e.g.*, radion-Higgs mixing, two composite H, ...)

$\Rightarrow$ **many possibilities!** *Will the experiments be able to differentiate between models?*

Conclusions

- A warped extra dimension provides a nice explanation for the hierarchy problem (related to walking technicolor).

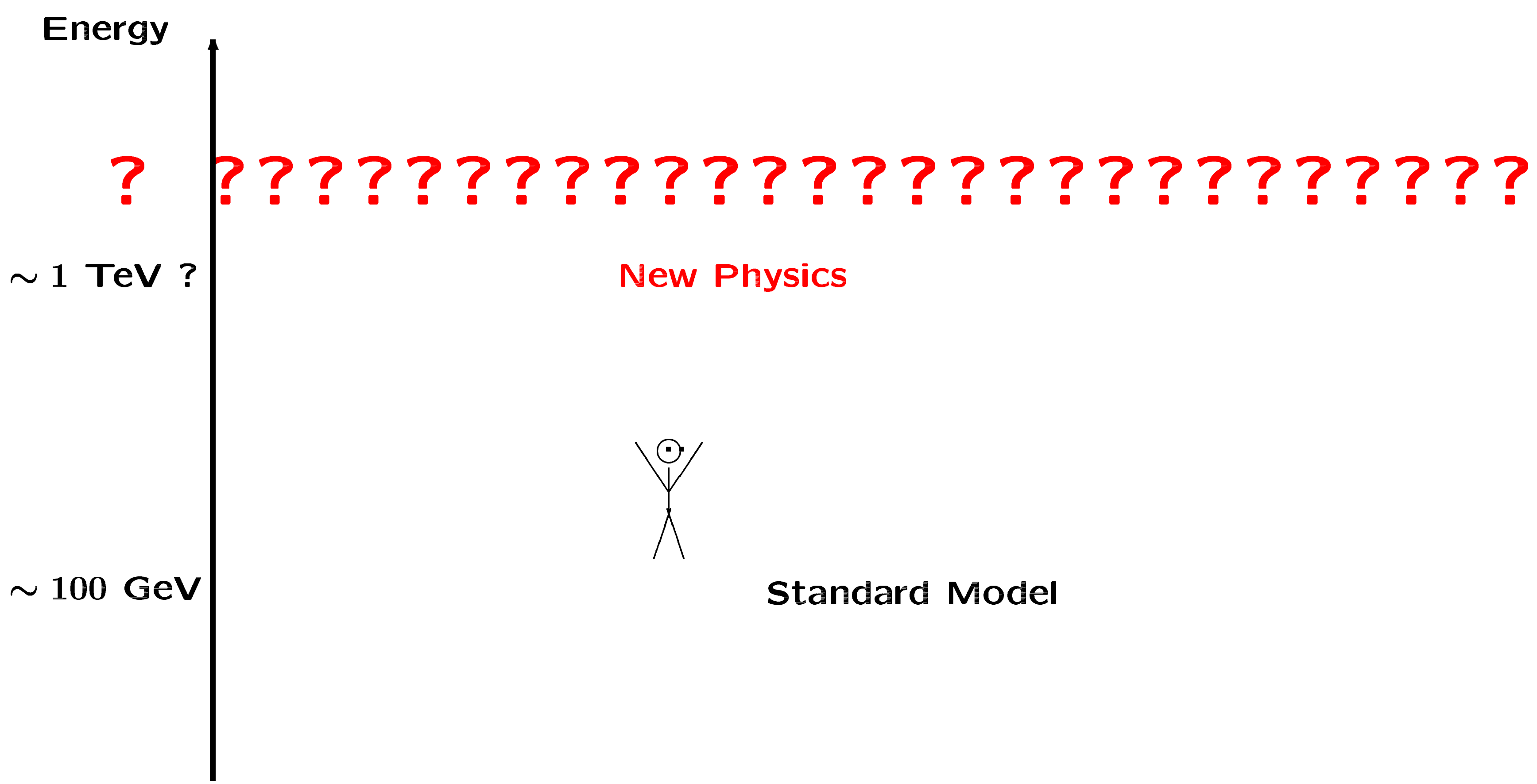
- Physics at the TeV scale may be strongly coupled. Extra dimensional theories provide some examples, as well as some tools for analyzing strong dynamics.

- Any particle that propagates in $D \geq 5$ would appear in experiments as a tower of heavy 4D particles. There are purely 4D theories with similar spectra and interactions, which are interesting whether or not extra dimensions exist in nature!

- Phenomenological implications of extra dimensions are highly model dependent. Even closely related theories, such as 1 versus 2 universal extra dimensions, predict quite different signals.

Final TASI Exam

Find out what theory describes physics at the TeV scale.



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