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TASI Lectures on Holographic Space-time, SUSY, and Gravitational Effective Field Theory

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Abstract

I argue that the conventional field theoretic notion of vacuum state is not valid in quantum gravity. The arguments use gravitational effective field theory, as well as results from string theory, particularly the AdS/CFT correspondence. Different solutions of the same low energy gravitational field equations correspond to different quantum systems, rather than different states in the same system. I then introduce *holographic space-time* a quasi-local quantum mechanical construction based on the holographic principle. I argue that models of quantum gravity in asymptotically flat space-time will be exactly super-Poincare invariant, because the natural variables of holographic space-time for such a system, are the degrees of freedom of massless superparticles. The formalism leads to a non-singular quantum Big Bang cosmology, in which the asymptotic future is required to be a de Sitter space, with cosmological constant (c.c.) determined by cosmological initial conditions. It is also approximately SUSic in the future, with the gravitino mass $K\Lambda^{1/4}$.

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1 Vacuum states in non-gravitational quantum field theory

QFTs in fixed space-time backgrounds, like Minkowski space, often exhibit the phenomena of degenerate and/or meta-stable vacuum states. In the semi-classical approximation these are solutions of the field equations that preserve all the isometries of the background, and for which there are no exponentially growing small fluctuations. Typically, this requires the model to contain fundamental scalar fields. The potential energy density is a function of these scalars, and multiple solutions occur when this function has multiple minima.

In the semi-classical approximation, this is evidence for multiple *superselection sectors* of the QFT: the Hilbert space breaks up into a direct sum of spaces, each associated with a different minimum. In the infinite volume limit, transitions between sectors vanish because the Hamiltonian is an integral of a local energy density. Actually, this is only true in perturbation theory around the true minima. When non-perturbative physics is taken into account, there are generally bubble nucleation processes, which signal an instability of all but the lowest energy minima. Superselection sectors only exist for minima which are exactly degenerate, including all quantum corrections to the energy (the energy differences between semi-classical vacua do not suffer from renormalization ambiguities).

A more non-perturbative view of these phenomena is afforded by the Wilsonian definition of quantum field theory. A general QFT is defined by a relevant perturbation of a CFT. CFT's in turn are defined by their spectrum of conformal primary operators and their operator product expansions (OPEs). In particular, this includes a list of all the relevant operators, which might be added as perturbations of the CFT, using the GellMann-Low formula to compute the perturbed Green's functions. The OPE allows us to perform these computations. Although there is no general proof, it is believed that these conformal perturbation expansions are convergent in finite volume.

The CFT has a unique conformally invariant vacuum state, which is the lowest energy state if the theory is unitary. However, in the infinite volume limit the Hilbert space of the perturbed theory might again separate into superselection sectors. It might also/instead have meta-stable states, but meta-stability always depends on the existence of a small dimensionless parameter, the life-time of the meta-stable state in units of the typical time scale in the model. In most explicit examples, this parameter is a semi-classical expansion parameter for at least some of the fields in the theory.

The following general properties of degenerate and meta-stable vacua in QFT, follow from these principles:

- The short distance behavior of Green's functions, and the high temperature behavior of the partition function of the theory are independent of the superselection sector. Both are controlled by the CFT. The partition function in finite volume V has the asymptotic form

  $$Z = e^{-cV^{\frac{2d-1}{d}} E^{\frac{d-1}{d}}},$$

  where d is the space-time dimension and E the total energy. This follows from scale invariance and extensivity of the energy. Extensivity follows from locality. The constant c, roughly speaking, measures the number of independent fields in the theory, at the UV fixed point.

- Tunneling from a meta-stable state produces a bubble, which grows asymptotically at the speed of light, engulfing any time-like observer[1] propagating in the false vacuum. Inside the bubble, the state rapidly approaches the true vacuum. If one excites a local

[1] We will often use the word observer in these lectures. We use it to mean a large quantum system with many semi-classical observables. Quantum field theories give us models for a host of such systems, whenever the volume is large in cutoff units. They are collective coordinates of large composites and have quantum fluctuations that fall off like a power of the volume. Quantum phase interference between different states of the collective coordinate falls off like the exponential of the volume, except for motions of the collective coordinates that excite only a small number of low lying states of the system. With this definition of the word, an observer has neither gender nor consciousness.

region of the false vacuum to sufficiently high energy, the tunneling rate goes to infinity and meta-stability is lost. This is because the energy density cost to produce a stable expanding bubble of true vacuum is finite.

- If there are two exactly degenerate quantum vacua, separated by a barrier in field space, then, with finite cost in energy, one can produce an arbitrarily large region of vacuum 1, in the Hilbert space of the model which consists of local operators acting on vacuum 2. If the region is very large, it is meta-stable and survives at least as long as the time it takes light to cross that region.

2 Are there vacuum states in models of quantum gravity?

One of the main contentions of this lecture series is that the answer to the above question is NO. In fact, in the end, we will contend that each possible large distance asymptotic behavior of space-time corresponds to a different Hamiltonian, with different sets of underlying degrees of freedom. This is true even if we are talking about two different solutions of the *same* set of low energy gravitational field equations. In the case of Anti-de Sitter asymptotics we will see that the models are literally as different from each other as two different QFTs, defined by different fixed points. The most conclusive evidence for this point of view comes from the Matrix Theory [1] and AdS/CFT [2] formulations of non-perturbative string theory, and ITAHO[2] it is overwhelming. However, we can see the underlying reasons for these differences from simple semi-classical arguments, to which this section is devoted.

The essential point is that general relativity is not a quantum field theory, and that the reasons for this can already be seen in the classical dynamics of the system. Again, it is worthwhile making a formal list of the ways in which this is evident[3].

- The classical theory has no conserved stress energy tensor. The covariant conservation law for the "matter" stress energy is not a conservation law, but a statement of local gauge invariance. There is no local energy density associated with the gravitational field. In particular, this implies that there is no gauge invariant definition of an analog of the effective potential of non-gravitational QFT.

- Correspondingly, when we try to define an energy in GR, which could play the role of the Hamiltonian in the quantum theory, we find that we have to specify the behavior of the space-time geometry on an infinite *conformal boundary*. Geometries restricted to such time-like or null boundaries often have asymptotic isometry groups, and the Hamiltonian is defined to be the generator of such an asymptotic isometry, whose associated Killing vector is time-like or null near the boundary. This feature of GR is the first inkling of the *holographic principle*, of which much will be said below. It

[2]ITAHO - In this author's humble opinion.

[3]This list will use language compatible with the idea that the quantum theory of GR is somehow the quantization of the variables that appear in the classical Einstein equations. This idea lies behind all attempts to define quantum gravity outside the realm of string theory, from loop quantum gravity to dynamical triangulations. We will argue below that this idea is wrong.

is already at this level that one begins to see that different solutions of the same low energy effective equations will correspond to different Hamiltonians and degrees of freedom in the quantum theory. I note in passing that asymptotic symmetry groups do not seem to be an absolute necessity in this context. For example, many of the Hamiltonians used in the AdS/CFT correspondence have perfectly well behaved time dependent deformations and one would suspect that these correspond to space-time geometries with no time-like asymptotic isometries.

- More generally, the principle of general covariance shows us that no model of quantum gravity can have local gauge invariant observables. This fact was discovered in string theory, and considered an annoyance by some, long before it was shown to be a model of quantum gravity. All known versions of string theory incorporate this fact. The observables are always defined on an infinite conformal boundary. ITAHO, the fact that other attempts to formulate a quantum theory of gravity do not have this property, is evidence that they are incorrect. Note that this property is in direct contradiction with claims that a proper theory of gravity should be *background independent*. We will argue below that the holographic principle does allow for a more local, background independent formulation of models of quantum gravity, but that this formulation is inherently tied to particular gauge choices.

