Gauge (+ Global)

Symmetry Extensions of the Standard Model

(1)

Gauge Extensions

- Why not? ~ Weak Scale: New Ph. expected
- Exact/Approx. Global Symmetries exist
- Larger Symmetry structure/ fewer parameters (unif.)
- Solution to Hierarchy problem (PGB Higgs, Ext. SUSY..)

Either $G_{SM} \times G_{NEW}$, $G_{NEW} \supset G_{SM}$, or in between

SM : $3 \times 2 \times 1$

q $(3, 2, +1/6)$

u^c $(\bar{3}, 1, -2/3)$

d^c $(\bar{3}, 1, +1/3)$

ℓ $(1, 2, -1/2)$

e^c $(1, 1, +1)$

h $(1, 2, -1/2)$ — left-handed Weyl spinor (anti-particle of RH quark)

$u^c = i\sigma^2 u_R^*$

Actually, $U(1)_Y$ is predetermined by $\mathcal{L}$:

$$\mathcal{L} = \frac{1}{2} G^{a}_{\mu\nu} G^{\mu\nu a} + \sum_{a = q, u^c, d^c, \ell, e^c} \Psi_i^{a\dagger} \bar{\sigma} \cdot D \Psi_i^a + |Dh|^2$$

$$+ \left[Y^u_{ij} h^\dagger q_i u^c_j + Y^d_{ij} h q_i d^c_j + Y^e_{ij} h \ell_i e^c_j + \frac{Y^\nu_{ij}}{\Lambda^c} h^\dagger \ell_i h^\dagger \ell_j \right] + \text{h.c.}$$

Non-trivial mixing of $q_{1,2,3}$ → All $U(1)_Y$ must be equal

- thus true for $u^c_{1,2,3}$ & $d^c_{1,2,3}$

→ Same in ν sector

Charges q, u, d, ℓ, e, h — 6 unknowns

$$\left.\begin{aligned} q + u - h &= 0 \\ q + d + h &= 0 \\ \ell + e + h &= 0 \\ 2\ell - 2h &= 0 \end{aligned}\right\} \text{4 eq.s} \rightarrow \text{1 unknown left}$$

1 overall scaling

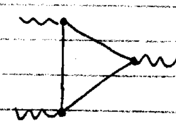


DIAGRAM DOES NOT VIOLATE SYMMETRY
— REGULATOR DOES

$SU(2) \times SU(2) \cdot U(1)$ **Anomaly**

$$\mathrm{Tr}\left[\{T^a, T^b\} Y\right] \propto 3_{\text{gen.}} \times \left[3_{\text{colors}} \times q + 1 \times \ell\right] \stackrel{!}{=} 0$$

$\ell = -3q$

$q + u - \ell = 0$	$4q + u = 0$
$q + d + \ell = 0$	$-2q + d = 0$
$2\ell + e = 0$	$-6q + e = 0$
$\ell = -3q$	

q	u^c	d^c	ℓ	e^c
q	$-4q$	$2q$	$-3q$	$6q$
$\frac{1}{6}$	$-\frac{2}{3}$	$+\frac{1}{3}$	$-\frac{1}{2}$	$+1$

W/ SM content + ν-masses (Maj.), there are NO continuous Global symmetries.

(Exer: check $SU(3)^2 U(1)_Y$ & $U(1)_Y^3$)

New gauge groups either spontaneously broken or new matter is required

Kaplan, Lectures 1, 2, + 3

Simplest example –

ν-masses are Dirac

New RH neutrino, ν^c

All Yukawas (incl. $y^{\nu}_{ij} h^{\dagger} l_i \nu^c_j$) preserve L, B

$SU(3)^2 U(1)_{B-L}$: $3\times\left[2\times(\frac{1}{3}) + 1\times(\frac{-1}{3}) + 1\times(\frac{-1}{3})\right] = 0$

$SU(2)^2 U(1)_{B-L} = 3\times\left[3\times\frac{1}{3} + 2\times(-1)\right] = 0$

⋮

$U(1)^3_{B-L}$ Req. ν^c!

(Ex: Show Any linear combination of $U(1)_Y$ + $U(1)_{B-L}$, $\alpha Y + \beta Q_{B-L}$ satisfies All anomaly cond. incl. $U(1)^2_Y U(1)_{B-L}$, etc..)

→ B−L could be unbroken (w/ tiny coupling const.)

Other Example (the "E_6" $U(1)$)

q, u^c, d^c, l, e^c : charge +1

(two higgses, opp. hypercharge) h_1, h_2 : charge −2

$3\times D^c, \bar{D}$: charge −2

$3\times L, \bar{L}$: charge −2

$3\times X$: charge +4

} Vectorlike under SM — Must Get Mass when E_6 breaks

ASIDE:

No Global symmetries,

No vector-like fermions,

No scalars – Naturalness?

Supersymmetrizing

– Using all LH Weyl spinors allows instant conversion to chiral superfield

$$W = H^c Q U^c + HQD^c + HLE^c + \mu H H^c$$

Extra Higgs often required for couplings / anomalies

New gauge group, New scalar potential ("D-Term")

$$\frac{1}{g^2_{BL}}\left[\frac{1}{3}|\tilde{q}_i|^2 - \frac{1}{3}|\tilde{u}^c_i|^2 - \frac{1}{3}|\tilde{d}^c_i|^2 - |\tilde{\ell}|^2 + |\tilde{e}^c|^2 + |\tilde{\nu}^c|^2\right]$$

($U(1)_X$, linear combo of B−L & Y allows h to be charged)

Froggatt-Nielsen Models –

~~Yukawas forbidden~~

$$\left(\frac{\phi}{M}\right)^n q_1 d^c h + \left(\frac{\phi}{M}\right)^m q_1 s^c h + q t^c h^{\dagger} + \ldots$$

$\epsilon = \langle\phi\rangle / M \sim 1/5$

ϕ carries $U(1)$ charge −1

q_1, d^c, h, etc... carry + charges

Yukawa

$$q\begin{pmatrix} \epsilon^7 & \epsilon^6 & \epsilon^4 \\ & \epsilon^5 & \epsilon^4 \\ & & \epsilon^2 \end{pmatrix} \quad (d^c)$$

→ But Flavor Scale Should not be too close to 1 TeV

[GROSSMAN'S TALK]

Non-Abelian: The Left-Right Model

$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$Q: \begin{pmatrix} u \\ d \end{pmatrix} \quad (3,2,1,1/3)$

$Q^c: \begin{pmatrix} d^c \\ u^c \end{pmatrix} \quad (\bar{3},1,2,-1/3)$

$L: \begin{pmatrix} \nu \\ e \end{pmatrix} \quad (1,2,1,-1)$

$L^c: \begin{pmatrix} e^c \\ \nu^c \end{pmatrix} \quad (1,1,2,+1)$

Higgses: $\phi: (1,2,2,1)$
$\Delta: (1,3,1,+2)$
$\bar{\Delta}: (1,1,3,+2)$

Potential for spontaneously breaking L-R parity ($\neq SU(2)_R$)

$V(\Delta) + V(\bar{\Delta}) + \lambda \Delta^2 \bar{\Delta}^2$ (Ex: check if this works, conditions)

$\langle \bar{\Delta} \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$

$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}$

$\bar{\Delta} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$ ← Doubly charged

$SU(2)_R$ TRANS $\times U(1)_{B-L}$

$\delta\phi \sim \phi \frac{\delta\alpha^a \sigma^a}{2}$

$\delta\bar{\Delta} \sim \left[\frac{\delta\alpha^a\sigma^a}{2}, \bar{\Delta}^c\right] + \delta\beta \cdot 2\bar{\Delta}$ ← B-L

only $\delta\alpha^3$

$\sim \begin{pmatrix} \alpha/2 & \\ & -\alpha/2 \end{pmatrix}\bar{\Delta} - \bar{\Delta}\begin{pmatrix} \alpha/2 & \\ & -\alpha/2 \end{pmatrix} + 2\beta\bar{\Delta}$

$= \begin{pmatrix} 2\beta \bar{\Delta}^+/\sqrt{2} & (\alpha+2\beta)\bar{\Delta}^{++} \\ (-\alpha+2\beta)\bar{\Delta}^0 & 2\beta(-\frac{\bar{\Delta}^+}{\sqrt{2}}) \end{pmatrix}$

FOR $\beta = \alpha/2$, V_3 left invar.

$Y = I_{3_{Right}} + \frac{B-L}{2}$

$\mathcal{L} = y_q q^T \phi \sigma_2 q^c + \tilde{y}_q q^T \sigma_2 \phi^* q^c$
$+ y_\ell \ell^T \phi \sigma_2 \ell^c + \tilde{y}_\ell \ell^T \sigma_2 \phi^* \ell^c$
$+ y_\Delta (\ell^T \sigma_2 \Delta \ell + \ell^{cT} \sigma_2 \Delta^c \ell^c)$
$+ V(\Delta) + V(\Delta^c) + \lambda \Delta^2 \Delta^{c2} + \tilde{V}(\phi)$

$\langle \phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} \quad v = \sqrt{v_1^2 + v_2^2}$

Neutrinos: $\begin{array}{c|cc} & \nu & \nu^c \\ \hline \nu & 0 & \sim y_\ell v \\ \nu^c & \sim y_\ell v & y_\Delta v_R \end{array}$ $\quad m_{\nu_L} \sim \frac{y_\ell^2 v^2}{y_\Delta v_R} \sim \frac{m_\ell^2}{y_\Delta v_R} \ll m_\ell$

Supersymmetrize

$Q: (3,2,1,+1/6)$
$Q^c: (\bar{3},1,2,-1/6)$
$L: (1,2,1,-1/2)$
$L^c: (1,1,2,+1/2)$

Anomalies — Double Higgses

$\phi_{1,2}: (1,2,2,0)$
$\Delta: (1,3,1,+2) \quad \bar{\Delta}: (1,3,1,-2)$
$\bar{\Delta}^c: (1,1,3,-2) \quad \Delta^c: (1,1,3,+2)$

$W = y_a^q Q \phi_a Q^c + y_a^\ell L \phi_a L^c$
$+ y_\Delta (L\Delta L + L^c \Delta^c L^c)$
$+ \mu_{ab} \operatorname{Tr} \phi_a \phi_b + \mu_\Delta (\Delta\bar{\Delta} + \Delta^c \bar{\Delta}^c)$

→ No R-parity violation allowed!