- More important than all of these formal properties is the nature of the space of solutions of gravitational field theories. It is well known that the mathematical theory of quantization begins by identifying a symplectic structure on the space of solutions, choosing a polarization of that symplectic structure, and identifying a family of Hilbert spaces and Hamiltonians whose quantum dynamics can be approximated by classical dynamics on that phase space. The general structure of ordinary QFT is that the space of solutions is parametrized, according to the Cauchy-Kovalevskaya theorem, in terms of fields and canonical momenta on a fixed space-like slice. The corresponding formulation of GR was worked out by Arnowitt, Deser and Misner (ADM), but it runs into a serious obstacle. Almost all solutions of GR are singular, and in order to define the phase space one must decide which singular solutions are acceptable. There are no global theorems defining this class, but there is a, somewhat imprecise, conjecture, called *Cosmic Censorship*. Here is what I think of as a precise formulation of this conjecture for particular cases:

  Start with a Lagrangian which has a Minkowski or AdS solution with a positive energy theorem. Consider a space-time with a boundary in the infinite past on which it approaches Minkowski or Anti-deSitter space, with a finite number of incoming wave packets corresponding to freely propagating waves of any of the linearized fluctuations around the symmetric solution[4]*. The amplitudes of these incoming waves are restricted to be small enough so that the following conjecture is true*[5]*. The conjecture is that to each such asymptotic past boundary condition there corresponds a solution which obeys*

[4]More properly, in the Minkowski case we should probably restrict ourselves to linearized waves that we expect to correspond to stable quantum states in the quantum theory.

[5]Recall that in the quantum theory, the classical field corresponding to a single particle has an amplitude which formally goes to zero in the classical limit.

Cosmic Censorship: the future evolution is non-singular, except for a finite number of finite area black holes. The asymptotic future solution corresponds to a finite number of outgoing wave packets plus a finite number of finite area black holes.

The last item focusses attention on the starring actor in the drama that will unfold in these lectures, the black hole. Our basic contention is that it is the answer to the age old question: How many angels can fit on the head of a pin? In modern language this is phrased: *How many bits ($\log_2$ of the number of quantum states) can fit into a given space-time region?* This is the content of what I will call the Strong Holographic Principle, and we will eventually view it as a crucial part of the *definition* of space-time in terms of quantum concepts.

For the moment, we stick to semi-classical arguments, and revisit our itemized list of the properties of the QFT concept of multiple vacua, but now with a view towards understanding whether this concept makes sense in a theory of quantum gravity.

- As a consequence of general covariance, no quantum theory of gravity can have gauge invariant correlation functions which are localized at a point in space-time. The physical reason for this is the existence of black holes. Quantum mechanics tells us that localized measurements require us to concentrate a large amount of energy and momentum in a small region. General relativity tells us that when the Schwarzschild radius corresponding to the amount of mass (as measured by an observer at infinity) enclosed within a sphere of radius R, exceeds R, the space-time geometry is distorted and a black hole forms. Bekenstein and Hawking [5] made the remarkable observation that one can calculate the entropy of the resulting black hole state in terms of classical properties of the geometry. It is given by one quarter of the area of the horizon of the black hole, measured in Planck units. This is in manifest contradiction with local quantum field theory, in which the entropy scales like the volume of the sphere. This is, in some sense, the *reason* that there are no local gauge invariant Green's functions. The region "inside the black hole" only has a space-time description for a very limited proper time, as measured by any observer in this region. We will see that a more fundamental description is in terms of a quantum system with a finite number of states, determined by interpreting the BH entropy as that of a micro-canonical density matrix. The internal Hamiltonian of this system is time dependent and sweeps out the entire Hilbert space of states an infinite number of times as the observer time coordinate approaches the singularity. From the point of view of an external observer this simply means the system thermalizes. The external description can be studied semi-classically and is the basis for Hawking's famous calculation of black hole radiation. Note by the way that Hawking radiation in asymptotically flat space-time removes the asymmetry in our description of the classical phase space. Black hole decay implies that once quantum mechanics is taken into account the final states in scattering amplitudes coincide with the initial states.

  At any rate, none of the points in a local Green's function can have a definite meaning, because we cannot isolate something near that point without creating a black hole that envelopes the point. It is easy to see that the most localization we can achieve in a theory of quantum gravity is holographic in nature. That is, if we introduce infinitesimal localized sources on the conformal boundary of an infinite space-time, then straightforward perturbation theory shows that, as long as we aim the incoming beams to miss each other (impact parameter much larger than the Schwarzschild radius corresponding to the center of mass energy, for each subset of sources[6]), there is a non-singular solution of the classical field equations. When these criteria are not satisfied, one can prove that a trapped surface forms [7], and a famous theorem of Hawking and Penrose guarantees that the solution will become singular. The Cosmic Censorship conjecture implies that this singularity is a black hole, with a horizon area bounded from below by that of the trapped surface.

In quantum field theory, the regime of scattering in which all kinematic invariants are large, is dominated by the UV fixed point. In this regime the differences between different vacuum states disappear. In quantum gravity by contrast, this is the regime in which black holes are formed. In asymptotically flat space, the specific heat of a black hole is negative, which means that at asymptotically high energies, the black hole temperature is very low. Thus, the spectrum of particles produced in black hole production and decay depends crucially on the infrared properties of the system. Different values of the *moduli*, the continuous parameters that characterize all known asymptotically flat string theory models, correspond to different low energy spectra. So in theories of quantum gravity, scattering at large kinematic invariants depends on what some would like to call the vacuum state. This is our first indication that these parameters correspond to different models, not different quantum states of the same system.

Black holes also falsify the claim that the high temperature behavior of the partition function is dominated by a conformal fixed point. In fact, all conformal field theories have positive specific heat and a well defined canonical ensemble. The negative specific heat of black holes in asymptotically flat space-time implies that their entropy grows too rapidly with the energy for the canonical partition function to exist. Although black holes are unstable, they decay by Hawking radiation, and the Hawking temperature goes to zero as the mass of the hole goes to infinity. Thus the high energy behavior of the micro-canonical partition function in asymptotically flat space would appear to be dominated by black holes, and cannot be that of a CFT.