GAUGE EXTENSIONS OF THE SM: UNIFICATION

RECAP – LR MODEL

$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$\phi : (1,\ 2,\ 2,\ 0)$

$\Delta : (1,\ 3,\ 1,\ +2)$

$\Delta^c : (1,\ 1,\ 3,\ +2)$

"Adjoint Rep"

How Does Δ^c transform under $SU(2)_R \times U(1)_{B-L}$?

e.g. $SU(2)_L$ Doublet

$\psi^i \to U^i_j \psi^j$

$= e^{i\alpha^a \sigma^a/2} \psi$

Inf. $\to (1 + i\delta\alpha^a \frac{\sigma^a}{2}) \psi$

$\delta\psi = i\delta\alpha^a \frac{\sigma^a}{2} \psi$

$\phi^{ia} = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}$; $\phi^{ia} \to U^a_b \phi^{ib}$

$= \phi\, e^{i\alpha^a \sigma^{aT}/2}$

Inf. $\delta\phi = \phi \frac{\sigma^{aT}}{2} (i\delta\alpha^a)$

$\Delta^c = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$; $\Delta^c \to U \Delta^c U^\dagger$

$= e^{i\alpha^a \sigma^a/2} \Delta^c e^{-i\alpha^a \sigma^a/2}$

Inf. $= (1 + i\delta\alpha^a \sigma^a/2) \Delta^c (1 - i\delta\alpha^a \sigma^a/2)$

$\delta\Delta^c = i\delta\alpha^a [\sigma^a/2, \Delta^c]$

UNDER $U(1)_{B-L}$ $\Delta^c \to e^{i\beta \cdot 2} \Delta^c$

$\delta\Delta^c = i\beta 2 \Delta^c$

What is left unbroken if

$\langle \Delta^c \rangle = \begin{pmatrix} 0 & 0 \\ V_3 & 0 \end{pmatrix}$ (i.e., what leaves V_3 invar.)

$\delta\Delta^c = i\delta\alpha^a \frac{\sigma^a}{2} \Delta^c - i\delta\alpha^a \Delta^c \frac{\sigma^a}{2} + i\,\delta\beta\, 2\Delta^c$

look at $a = 3$

$= i\delta\alpha^3 \begin{pmatrix} 1/2 & \\ & -1/2 \end{pmatrix} \begin{pmatrix} & \\ V_3 & \end{pmatrix} - i\delta\alpha^3 \begin{pmatrix} & \\ V_3 & \end{pmatrix} \begin{pmatrix} 1/2 & \\ & -1/2 \end{pmatrix} + i\delta\beta \begin{pmatrix} & \\ 2V_3 & \end{pmatrix}$

$= i \begin{pmatrix} 0 & 0 \\ (2\delta\beta - \delta\alpha^3)V_3 & 0 \end{pmatrix}$

$\Rightarrow$ if $\frac{\delta\alpha^3}{2} = \delta\beta$

$$\boxed{Y = I_{3_R} + \frac{B-L}{2}}$$

UNIFICATION – WARM UP

$SU(3)_L \supset SU(2)_L \times U(1)_Y$ — DEEPER EXPLANATION FOR CHARGE QUANT

8 generators (Gellmann Matrices)

$T^a_{1,2,3} = \frac{1}{2} \begin{pmatrix} \sigma^a_{2\times 2} & 0 \\ 0\ 0 & 0 \end{pmatrix}$ $T^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} -1/2 & & \\ & -1/2 & \\ & & +1 \end{pmatrix}$

3 of $SU(3)_{L}$ — NORMALIZED so $\mathrm{Tr}\, T^a T^b = \frac{1}{2}\delta^{ab}$ ← "Dynkin Index"

$\psi^i = \begin{pmatrix} l \\ e^c \end{pmatrix}$

one for each generation

Breaking $SU(3) \to SU(2) \times U(1)$

adjoint or "8" of $SU(3)$: $\langle \Phi^i_j \rangle = \begin{pmatrix} -f/2 & & \\ & -f/2 & \\ & & +f \end{pmatrix}$ ← $\Phi \to U \Phi U^\dagger$ ($T^{a=1,2,3,8}$ leaves invariant)

$|D\Phi|^2 = \mathrm{Tr}|\partial\Phi + ig[A^a T^a, \Phi]|^2$

$\to g^2 |\mathrm{Tr}[T^a, T^8] A^a|^2 \cdot 3f^2$

$a = 4,5,6,7 \to$ Get Mass

$A_\mu = \begin{pmatrix} W_\mu, B_\mu & X_\mu \\ X_\mu & B_\mu \end{pmatrix}$

WHAT ARE THE RESULTING GAUGE COUPLINGS?

look at $\psi^\dagger \sigma \cdot D \psi$ (or $\bar{\Psi} \not{D} P_L \psi$)

$\to (\partial_\mu + i g_3 T^a A^a_\mu + i g_3 T^8 A^8_\mu) \psi$ ($A^a_\mu \to W_\mu$, $A^8_\mu \to B_\mu$)

look at l: $(\partial_\mu + i g_3 \frac{\sigma^a}{2} W^a_\mu + \frac{g_3}{\sqrt{3}} (\frac{-1}{2}) B_\mu) l$

$g_2 = g_3 = g$; $g_1 = g_3 = \sqrt{3} g'$

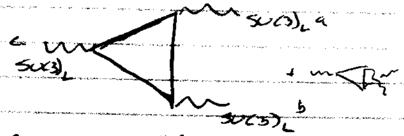
$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2}$

$= \frac{g_3^2/3}{g_3^2 + g_3^2/3} = \frac{1}{4}$

COMPARE W/ .23!

$\boxed{g = g_3}$ $\boxed{g' = g_3/\sqrt{3}}$

Issues – Anomalies



Note: $SU(3)_c^2\, SU(3)_L = 0$ etc...

~~Could add $\psi^c(\bar{3})$, but that~~ would be vector-like

$Tr\left[\{T_a, T_b\} T_c\right] \equiv d_{abc}$

(non-zero for a single triplet)

$Tr\left[\{T_a^r, T_b^r\} T_c^r\right] = A(r)\, d_{abc}$

T^r: $n \times n$, n is the dim; $A(r)$: the "Anomaly" of a rep

Can prove

$A(\bar{r}) = -A(r)$

$A(r_1 \oplus r_2) = A(r_1) + A(r_2)$

$A(r_1 \otimes r_2) = \dim(r_1) A(r_2) + \dim(r_2) A(r_1)$

Building other reps

$$\psi^i \chi^j = \left(\frac{\psi^i\chi^j + \psi^j\chi^i}{2}\right) + \left(\frac{\psi^i\chi^j - \psi^j\chi^i}{2}\right)$$

9 things; Sym. → 6, S^{ij}; Antisym. → $\bar{3}$, A^{ij}

Transformations

"fundamental" (3) $F^i \to U^i{}_j F^j$

Inf $\to (\delta^i{}_j + i\delta\alpha^a (T^a)^i{}_j) F^j$

$(\bar{3})$ $\bar{F}_i \to \bar{F}_j (U^\dagger)^j{}_i$

Inf $\to (\delta^{ij} - i\delta\alpha^a (T^{a*})_i{}^j)\bar{F}_j$

(6) $S^{ij} \to U^i{}_k U^j{}_l S^{kl}$

$(\bar{3})$ $A^{ij} \to U^i{}_k U^j{}_l A^{kl}$

$F^i \bar{F}_i$ invariant

$\bar{F}_i \bar{F}_j S^{ij}$ invariant

$A^{ij} F^k \epsilon_{ijk}$ "

Ex: Prove ϵ_{ijk} is SU(3) invariant

Could write $\bar{A}_k = A^{ij}\epsilon_{ijk}$ ← $\bar{F}$ $\bar{3}$

$$3 \otimes 3 = 6 \oplus \bar{3}$$

6 of SU(3) → The Anomaly?

$A(3\otimes 3) = 3A(3) + 3A(3)$
$= A(6\oplus\bar{3}) = A(6) + A(\bar{3}) = A(6) - A(3)$

$A(6) = 7A(3) = 7$

Anomaly Free SU(3): a 6 and ~~7~~ 7 $\bar{3}$'s

What's in a 6 (in terms of $SU(2)\times U(1)$)

$(3\otimes 3) = (2_{-1/2}\oplus 1_{+1})\otimes(2_{-1/2}\oplus 1_{+1})$

(ℓ, e^c) $6\oplus\bar{3} = 3_{-1}\oplus 1_{-1}\oplus 2_{+1/2}\oplus 2_{+1/2} + 1_{+2}$
$= (3_{-1}\oplus 2_{+1/2}\oplus 1_{+2}) \oplus (2_{+1/2}\oplus 1_{-1})$
6 $\bar{3}$

Nothing useful here

Hypercharge comes in units of 1/2 → bad for quarks

Try a different ~~em~~ embedding

$\begin{pmatrix} a & & \\ & a & \\ & & -2a \end{pmatrix}$

Look @ ~~3⊗3~~ $3^i \times \bar{3}_j = 8 + 1$

$\Sigma^i{}_j = \tilde{\Sigma}^i{}_j + \frac{1}{3}\delta^i{}_j Tr(\Sigma)$

$(2_{a} + 1_{-2a}) \times (2_{-a} + 1_{+2a})$

if $a = 1/6$: ℓ ← $= (3_0 \oplus 1_0 \oplus 2_{3a} \oplus 2_{-3a}) \oplus 1_0$
8 1

Ex: find a spot for all of $q\, u^c d^c \ell\, e^c$

Pati-Salam

parity symmetric

$$SU(4)_c \times SU(2)_L \times SU(2)_R$$

↳ "Lepton number as the fourth color"

$T^{a}_{1\ldots 15} \to \begin{pmatrix} SU(3) & \vdots \\ \cdots & \cdot \end{pmatrix}$ (4x4)