It is interesting to carry out the corresponding black hole entropy calculation in the other two maximally symmetric space-times, with positive or negative values of the c.c. . The modified Schwarzschild metric is

$$ds^2 = -(1 - V_N(r) \pm (\frac{r}{R})^2)dt^2 + \frac{dr^2}{(1 - V_N(r) \pm (\frac{r}{R})^2)} + r^2 d\Omega^2,$$

where $V_N(r)$ is the Newtonian potential in d space-time dimensions,

$$V_N(r) = c_d \frac{G_N M}{r^{d-3}},$$

R the radius of curvature of the de Sitter or AdS space, and the $+$ sign is for the AdS case. In that case, the horizon radius is the unique zero of g_{tt}. When it is much larger

[6]Here we use language appropriate for asymptotically flat space-time. The corresponding scattering theory for asymptotically AdS spaces has been studied in [6].

than R it is given approximately by

$$R_S^{d-1} \sim c_d M R^2 L_P^{d-2},$$

where the Planck length is defined by $G_N = L_P^{d-2}$, in units where $\hbar = c = 1$. The area of the horizon is $A_d R_S^{d-2}$, so the BH entropy is

$$B_d (MR)^{\frac{d-2}{d-1}} (\frac{R}{L_P})^{\frac{d-2}{d-1}}.$$

$B_d = A_d c_d^{\frac{d-2}{d-1}}$. Remarkably, this looks like the entropy formula for a conformal field theory in $\mathbf{d-1}$ dimensions, living on a space with volume $\sim R^{d-2}$. In this interpretation, the quantity $(\frac{R}{L_P})^{\frac{d-2}{d-1}}$ plays the role of "the number of independent fields" in the CFT. This formula is one of the key elements of the AdS/CFT correspondence [2] [32] [4]. Note in particular *the dependence of the high energy density of states on the c.c.*. In bulk QFT, which motivates the idea of different vacuum states, the c.c. is a low energy property of the theory and the high energy density of states does not depend on it. We will see that the manifold examples of the AdS/CFT correspondence make it abundantly clear that different solutions of the bulk field equations correspond to different quantum Hamiltonians; different models of quantum gravity rather than different states in a given model.

The dS case is even more striking. g_{tt} has two zeroes, the larger of which is the cosmological horizon, which persists even when the black hole mass goes to zero. The sum of the areas of those two horizons is always less than that of the cosmological horizon of "empty dS space", and in fact decreases as the black hole mass increases. There is a maximal mass (Nariai) black hole, whose two horizons have equal area. When combined with the result of Gibbons and Hawking [8], that the dS vacuum state is a thermal state for the local observer in a maximal causal diamond of dS space, this result leads to the conclusion [9] that a quantum theory of a stable dS space must have only a finite number of quantum states.

- The semi-classical theory of quantum tunneling in the presence of gravity begins with the seminal paper of Coleman and De Lucia [10]. It confirms the picture of different solutions corresponding to different models, rather than different states, although almost all of the literature is couched in the language of vacuum decay. I will use the terms *true and false minima* rather than *true or false vacua* in order to emphasize that the conventional interpretation is wrong. The characteristics of gravitational tunneling depend crucially on the values of the energy density at the true and false minima. Let us begin with the case where the true minimum has negative c.c. . One of the most important results in [10] is that in this case, the classical evolution after tunneling does not settle down to the AdS solution with the field sitting at the true minimum. Instead, the geometry undergoes a singular Big Crunch. There is no conserved energy, and as the universe inside the bubble contracts, the energy of the scalar field gets larger and larger. The field explores its entire potential and does not remain near the "true minimum". More importantly, the semi-classical approximation breaks down.

Even in quantum field theory, particle production occurs and one might imagine that fluctuations in the energy density could lead to black hole formation. We will reserve to a later section a conjecture about what the real physics of the singularity is. For now we only note that the maximal causal diamond in this crunching geometry has only finite proper time between its past and future tips, as well as a maximal finite area for any space-like $d-2$ surface on its boundary.

The main point here is that there is no sense in which this semi-classical approximation describes decay to a well understood ground state. Below, by using the holographic principle, we will find a sensible interpretation of some of these processes (but not as decays) and present arguments that others simply can't occur in well defined models of quantum gravity. This is in stark contrast to the situation in QFT, where of course the value of the potential at its minimum is unobservable. Notice that none of this has anything to do with the AdS solution, which one gets by fixing the scalar field at its true minimum. This solution may or may not represent a sensible model of quantum gravity, but it certainly has no connection to the hypothetical model in which the CDL instanton describes some kind of transition.

When the true minimum has positive c.c., the situation is much better. Classical evolution of the scalar field after tunneling, rapidly brings it to rest at the true minimum. Furthermore, the resulting space-time has an (observer dependent) cosmological horizon. Inside an observer's horizon volume, all fields rapidly approach the empty dS configuration. We will see below that in this case of dS to dS tunneling, more can be gleaned from the nature of the semi-classical CDL solution, and it is all consistent with the idea that the quantum theory of stable dS space has a finite number of of states.

The case of a true minimum with vanishing c.c., whether this is achieved at a finite point in field space, or at asymptotically infinite scalar field, is much more ambiguous. If the falloff of the potential is that found in all asymptotic regions of string theory moduli space[7] then the future causal boundary of the universe is similar to that of Minkowski space: the maximal causal diamond has infinite area holographic screen, and at finite points within that diamond, at late times, the space-time curvature goes to zero, and the scalar field asymptotes to the zero c.c. point. On the other hand, this is NOT an asymptotically flat space. Furthermore, if one takes the analogues of outgoing scattering states for this universe, then most do not extrapolate back to smooth perturbations of the instanton geometry. The meaning of this kind of situation is the central issue in trying to establish the existence of the String Landscape. We will explore these issues, which are far from settled, in section 4 below.

To summarize, CDL tunneling provides abundant evidence for the fact that AdS solutions of gravitational field equations are NOT part of the same model as other stationary points of the same effective action. *One never tunnels to AdS space.* It also suggests that there can be models of quantum gravity with a finite number of states, which describe stable dS space. We will complete that discussion in section 6. Simi-

[7]As we will emphasize below, the notion of a potential on string theory moduli space is a problematic one. Nonetheless, if one accepts the validity of the concept one can use the symmetries of string theory to establish bounds on the behavior of the potential at infinity [12].

larly, there is no tunneling to asymptotically flat solutions of the field equations, which again must be regarded as (possibly) defining separate models of quantum gravity. We will argue below that there is no tunneling *from* AdS minima either, and that tunneling from asymptotically flat minima leads to a bizarre picture of the final state.

- Finally, we revisit the question of creating large meta-stable regions of space, which are in "another vacuum". If we start from an asympotically flat, or AdS minimum, and the potential is everywhere much less than the Planck scale and varies on a field space scale $\leq m_P$, then it is easy to find finite energy incoming configurations which move the field into another minimum over a sphere of radius R. However, if there is any potential barrier at all between the asymptotic minimum of the potential and the field value inside R, then the domain wall energy will scale like R^{d-2} and the Schwarzschild radius of the configuration will be $> R$. In other words, a black hole will form before the false vacuum bubble gets too big. Notice that if the false vacuum is a dS space, there will be an additional, volume contribution to the Schwarzschild radius. This guarantees that the black hole ALWAYS forms before the bubble can inflate[8] Thus, while auxiliary minima of a sub-Planckian effective potential do allow the creation of meta-stable states, they are not false vacua. The meta-stable regions that resemble homogeneous vacuum solutions are of limited size. Anything above that size is a black hole, which is to say, a thermodynamic equilbrium state indistinguishable from any other state of the theory that maximizes the entropy within the region $\leq R_S$. Notice also that there is no sense in which the decay of the meta-stable states created here is related to the instanton transitions discussed above. These are localized excitations of the true vacuum state, and will decay back to it by radiating particles off to infinity.

The conclusion is that rather simple classical considerations show that, whatever the theory of quantum gravity is, *it is not a QFT and the QFT concept of a vacuum state does NOT generalize to QG. Different solutions of the same low energy effective gravitational field equations, can correspond to different models of QG, rather than different states of the same model.*

3 Matrix Theory and the AdS/CFT correspondence

Indeed, all of our non-perturbative constructions of quantum gravity have this property in spades. In this section I'll quickly review these constructions, starting with the case of asymptotically flat space.

[8] A lot of confusion is caused by solutions of the equations of GR which describe an arbitrarily large region of dS space or slow roll inflation, connected to an asymptotically flat or AdS region with a small black hole in it. These solutions cannot evolve from data that is non-singular in the past and in particular from incoming scattering data in a space-time with a well defined past conformal infinity. If they represent anything in a real quantum theory of gravity it is an artificially entangled state of two, generally different, quantum Hilbert spaces. In the present discussion we approach localized regions of false minimum by starting from small regions that do not inflate and boosting the incoming energy continuously. In that case the black hole mass is bounded from below by the integrated energy density of the false minimum and the black hole always forms before inflation can occur.

3.1 Matrix theory

We have seen that in d dimensional asymptotically flat space, the entropy grows like $E^{\frac{d-2}{d-3}}$, so that conventional constructions of the partition function and the path integral fail. However, at least if $d > 4$, the light-front partition function at fixed longitudinal momentum

$$\mathrm{Tr} e^{-\beta P^-},$$

should be well defined, and we might hope to discover a more or less standard Lagrangian formulation of gravity. The Lagrangian for a single supersymmetric particle (superparticle) in 11 dimensions, is

$$\int dt \, \frac{p}{2}\dot{\mathbf{x}}^2 + i\theta\dot{\theta}.$$

Here $p \geq 0$ is the longitudinal momentum, which is treated as a fixed constant, and the time variable is light front time. $\mathbf{x}$ is a transverse 9-vector and θ_a a 16 component light front spinor. The system is quantized in terms of 9 commuting transverse momentum variables $\mathbf{p}$ and the 16 θ_a, with commutation relations

$$[\theta_a, \theta_b]_+ = \delta_{ab}.$$

The SUSY generators and Hamiltonian are

$$q_a = \theta_a,$$

$$Q_a = (\gamma \cdot \mathbf{p})_{ab}\theta_b,$$

$$P^- = \frac{(\mathbf{p})^2}{2p}.$$

The θ_a don't appear in the Hamiltonian, which describes a single massless relativistic particle. However, they give this massless state a degeneracy, with precisely the spin content of the 11 dimensional SUGRA multiplet.

Notice that this procedure only makes sense when p is strictly greater than zero. Particles with zero longitudinal momentum are non-dynamical. However, when the longitudinal momentum is continuous, the region of low longitudinal momentum becomes singular and one must exercise great care in treating it in order to extract correct results. In QFT this is often done by the method of Discrete Light Cone Quantization (DLCQ), in which the longitudinal direction is formally compactified so that p takes on only the discrete values $\frac{n}{R}$, with n a positive integer. One then studies the limit $R \to \infty$, by considering wave packets made from states including large values of n, so that they are localized in the longitudinal direction and can become independent of R. One convenience of this procedure, often exploited in QFT is that for fixed total longitudinal momentum $\frac{N}{R}$, a multi-particle state can have only a finite number of particles in it, so that in DLCQ field theory is approximated by the quantum mechanics of a finite number of particles.

The word approximated in the previous paragraph has to be stressed. The real system is obtained only in the limit when N is strictly infinite. Thinking about multi-particle states, we see in particular that, at fixed R, only those states with light cone energies $\sim \frac{1}{N}$ will survive in the limit $N \to \infty, R \to \infty$, with $p = \frac{N}{R}$ fixed. This introduces a degree of ambiguity into DLCQ, which can be exploited to simplify the limit. This ambiguity, well known in QFT, has mostly been ignored in the gravitational case, because Seiberg, following work of Sen and Susskind [13] found a particularly compelling form of DLCQ using string dualities.

Matrix Theory proceeds from this kinematical framework, by introducing an alternative to the Fock space treatment of identical particles. Instead, we generalize the variables $\mathbf{x}$ and θ_a to simultaneously diagonalizable $N \times N$ matrices. These can be written as

$$\mathbf{X} = \sum \mathbf{X}^I e_I, \qquad \Theta_a = \sum \theta_a^I e_I,$$

where $e_I^2 = e_I$ and $\mathrm{Tr} e_I = n_I$, $\sum n_I = N$. This representation is redundant if some of the n_I are the same, and we have a gauge symmetry permuting the e_I with equal trace, which is precisely the gauge symmetry of particle statistics. The fact that half integral spin is carried by the anti-commuting variables Θ_a guarantees that the spin-statistics connection is the conventional one. The Lagrangian is

$$L = \frac{1}{R}\mathrm{Tr}\ \big[\frac{1}{2}\dot{\mathbf{X}}^2 + i\Theta\dot{\Theta}\big],$$

and as we run over all possible choices of the e_I we reproduce the Lagrangians for $k \leq N$ supergravitons with all configurations allowed in DLCQ, and total momentum $\frac{N}{R}$.

If we insist on preserving all the SUSY, as well as the $SO(9)$ symmetry of this Lagrangian, there is a unique way of modifying it that allows for interaction between the supergravitons. To see what it is, we note that the Lagrangian we've written down is the dimensional reduction of $N = 4$ super Yang-Mills theory with gauge group $U(1)^N \ltimes S_N$, which is the low energy effective Lagrangian on the maximally Higgsed Coulomb branch moduli space of $U(N)$ SYM theory. The Lagrangian is written in temporal gauge (with the time of the gauge theory identified with light cone time) and the restriction to Bose or Fermi statistics for the particles is just the residuum of the Gauss Law of the non-abelian gauge theory. The full non-abelian Lagrangian is

$$L = \frac{1}{R}\mathrm{Tr}\ \big(\frac{1}{2}(D_t\mathbf{X})^2 - g^2[X^i, X^j]^2 + i\Theta D_t\Theta + g[\gamma \cdot \mathbf{X}, \Theta]\big),$$

where the adjoint covariant derivatives are $D_0 Y = \partial_t Y + g[A_t, Y]$. The constraints are now obtained by varying w.r.t. A_t and then setting $A_t = 0$. We'll describe how the SYM coupling is determined in terms of the Planck length below.

Before doing so, we note that this Lagrangian can also be shown to be the world volume Lagrangian of D0 branes in ten dimensional Type IIA string theory. The excitations on D-branes are open strings satisfying the appropriate mixed Dirichlet/Neumann boundary conditions. For N D-branes, the lowest excitations in open superstring theory have the quantum numbers of the maximally supersymmetric $U(N)$ Yang-Mills multiplet. If all of the spatial boundary conditions are Dirichlet, then the low energy world volume Lagrangian is unique and is given by the above formula. This idea led Seiberg, following Susskind and Sen, to argue that the compactified theory was just given by the D0 brane Lagrangian on the compact space. This conjecture is valid if we preserve at least 16 supercharges. It identifies the correct degrees of freedom, and their Lagrangian is completely determined by symmetries.

The D0 brane picture tells us how to identify the Yang-Mills coupling. Interactions are determined by the open string coupling, g_S, so $g_{YM}^2 = g_S$. On the other hand, Type IIA string theory is the compactification of M-theory (the quantum theory whose low energy limit is 11D SUGRA), on a circle whose radius is small in Planck units. D0 brane charge is Kaluza-Klein momentum. So we have the identification

$$\frac{l}{l_S g_S} \propto \frac{1}{R},$$

where we've equated the string theory formula for the D0 brane mass to the KK formula. In the duality between M-theory and Type IIA string theory, the string is viewed as an M2 brane wrapped on the small circle, so

$$l_S^{-2} \propto l_P^{-3} R.$$

Combining the two formulae we find $g_S \propto (R/l_P)^{3/2}$. All of these formulae are actually exact consequences of SUSY, so the constants we have omitted can be calculated exactly.