$Q: (4, 2, 1)$

$Q^c: (\bar{4}, 1, 2)$

$Q: \begin{pmatrix} q \\ \ell \end{pmatrix}$ ($SU(3)$, $SU(2)_L$) $\qquad \begin{pmatrix} d^c & u^c \\ e^c & \nu^c \end{pmatrix}$ ($SU(2)_R$)

Hypercharge must contain T^{15} — must contain T^3_R too

$$T^{15} = \sqrt{\tfrac{3}{2}} \begin{pmatrix} 1/6 & & & \\ & 1/6 & & \\ & & 1/6 & \\ & & & -1/2 \end{pmatrix}$$

$$\delta Q^c = i\delta\alpha T^{15} Q^c - i\delta\beta Q^c T^3_R$$

ν^c should have $Y=0$

$$\left[\delta\alpha \sqrt{\tfrac{3}{2}} \begin{pmatrix} 1/6 & & & \\ & 1/6 & & \\ & & 1/6 & \\ & & & -1/2 \end{pmatrix}\begin{pmatrix} \vdots & \vdots \\ \cdot & \nu^c \end{pmatrix} - \delta\beta \begin{pmatrix} \vdots & \vdots \\ \cdot & \nu^c \end{pmatrix}\begin{pmatrix} 1/2 & \\ & -1/2 \end{pmatrix}\right]_{4,2}$$

(RH ν^c component)

$$= -\sqrt{\tfrac{3}{2}}\frac{\delta\alpha}{2} + \frac{\delta\beta}{2} \overset{?}{=} 0 \Rightarrow \delta\beta = \sqrt{\tfrac{3}{2}}\,\delta\alpha$$

$$Y = \sqrt{\tfrac{2}{3}}\, T^{15} + T^3_R$$

→ CHECK hypercharge of e^c, d^c, u^c (Ex:)

$H: (1,2,2)$

$\Phi^c: (\bar{4},1,2)$ ← VEV here

$\Phi: (4,2,1)$

ASIDE: This is the "minimal" setup. Other Higgs reps can be used

$\to \mathrm{Tr}\left[\begin{pmatrix} q \\ \ell \end{pmatrix}(h_1 h_2)\begin{pmatrix} -d^c & e^c \\ u^c & \nu^c \end{pmatrix}\right]$

$$\mathcal{L} \sim (y\, Q H Q^c + \tilde{y}\, Q \sigma_2 H^* \sigma_2 Q^c) + h.c. + V(\Phi, \Phi^c, H)$$

(Z_2-symmetric)

~~Exercise:~~

Ex: Supersymmetrize

Couplings $\quad g_4,\ g_L = g_R \qquad$ VEV $\langle \Phi^c \rangle = \begin{pmatrix} & \\ & v \end{pmatrix}$

$D\Phi^c$

$\to \left(g_4 \sqrt{\tfrac{3}{2}}\left(\tfrac{1}{2}\right) A^{15}_\mu + g_{LR}\left(\tfrac{1}{2}\right) A^3_R\right)$ gets a mass

NORMALIZE

$$\to A_{Heavy} = \frac{\sqrt{\tfrac{3}{2}}\, g_4 A^{15} + g_{LR} A^3_R}{\sqrt{\tfrac{3}{2} g_4^2 + g_{LR}^2}}$$

$$A_Y = \frac{+g_{LR} A^{15} + \sqrt{\tfrac{3}{2}}\, g_4 A^3_R}{\sqrt{\tfrac{3}{2} g_4^2 + g_{LR}^2}}$$

Can compute SM couplings

$\to g_3 = g_4\ ;\ g = g_{LR}$

To get g', look at gauge coupling of e^c

$$DQ^c \to (i g_4 T^{15} A^{15}) Q^c + (i g_{LR} Q^c A^3_R T^3_R)$$

$$\Rightarrow \left(i g_4 \sqrt{\tfrac{3}{2}}\left(\tfrac{1}{2}\right) A^{15} + i g_{LR}\left(\tfrac{1}{2}\right) A^3_R\right) e^c$$

$$= i\frac{g_{LR}\sqrt{\tfrac{3}{2}}\, g_4}{\sqrt{\tfrac{3}{2} g_4^2 + g_{LR}^2}}\, e^c \equiv i g' e^c$$

$$\frac{g^2}{g'^2} = \cot^2\theta_W = 1 + \frac{2}{3}\frac{g^2}{g_3^2}$$

$\to \sin^2\theta_W \sim .45$ running...

(7′)

Proton Decay? $\not{L}$ but not $\not{B}$

$\mathcal{L}$ has a classical global $U(1)_X$ symmetry

Q: X charge +1

Q^c: X charge −1

$$\boxed{B = \sqrt{6}\, T^{15} + X}$$

SU(5): "We present a series of hypotheses and speculations leading inescapably to the conclusion that SU(5) is the gauge group of the world." –

T^{ij} (10) (anti-sym): $\begin{pmatrix} 0 & u^c_3 & -u^c_2 & u & d \\ & 0 & u^c_1 & u & d \\ & & 0 & u & d \\ - & & & 0 & e^c \\ & & & & 0 \end{pmatrix} \sim \begin{pmatrix} u^c & q \\ -q & e^c \end{pmatrix}$

$\bar{F}_i$ ($\bar{5}$): $\begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ e \\ \nu \end{pmatrix} \sim \begin{pmatrix} d^c \\ \ell \end{pmatrix}$

Anomaly Free?

Look at SU(3) subgroup

$5^i \times 5^j = \underset{sym}{15} + \underset{asc}{10}$

$(3+1+1)\times(3+1+1) = 6 + \bar{3}_{a.s.} + 3 + 3 + 3 + 3 + 1 + 1 + 1 + 1$ → 15, 10

$A(10) = A(\bar{3}+3+3+1)$ ($SU(5)^3$, $SU(3)^3$)

$= A(\bar{3}) + A(3) + A(3) = A(3) = 1$

$A(\bar{5}) = A(\bar{3}) = -1$

✓ Anom. Free!

(8′)

No Extra Matter (singlet could be added)

$H(\bar{5})$: $\begin{pmatrix} h_T \\ h \end{pmatrix}$ ← extra color higgs h_T: $(\bar{3}, 1, +1/3)$

$\Phi(24)$: ⟶ $\langle\Phi\rangle = \begin{pmatrix} v_5 & & & & \\ & v_5 & & & \\ & & v_5 & & \\ & & & -\tfrac{3}{2}v_5 & \\ & & & & -\tfrac{3}{2}v_5 \end{pmatrix}$

$T^{24} = \sqrt{\tfrac{3}{5}} \begin{pmatrix} -1/3 & & & & \\ & -1/3 & & & \\ & & -1/3 & & \\ & & & +1/2 & \\ & & & & +1/2 \end{pmatrix}$

$\mathcal{L} \sim y^u T^{ij} T^{kl} H^m \epsilon_{ijklm} + y^d T^{ij} \bar{F}_i H_j + V(H, \Phi)$ (↑ invar.)

RH ν → $y^\nu \bar{F}_i N H^i + M N N$

– all yukawas allowed!

Generators

$\begin{pmatrix} SU(3) & X, Y \\ & SU(2) \end{pmatrix}$

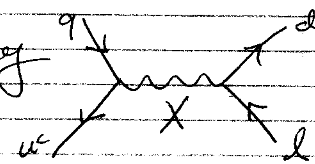
$5 \times \bar{5} = 24 + 1$

$[(3,1)_{-1/3} \oplus (1,2)_{1/2}] \times [(\bar{3},1)_{+1/3} \oplus (1,2)_{-1/2}]$

$= [(8,1)_0 + (1,1)_0 + (3,2)_{-5/6} + (\bar{3},2)_{+5/6} + (1,3)_0] + (1,1)_0$

($G^{\mu\nu}$, $B^{\mu\nu}$, X,Y, $W^{\mu\nu}$; 24)

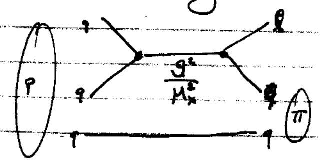
$\not{B}, \not{L}$ ⟹ Proton Decay



$p \to \pi^+ \nu$

ASIDE: B−L does NOT commute w/ SU(5)

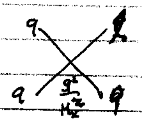
proton decay rate



$\Gamma \sim \frac{g^4}{8\pi}\frac{m_p^5}{M_X^4}$ × overlap

$\tau_{p\to\pi e} \simeq \frac{1}{\Gamma} \sim 2\times10^{34} \times \left(\frac{M_X}{10^{16}\,\mathrm{GeV}}\right)^4$ yr.

DIM 6



Bound $\tau > 10^{32}$ y   ROUGH!

$(M_X \gtrsim 10^{15}\,\mathrm{GeV})$

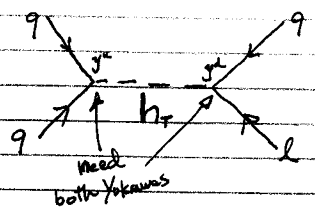
The extra color-Higgs:

$y^u T^{ij} T^{kl} H^m \epsilon_{ijklm}$ +

$q u^c h^\dagger + u^c e^c h_T^\dagger + q q h_T^\dagger \epsilon_{ijk}$ ← $SU(3)$

$y^d T^{ij} \bar{F}_i H_j^\dagger$

$q d^c h + \ell e^c h + q \ell h_T + u_i^c d_j^c h_{T_k} \epsilon^{ijk}$



$(u)ud \to (u)s\bar\nu$ $p \to K^+\bar\nu$

$\Rightarrow \mathcal{L} \sim \frac{y^u y^d}{M_{H_T}^2}\, q\, u^i d^j s^k \nu\, \epsilon_{ijk}$

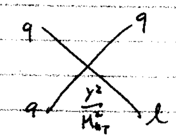
$y^u, y^s \ll 1$ $\frac{(y_u y_s)^2}{\ldots}$

$\tau_{p\to K\nu} \sim 2\times10^{34} \left(\frac{M_{H_T}}{10^{16}\,\mathrm{GeV}}\right)^4 \times 4\times10^{-16}$ yr

$\sim 10^{19} \left(\frac{M_{H_T}}{10^{16}\,\mathrm{GeV}}\right)^4$ y

$= 10^{31} \left(\frac{M_{H_T}}{10^{13}\,\mathrm{GeV}}\right)^2$ yr

DIM 6



ROUGH

However, $M_{H_T} \gg M_H$
← SAME MULTIPLET ~ "Doublet-Triplet splitting problem"

Non-susy, Hierarchy problem
~ was h tuned to be light?