Seiberg's prescription tells us that if we want to find the DLCQ of M-theory compactified on a torus or K3 manifold, we should study the world volume Lagrangian of D0 branes moving on that manifold. If the manifold has size of order the 11D Planck scale, then it is very small in string units, and we should do a T-duality transformation to find a description that is under greater control. For a torus of less than four dimensions, this gives us SYM theory compactified on the dual torus. These are all finite theories and the prescription is unambiguous. Many exact results, including some famous string dualities can be derived from this prescription, and agree with calculations or conjectures that one already had in supergravity or string theory. Other calculations, not protected by supersymmetry non-renormalization theorems are only supposed to be correct when takes the $N \to \infty$ limit, keeping only states whose light cone energy scales like $\frac{1}{N}$.

For a four torus or a K3 manifold, one naively gets the four dimensional SYM theory, which is not renormalizable. However, the T-dual string coupling is large, so we should really be studying the D4 branes (into which the D0 branes are converted by T-duality) in the strong coupling limit. In this limit, D4 branes become M5 branes. The world volume theory on N M5 branes is a maximally superconformally invariant 6 dimensional theory. It is compactified on T^5 or $K3 \times S^1$. Again, the prescription is finite and makes a number of correct exact predictions. It is however more difficult to calculate with since not much is known about the $(2,0)$ superconformal field theory.

If we add one more circle to either of these constructions, we obtain *little string theory*. This is the world volume theory of N NS5 branes in the zero string coupling limit. Even less is known about this model than about the $(2,0)$ superconformal field theory, and there have even been questions raised about whether it really exists. With six or more compact dimensions, the Seiberg construction fails and we do not have a working definition of the DLCQ of M-theory with 4 or 5 asymptotically flat dimensions.

Among the most striking features of these constructions is that each different gravitational background gives rise to a different quantum Hamiltonian. Even two versions of M-theory with values of continuous moduli that differ by a finite amount, correspond to the same field theory Hamiltonian on different compactification manifolds. And remember that the canonical variables of this Hamiltonian do not include a gravitational field. The geometry of the compactification manifold is not a dynamical variable in the Matrix theory Hamiltonian.

3.2 The AdS/CFT correspondence

The correct statement of the AdS/CFT correspondence is that in certain quantum field theories in $d-1$ space-time dimensions, there is a regime of large parameters, in which three important properties are satisfied:

- The high temperature behavior of the partition function on a spatial sphere of radius R is $c(RT)^{d-2}$, with $c \gg 1$.
- The dimension of most operators at the UV fixed point which defines the theory go to infinity.
- The Green's functions of those operators whose dimension remains finite can be computed approximately by solving the classical field equations of a $d + D$ dimensional gravitational Lagrangian, with boundary conditions first outlined by [32]. The space-time metric has a conformal boundary identical to that of $AdS_d \times K$, where K is a compact manifold. If the non-compact space-time is exactly AdS_d then the boundary field theory is conformal.

As a consequence of the last property, we consider such QFTs to be *definitions* of models of quantum gravity, with fixed asymptotic background. The idea that AdS/CFT defines a duality between two independently defined theories, is probably without merit. For a subclass of these theories, one of the large parameters is an integer N which controls the size of the the gauge group of the boundary field theory, and the model has a conventional large N expansion. In this case there is a weak coupling string theory description of the model, which goes beyond the classical gravity expansion described above. In these cases, the models have at least two adjustable parameters. One, N, controls the standard planar expansion of the theory, which can be recast as an expansion in world sheet topology. The other, loosely called the *'t Hooft coupling*, is continuous (at least in the large N limit). When it is large, the solution of the theory in terms of classical gravitational equations is valid. When the 't Hooft coupling takes on moderate or small values there is a calculation of the correlation functions of all operators whose dimensions are finite in the large N limit, in terms of a world-sheet quantum field theory. In most of the interesting cases[9] the world sheet theory is hard to solve, but enormous progress has been made in establishing the conjecture.

However, even if we were able to calculate everything, including all higher genus contributions in the world sheet theory, this would not constitute an independent definition of the "other side" of the "AdS/CFT duality". String perturbation theory is a non-convergent asymptotic expansion. We know plenty of examples where its existence and finiteness to all orders is *not* a guarantee of the existence of a real quantum model of gravity. Bosonic

[9] The models of [14] are an exception.

matrix models related to $1+1$ dimensional string theories are a calculable example [15]. A continuous infinity of other examples is provided by moduli spaces of 4 dimensional compactifications of string theory with $N = 1$ SUSY. These have well defined perturbation expansions. However, general symmetry arguments, as well as many explicit instanton calculations show that there must be a non-perturbative superpotential on this moduli space, if this set of models makes sense at all. This means that all of the perturbation expansions except *perhaps* for a discrete set of points in moduli space do not correspond to well defined models. Furthermore, even if the wildest conjectures about the string theory Landscape are correct, most of these discrete points correspond to space-times with non-zero c.c.. This means that the flat space S-matrix elements one calculates in string perturbation theory do not correspond, even qualitatively, to the correct set of observables of the hypothetical underlying model. We will return to this point when we discuss the string Landscape below. Our conclusion here is that the AdS/CFT correspondence is a *definition* of a class of models of QG, in terms of QFTs defined on the conformal boundary of AdS space.

It is important to emphasize that most QFTs fit into neither of these categories, even when they have a large N expansion. All large N models, and many other examples, such as the tensor product of any large collection of mutually non-interacting QFTs (or theories that are small perturbations of such a collection) satisfy the first of our criteria above. Referring back to the formula for black hole entropy in AdS space, we see that this criterion can be rephrased as: *AdS/CFT gives a rigorous justification of the BH entropy formula for asymptotically AdS space-times.* Comparison of the two formulae leads to the conclusion that the constant c is a measure of the ratio of the AdS curvature radius to the Planck length. Obviously, any classical space-time interpretation of the model will be valid only when this parameter is large, but this is only a necessary condition for the classical gravity approximation to be valid.

To understand better what is going on, let's recall the basic equations of the AdS/CFT correspondence. The Euclidean[10] AdS metric is

$$ds^2 = (1 + \frac{r^2}{R^2})d\tau^2 + \frac{dr^2}{1 + \frac{r^2}{R^2}} + r^2 d\Omega_{d-2}^2.$$

It follows that, at large r, solutions of the Klein-Gordon equation behave like $r^{\lambda_\pm} J(\tau, \Omega)$, with

$$\lambda(\lambda + d - 1) = m^2 R^2.$$

The $\pm$ signs refer to the two roots of these equations. The AdS/CFT prescription is to solve the coupled non-linear Einstein matter equations, with the boundary conditions that the fields behave like the larger root of this equation, and arbitrary source function J. Analogous boundary conditions are imposed on the metric and other higher spin fields. The action as a functional of the source is the generating functional for conformally covariant Green's functions on the boundary.

A consequence of this prescription is that every primary operator in the boundary CFT corresponds to a different field in the bulk. The mass of small fluctuations is related to the

[10] The Euclidean rotation familiar from QFT is not valid for QG in asymptotically flat space, because the density of states blows up too rapidly for the finite temperature partition function to be well defined. However, in AdS space the quantum theory is a boundary QFT and the Wick rotation makes sense.

dimension of the primary. Thus, the bulk theory will have, generically, an infinite number of fields. The only known way to write an approximately local field theory with an infinite number of fields in AdS space, is to consider field theory with a finite number of fields on $AdS \times K$, where K is a compact manifold. The infinity then corresponds to a complete set of functions on K. The degeneracy of the Laplacian on K for high eigenvalues is power law in the eigenvalue, so this prescription could at most give us a power law growth of the number of fields of mass m, as $m \to \infty$.