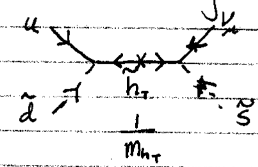
$\left(\frac{M_h}{M_{GUT}}\right)^2 \sim 10^{28}$

If a symmetry protects h, why not h_T?

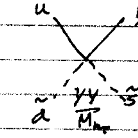
Now, SUPERSYMMETRIZE

$W = y^u T T \bar{H} + y^d T \bar{F} H + \mu H \bar{H}$ ← D-T split?

Proton Decay

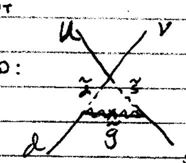


→



DIM 5!

Close loop:



$\Gamma \sim \frac{m_p^5}{8\pi M_{H_T}^2} \left(\frac{m_{\tilde{g}}}{m_{\tilde{q}}^2}\right)^2 \left(\frac{\alpha_s}{4\pi}\right)^2 (m_s m_d G_F)^2$ × (fudge)

for $m_{\tilde{g}} \sim m_{\tilde{q}} \simeq M_{SUSY}$

$\tau \sim 10^{28}\,\mathrm{y} \times \left(\frac{M_{H_T}}{10^{16}\,\mathrm{GeV}}\right)^2 \left(\frac{M_{SUSY}}{1\,\mathrm{TeV}}\right)^2$

Powers of $\left(\frac{f_\pi}{m_p}\right)$

$\langle 0 | uud | p \rangle \sim 5\times10^{-3}$ GeV

$\to \tau \sim 10^{31\pm2}\,\mathrm{y} \left(\frac{M_{H_T}}{10^{16}\,\mathrm{GeV}}\right) \left(\frac{M_{SUSY}}{1\,\mathrm{TeV}}\right)^2$

$\begin{pmatrix} s_{s_{s_s}} \end{pmatrix}$ $\begin{pmatrix} \theta \end{pmatrix}$ → fine tune

D-T splitting → $\bar{H} S H + \bar{H} \Theta H$

$\uparrow 24$

OR

e.g. Break SU(5) w/ 75, 50, $\overline{50}$

$\Sigma^{\alpha\beta}_{\gamma\delta}$ (Traceless) $\Theta^{\alpha\beta}_{\gamma\delta\sigma}$ (Traceless, Anti-symmetric) $\bar{\Theta}$

$$W = \lambda \Theta^{\alpha\beta}_{\gamma\delta\sigma} \Sigma^{\gamma\delta}_{\alpha\beta} \bar{H}^{\sigma} + \lambda' \bar{\Theta}^{\gamma\delta\sigma}_{\alpha\beta} \Sigma^{\alpha\beta}_{\gamma\delta} H_{\sigma} + M \Theta\bar{\Theta} + f(\Sigma)$$

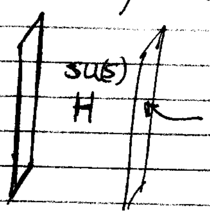
(3×2)

$\Sigma \supset (1,1) + (3,1) + (3,2) + (\bar{3},1) + (\bar{3},2) + (8,\bar{3}) + (6,2) + (\bar{6},2) + (8,1)$

$\Theta \supset (8,2) + (6,3) + (\bar{6},1) + (3,2) + (\bar{3},1) + (1,1)$

many H_T w/ these but there's no (1,2)

Real way to do it (DOBRESCU)



BOUNDARY CONDITIONS - ONLY 3×2×1 Preserved

X_μ, h_T : Dirichlet B.C.

(DOBRESCU)

The Weak mixing angle in SU(5)

$g = g_5$; $g' = \sqrt{\frac{3}{5}} g_5$ from T^{24}

$$\sin^2\theta_W = \frac{3/5}{1 + 3/5} = 3/8 \quad \text{(They quoted)}$$

Gauge Coupling Unification

$t = \ln\mu$ (renorm. scale) $\quad \frac{d\alpha_i}{dt} = \frac{1}{2\pi} b_i \alpha_i^2$

b_i = gauge bosons + matter/higgses

$= -\frac{11}{3} N + \frac{2}{3} \sum_f T_f(r)$ WEYL FERMION

$+ \frac{1}{3} \sum_s T_s(r)$ COMPLEX SCALAR

Dynkin Index with $T(r) = \frac{1}{2}$ for the fund. rep. $= N$ for the adjoint

$SU(3)_c$

$b_3 = -\frac{11}{3}\cdot 3 + \left[\frac{2}{3}\cdot\frac{1}{2}\cdot 12\right]$

$= -11 + 4 = -7$

$SU(2)_L$

$b_2 = -\frac{11}{3}\cdot 2 + \frac{2}{3}\cdot\frac{1}{2}\cdot 4 \times 3 + \frac{1}{3}\cdot\frac{1}{2}$ (Higgs; Dynkin; q_i, ℓ_i; gen.)