It is well known that the number of primary operators of dimension D grows exponentially with a power of dimension, which implies an exponentially growing number of *fields*, in the approximate local field theory describing fluctuations around the hypothetical $AdS \times K$ background. Kaluza-Klein compactification on K gives rise only to a spectrum of masses that grows like a power of the mass (in $1/R$ units, where R is the radius of curvature of AdS, typically of the same order of magnitude as that of K). In examples where the CFT is dual to a weakly coupled string theory, such an exponential growth *is* seen among string states. So, for a generic CFT, one needs parametrically large entropy in order to claim that the geometrical radii are larger than the Planck length, but also another large parameter to guarantee that geometrical radii are larger than the length defined by the string tension.

It should be emphasized that very few CFTs actually correspond to weakly coupled string theories. The necessary and sufficient condition is that the theory have a conventional *matrix* $1/N$ expansion. This is what is necessary to have both a free string limit, and a topological structure of interactions that corresponds to a sum over world sheet topologies. Neither vector large N limits, nor the topological expansions typical of theories with comparable numbers of flavors and colors, or matter in other large representations of $SU(N)$[11], have a free string interpretation. Thus, for many CFTs, there seems to be no interpretation of their correlation functions as a set of observables corresponding to objects propagating in an AdS space[12].

In all rigorously established examples of the AdS/CFT correspondence the large parameter is an analog of the 't Hooft coupling of a large N gauge theory, a parameter which is continuous in the planar limit. In the two and three dimensional examples the 't Hooft coupling is really a ratio of two large integers, while in four dimensions it is the rescaled Yang Mills coupling. It is important that the theory is conformally invariant for *every* value of the 't Hooft coupling. In the limit when the coupling is large some dimensions remain of order 1, while others go to infinity. Furthermore, the multiplicity of operators with order 1 dimension grows only like a power of the dimension, consistent with a bulk space-time interpretation on a background of the form $AdS \times K$. All of the examples where this behavior has been established are exactly supersymmetric.

Non-supersymmetric marginal perturbations of these theories *all* lead to models with at most isolated fixed points at 't Hooft coupling of order one. One can also consider

[11] The second rank symmetric and anti-symmetric tensor representations of $O(N)$, *do* appear in orientifold projections of large N gauge theories and have a string loop expansion.

[12] Even when the CFT has an entropy and dimension spectrum corresponding to an AdS radius that is large compared to both the "string length" and the Planck length, in the sense described above, it may not have a simple space time interpretation. A simple example is maximally supersymmetric $SU(N) \times SU(M)$ Yang Mills theory with both 't Hooft couplings large, or a perturbation of it by an exactly marginal operator constructed as a product of relevant operators from the individual theories.

orbifolds of the $\mathcal{N} = 4$ SYM theory, whose planar diagrams coincide with the original theory, and are conformally invariant for all values of the 't Hooft coupling. However, the leading non-planar corrections to the beta functions of several couplings are non-zero and depend explicitly on the 't Hooft coupling. The theories will be conformal, if at all, only at 't Hooft coupling of order one. These theories provide interesting analogs of tachyon free non-supersymmetric string theories in flat space-time. Those asymptotically flat models seem completely sensible at string tree level, but the loop diagrams are divergent. If one tries to invoke the Fischler-Susskind mechanism to cancel these divergences, one finds perturbations of the space-time geometry which are singular in either the remote past or future or both. The string perturbation expansion breaks down. There is no evidence that these models really exist. The same is true for the non-supersymmetric orbifold theories. At leading order in the planar expansion, we have a free string theory on an AdS space-time. Finite string coupling corrections destroy this interpretation, except perhaps for a particular AdS radius of order the string scale. The question of whether the model at this particular radius makes sense is the question of whether the leading non-planar beta function has a finite coupling fixed point. In fact, that only guarantees that string perturbation theory in AdS space will make sense at that radius, and one must confront the resummation of the divergent $\frac{1}{N}$ expansion.

3.3 Domain walls and holographic renormalization group flow

When a flat space QFT has two isolated degenerate vacua, $\phi^i_{\pm}$ it also has domain wall solutions in which the scalar fields vary only in a single coordinate $\phi^i(z)$, and $\phi^i(\pm\infty) = \phi^i_{\pm}$. These solutions are stable and have a finite surface energy density, called the tension of the domain wall. They are limits of meta-stable finite energy states of the field theory with spherical domains of one vacuum inside the other. We have already argued that no such limit exists in theories with gravity. If the spherical domain wall becomes too big it collapses into a black hole.

There are however many examples of infinite hyper-planar domain wall solutions of Lagrangians with gravity, and the AdS/CFT correspondence gives us a novel interpretation of them. Consider a scalar field coupled to gravity with a potential having two stationary points, one a maximum and one a minimum, both with negative c.c. . There are AdS solutions corresponding to each of these points, and it is possible for both of them to be stable. Indeed Breitlohner and Freedman showed that tachyonic scalar fields are allowed in AdS_d space, as long as the tachyonic mass satisfies

$$4m^2R^2 > (d-1)^2.$$

Referring to the dictionary relating bulk masses to boundary dimensions, we see that this is the condition for dimensions to be real and that B-F allowed tachyons are dual to *relevant* operators.

The equations for a domain wall solution connecting the two stationary points are

$$\phi''(z) + (d-1)\frac{\rho'}{\rho}\phi'(z) = \frac{dV}{d\phi}.$$

$$2\rho'^{\,2} = \epsilon^2\rho^2(\phi'^{\,2} - V).$$

We have rescaled fields and coordinates so that everything is dimensionless (see the discussion of instanton solutions below) and ϵ is a measure of the rate of variation of the potential V in Planck units. The metric is

$$ds^2 = dz^2 + \rho^2(z)d\mathbf{x}^2,$$

corresponding to a hyper-planar domain wall geometry. Our boundary conditions are that $\phi(\pm\infty)$ be the positions of the two stationary points. ρ then interpolates between the two different AdS geometries.

Near the AdS maximum of the potential, the two solutions of the linearized equation both fall off at infinity, so we only use up one boundary condition by insisting that the solution approach the maximum as $z \to \infty$. The solution then contains both possible power law behaviors and thus, *from the point of view of the AdS/CFT correspondence at the maximum, it corresponds to a perturbation of the Lagrangian of the boundary field theory by a relevant operator.* This is a novelty compared to traditional QFT. The domain wall is not an infinite energy state in the original model, but a perturbation of its Hamiltonian. It becomes clear that we should be trying to view the domain wall solution as the "anti-holographic" representation of a boundary renormalization group flow between two CFTs.

In general there is no such solution. The problem is that one of the linearized solutions of the fluctuation equations around the AdS minimum blows up at infinity. Thus we need two boundary conditions to ensure that the solution approaches the minimum as $z \to \infty$ *and* that its derivative goes to zero there. Having used up one parameter on the other side of the wall, we do not have this freedom. However, we can always find a solution by fine tuning one parameter in the potential, in order to set the coefficient of the growing mode to zero. Thus, the space of potentials with static domain wall solutions connecting two AdS stationary points is co-dimension one in the space of all potentials with two such stationary points[13]. Note that the fact that the second stationary point is a minimum is consistent with, and implied by, the RG interpretation. An RG flow should always approach its IR fixed point along an irrelevant direction in the space of perturbations of that fixed point. The AdS/CFT dictionary translates *irrelevant* as *positive mass squared*.

Having found such an RG flow we are almost ready to declare that we have a self consistent discovery of a new CFT with a large radius AdS dual. However, consistency requires that we check all directions in the bulk scalar field space, to determine if there are any tachyonic modes that violate the B-F bound. One way to guarantee both the existence of the domain wall solution and its B-F stability is to work in SUGRA, and insist that both stationary points preserve some SUSY. A host of solutions of this type have been found, that interpolate between fixed points with different numbers of supercharges in their super-conformal algebra.

Remarkably, when we perturb a supersymmetric CFT with a large radius dual by a relevant operator that violates all supersymmetry, we have yet to find a consistent solution. There are a number of smooth domain wall solutions of this type, but one always finds tachyons that violate the B-F bound in the spectrum of scalar fluctuations of the new minimum. There is, as yet, no theorem that this is *always* the case, but when combined with the failure to find non-supersymmetric large radius AdS spaces by orbifolding one is led to

[13] For future reference, we note that the parameter counting remains the same when we search for a domain wall connecting an asymptotically flat minimum to one with negative c.c., although the interpretation of the solution as an RG flow is no longer applicable.

suspect a connection between SUSY and the low curvature of space-time. We will see below that the construction of holographic space-time seems to imply that all consistent theories of gravity in asymptotically flat space are exactly supersymmetric. Many years of failure to find consistent perturbative string constructions, which violate SUSY in asymptotically flat space-time, have convinced most theorists that no such theories exist.

By contrast, the theory of the String Landscape, to which we will turn in a moment, suggests that there is no particular relation between the size of the cosmological constant and the scale of SUSY breaking. This effective field theory based scenario seeks to identify a huge set of string models with many independent small positive contributions to the effective potential. Adding these to a large negative contribution, one argues that if the number of positive contributions is of order 10^X with X significantly larger than 123, then there will be many of these models with positive c.c. of order the one we observe. One then invokes the "successful" anthropic prediction of the c.c. to explain why we happen to see only this special class of models. As a byproduct, this construction produces a huge set of models with very small negative c.c., without SUSY. Indeed the typical strategy is to find such a negative c.c. AdS solution and then add a single small positive contribution to get a model representing the real world.

It thus seems rather important to determine whether there are in fact non-supersymmetric CFTs with large radius AdS duals. This is a well defined mathematical problem, in stark contrast to the effective potential discussion of the landscape. It's my opinion that more people should be working on it.

4 Is there a string theory landscape?

The basic idea of the string landscape is easy to state. If one looks at compactifications of string theory to four dimensions, with $N \geq 2$ SUSY, we find moduli spaces of models of quantum gravity, with continuous parameters. The number of such parameters is related to the topological complexity of the compactification manifold. For example, in compactifications of Type IIA string theories on Calabi-Yau manifolds, we find a vector multiplet of N=2 SUSY for every non-trivial $(1,1)$ cycle[14] We find a massless hypermultiplet for every $(2,1)$ cycle. So complicated topologies have high dimensional moduli spaces.

When we consider compactifications with only N=1 SUSY, for example heterotic strings on CY_3, then we find a similar list of moduli at string tree level and to all orders in perturbation theory. However, there is no non-perturbative argument (in most cases) that these moduli spaces are an exact property of the theory. The fact that there are moduli spaces in perturbation theory is related to a continuous shift symmetry of the superpartner of the dilaton field. There are many non-perturbative effects that violate this symmetry. Thinking in terms of low energy effective field theory, we imagine a non-trivial superpotential on this moduli space, which leads to a non-trivial potential. A generic function on a space of dimension D is expected to have a set of local minima whose number is exponential in D. This is the most naive picture of the string landscape.

[14] Actually, it's a bit more complicated. One of the vector fields is part of the N=2 SUGRA multiplet. We get a number of non-gravitational vector multiplets equal to $h_{1,1} - 1$.

As we have described it, the landscape is not under any quantitative control. If one tries to write down a few terms in the expansion of the superpotential around weak coupling, one finds that non-trivial minima always lie at values of the string coupling where the expansion is invalid. A major step in the development of landscape ideas was the notion of *flux compactification* [16]. This was the study of solutions in which field strengths of p-form fields on the internal manifolds are turned on. The Dirac quantization condition tells us that these fluxes obey integer quantization rules, so we anticipate a large discrete lattice of solutions, for an internal manifold of complicated topology. A particularly simple set of solutions was found by [17] using the Lagrangian of Type IIB SUGRA. The internal manifold is conformal to a Calabi-Yau manifold, there are imaginary self dual fluxes of a combination of the Ramond-Ramond and Neveu-Schwarz 3-form fields. The flux superpotential fixes all the complex structure moduli and string coupling. The Kahler moduli remain moduli of these solutions. For some choices of fluxes, the fixed value of the string constant is numerically small, so one claims that one can still trust notions from weak coupling string theory.

One feature of this system that is quite general is the necessity for an orientifold in addition to classical super-gravity fields. All weak coupling string theory approaches to compactification will have to deal with the dilaton field. Apart from Calabi-Yau compactifications with no flux, (for which the Einstein Lagrangian vanishes on shell), there will always be sources for the dilaton field in the compact dimensions. The classical SUGRA contributions to the dilaton source are all positive, so we get an equation

$$\nabla^2 \phi = P.$$

Integrating this equation over a compact manifold, we get a contradiction [18]. The orientifold provides a negative source term, which allows for consistent solutions. Orientifolds are singular and do not belong in effective field theory, but they are certainly innocuous in weakly coupled string theory in flat space. As long as one has the weak coupling string theory formalism at one's disposal, one can imagine that this remains true in curved space. Orientifolds can be defined in a finite manner if one has a world sheet sigma model. We'll discuss this further below.

At tree level, the solutions preserve SUSY in Minkowski space if the value of the flux superpotential at the minimum vanishes. Other choices of fluxes, for which $W_0 \neq 0$ break supersymmetry. These solutions still have vanishing cosmological constant because the Kahler potential of Type IIB SUGRA for the Kahler moduli, has the so called no-scale form. However, quantum corrections to the Kahler potential or superpotential will change this. While the latter is exponentially small in the compactified Kahler moduli one can argue that for small W_0 it can still be the dominant effect at large Kahler moduli. One finds (AdS)-supersymmetric solutions with negative c.c. by tuning fluxes so that W_0 is small.

There are, in my opinion, two related things to worry about in these solutions. The first is the question of what it means for the string coupling to be small, and the second is what to do about the orientifold. The normal meaning of small string coupling is that there is a world sheet expression for observables, and an expansion in powers of the string coupling by summing over world sheet topology. If there were such an expansion, we would have no problem defining orientifolds as finite world sheet field theories. But there cannot be such an expansion in this context, because the string coupling is fixed by competition between different terms in the g_S expansion of the superpotential. So we must view these compactifications as constructs in effective field theory, but the orientifold is problematic in a long wavelength expansion. We know that orientifolds are perfectly finite in flat space perturbative string theory, and many orientifolds are related by dualities to smooth solutions of M-theory. So it is not so much the existence of the orientifold that is at issue, but rather whether its singularity could hide dependence on the fluxes which are the control parameters for these solutions.