$= -\frac{22}{3} + 4 + \frac{1}{6} = -\frac{19}{6}$

Matter runs the three couplings equally! ($b_{mat} = 4$)

$$\frac{d\alpha'}{dt} = \frac{1}{2\pi} b' \alpha'^2$$
$$\alpha' = \frac{3}{5}\alpha_1 \Rightarrow b_1 = \frac{3}{5} b'$$
$$\frac{d\alpha_1}{dt} = \frac{1}{2\pi} b_1 \alpha_1^2$$

b'

$= 0 + \frac{2}{3}\left[3\cdot 2\cdot(\frac{1}{6})^2 + 3\cdot(\frac{2}{3})^2 + 3\cdot(\frac{1}{3})^2 + 2\cdot(\frac{1}{2})^2 + 1\cdot(1)^2\right] \times 3$

(q, u, d, ℓ, e)

$+ \frac{1}{3} 2\cdot(\frac{1}{2})^2$

$= 2\cdot(1/6 + 4/3 + 1/3 + 1/2 + 1) + 1/6 = 20/3 + 1/6$

$b_1 = \frac{3}{5} b' = 4 + \frac{1}{10}$

Supersymmetry

SU(N) ; $T(\text{Adj.}) = N$

→ gauginos

* Every f has a s

$$b_i = -3N + \sum_r T(r)$$

← # of chiral superfields

$b_3 = -9 + 6 = -3$

$b_2 = -6 + 6 + 1 = +1$

$b' = 0 + 10 + 1 = +11$ ← 2 Higgs chiral S-fields

$b_1 = 0 + 6 + \frac{3}{5} = 33/5$

→ INCOMPLETE MULTIPLETS

$$2\pi \frac{d(\alpha_i^{-1})}{dt} = -b_i \Rightarrow \alpha_i^{-1}(M_Z) = \alpha_G^{-1}(M_{GUT}) + \frac{b_i}{2\pi} \ln \frac{M_{GUT}}{M_Z}$$

3 eq.s ; 2 unknowns

2nd →
$$\alpha_1^{-1} - \alpha_2^{-1} = \frac{(b_1 - b_2)}{2\pi} \ln \frac{M_{GUT}}{M_Z}$$

59.4 ± .1 ; 29.75 ; $\left(\frac{1}{2\pi}\frac{28}{5}\right)$

$$\Rightarrow \frac{M_{GUT}}{M_Z} \approx 2.8 \times 10^{14}$$

$M_{GUT} \simeq 2.6 \times 10^{16}$ GeV

5th →
$$\frac{\alpha_1^{-1} - \alpha_G^{-1}}{\alpha_2^{-1} - \alpha_G^{-1}} = \frac{b_1}{b_2}$$

$$\alpha_G^{-1} = \frac{\alpha_2^{-1} b_1 - \alpha_1^{-1} b_2}{b_1 - b_2}$$

$= 24.5$

$$\alpha_3^{-1}(M_Z) = 24.5 + \frac{b_3}{2\pi} \ln(2.8 \times 10^{14})$$

$\alpha_3(M_Z) = .116$

$\alpha_3^{exp} = .117 \pm .002$!

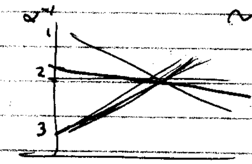
2-loop corrections important

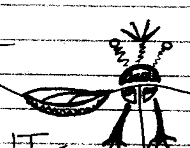
$\sim \alpha^2 \ln(\mu/M)$ (pushes ↑ $\alpha_3 \sim .130$!)

1-loop finite important → GUT thresholds

$\sim \alpha$ → same order

(since $\frac{\alpha}{2\pi} \ln(\mu/M) \sim 1$)





Other GUTs

$y TT\bar{H} + y T\bar{F}H$ ← d + l

y_b vs. y_τ → approximate unification

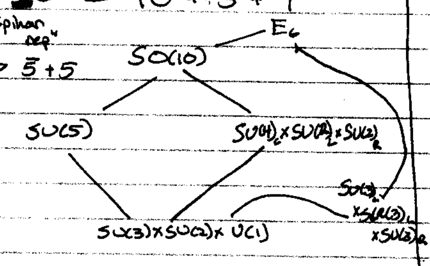
$y_{s,d}$ vs. $y_{\mu,e}$ → NOPE

SO(10)

16 ⊃ 10 + $\bar{5}$ + 1 (ν^c) [SU(5)]

"spinor rep"

10 ⊃ $\bar{5}$ + 5



$\frac{M_{GUT}}{M_{Pl}} \sim 10^{-2} - 10^{-3}$

→ higher dim. op.s could affect

E_6

27 ⊃ 16 + 10 + 1

BIGGER than E_6 → All vector-like (Real reps.)