Many of these ambiguities are removed, at the expense of a considerable loss in computational power, by looking at F-theory compactifications. F-theory is a rubric for a class of solutions of Type IIB string theory, in which the complex string coupling $\tau = a + i\frac{4\pi}{g_S}$ (a is the RR axion field) varies over a complex 3-fold base space of large volume. The ensemble defines an elliptically fibered CY_4 space, with τ describing the complex structure of the elliptic fiber. The orientifold solutions described above are special limits of F-theory compactifications, which were introduced in order to use weak coupling methods. In a general F-theory compactification the string coupling varies over the 3-fold base and is never weak everywhere. In the orientifold limit the region where the coupling is strong shrinks to the locus of the singular orientifolds. More generally, the only expansion parameter in F-theory is the volume of the 3-fold base in 10 dimensional Planck units. F-theory models with fluxes also exist and have been studied extensively in recent years [19] . While the flux induced superpotential for the complex structure moduli of the base has not been computed explicitly, there seems little doubt that for sufficiently generic fluxes all the moduli will be fixed, leaving only the Kahler moduli. For simplicity we can assume that there is just one Kahler modulus. There is at least one since the overall volume of the compact space will not be determined by the SUGRA action.

Thus it is extremely plausible that on a 3-fold base with large $h_{2,1}$ there will be a large number of smooth solutions of Type IIB SUGRA, with all moduli but the overall volume fixed. Below the Kaluza- Klein scale there will be an effective four dimensional theory with $N = 1$ SUGRA and a single chiral multiplet with a no-scale superpotential. The superpotential W_0 will be a flux dependent constant, and since there are many fluxes, it is plausible that it can be tuned to be much smaller than the KK energy scale, as one finds explicitly for the superpotential computed in the orientifold limit. The use of classical SUGRA is of course predicated on the assumption that the KK radius is much larger than the ten dimensional Planck length. These solutions preserve $N = 1$ SUSY only if the superpotential vanishes. (One way to guarantee this is to search for solutions that preserve a discrete R symmetry. The volume modulus will have R charge 0.) However, as a consequence of the no-scale Kahler potential, all of them will have four flat Minkowski dimensions.

This is not consistent if $W_0 \neq 0$. If it were, there would be a low energy effective action for the modulus in four dimensional $N = 1$ SUGRA, but corrections to the Kahler potential would change the cosmological constant, and there could not be a Minkowski solution. However, it does make sense to postulate the existence of a supersymmetric AdS solution. The condition for supersymmetry is

$$\partial_\rho W - \frac{1}{m_P^2}\partial_\rho K W,$$

where we have parametrized the Kahler modulus as

$$(Rm_{10})^{-4} = \frac{\text{Im } \rho}{m_P}.$$

The real part of ρ is an angle variable, so all corrections to the superpotential must be integer powers of $e^{2\pi i \rho}$[15]. Following KKLT one can then argue that if W_0 is small, the system is self-consistently stabilized at a large value of the imaginary part of ρ, where the corrections to the classical Kahler potential are small.

Our own analysis of this situation differs from that of KKLT in two ways. Rather than considering it a controlled calculation in string theory, we view it as a plausible self consistency check for the existence of a supersymmetric AdS model of quantum gravity, whose low energy Lagrangian and background configuration are those suggested by KKLT. The second difference is that we reject the idea that the weak coupling orientifold calculation is more controlled than the general F-theory set-up. The former has an orientifold singularity, whose effect can only be estimated if we have a systematic world sheet expansion. However, the model fixes the string coupling at a value that is not parametrically small, so no world sheet calculation is likely to exist. The only world sheet calculation one can attempt is an expansion around one of the Minkowski solutions of the classical string equations with the orientifold source. We know that if $W_0 \neq 0$, the string loop expansion leads to divergences in the integral over toroidal moduli space. One can attempt to cancel these divergences with the Fischler-Susskind mechanism [20], but this leads to a time dependent background, which is singular in either the past or the future or both. It does not correspond to the stable supersymmetric model whose existence we are asserting.

The only real calculational advantage of the orientifold limit of F-theory is the exact formula for the flux induced superpotential. Rather than pursuing the idea that weak coupling string perturbation theory can be used to calculate some useful property of the hypothetical supersymmetric AdS model, it would seem more profitable to try to find an analogous formula for the superpotential in general F-theory models, or at least to argue that a general model with generic fluxes will indeed stabilize all the complex structure moduli.

The bottom line of this discussion is that F-theory compactifications with generic fluxes seem to stabilize all complex structure moduli at the level of classical SUGRA. We use the phrase seem to because detailed calculations rely on the GVW superpotential, calculated at weak string coupling. Even in the orientifold limit of F-theory, there is no systematic string loop expansion of these models, when $W_0 \neq 0$. Classical solutions in which $W_0 = 0$ as a consequence of an anomaly free discrete R symmetry provide us with moduli spaces of asymptotically flat models of quantum gravity in four dimensions. The Kahler moduli are exact moduli of these models. When $W_0 \neq 0$ we have, at the classical SUGRA level SUSY violating asymptotically flat solutions. The classical SUGRA equations are formally exact

[15]If, in F-theory, the cycle associated with this Kahler modulus is wrapped by multiple seven branes, so that the CY_4 is singular on that cycle, then the shift symmetry of the angle variable induces a chiral transformation on the matter fields that couple to the 7-brane gauge group. This chiral symmetry is spontaneously broken by strong coupling gauge theory dynamics, which introduces a new discrete finite variable into the effective field theory, parametrizing the different field theory vacuum states. The result is fractional powers of $e^{2\pi i \frac{\rho}{m_P}}$ in the effective superpotential. Unless the 7-brane group is very large, this does not substantially change our argument.

in the limit of infinite Kahler moduli, but corrections to this approximation ruin asymptotic flatness. *If we assume the existence of large radius, supersymmetric AdS compactifications*, then effective field theory provides a self consistent solution. Such compactifications should have a low energy effective field theory, and symmetries constrain the superpotential and Kahler potential of that field theory. If $W_0 \ll 1$ (in Planck units) then we find a self consistent solution in which we only need to include the leading correction to the superpotential. For F-theory solutions whose CY_4 has large Betti numbers, of order 100, there are many solutions of this type and it seems plausible that one can find many examples with small W_0. The effective expansion parameter is $|\ln \frac{W_0}{m_{10}^3}|^{-1}$, and the scale of Kaluza-Klein excitations is parametrically larger than the inverse AdS curvature radius. It is believed that the Betti numbers of CY_4-folds are bounded, so the expansion parameter can never be really small.

The KKLT paper can thus be viewed as providing evidence for a large class of large radius supersymmetric AdS compactifications. This conjecture is subject to a rigorous test. One must find 3 dimensional superconformal field theories whose properties mirror those of the conjectured geometries via the AdS/CFT correspondence.

DeWolfe *et. al.* [21] have suggested another set of supersymmetric AdS solutions with a tunably small parameter. These are based on solutions of (massive) Type IIA string theory. They again purport to have small string coupling and a parametrically suppressed ratio between the compactification radius and the AdS radius. In these compactifications, the control parameter is a large flux, N. However, in [22] we provided evidence that the ever-present orientifolds in such weak coupling constructions hide a region of the compact manifold where the string coupling is large and the compactification radius scales like the AdS radius. The picture in [22] provides an explanation for the scaling of the entropy with N, which is not available in the weak coupling picture. Again, the real test of all of these conjectures is to find superconformal field theories with the properties implied by these geometries. This is particularly interesting in the Type IIA case, because [23] have exhibited non-supersymmetric versions of these compactifications, which look equally plausible. This implies the existence of a large class of non-supersymmetric fixed points with large radius AdS duals. As I've emphasized above, neither orbifolding nor holographic RG flow, both of which seem like plausible mechanisms for finding examples of such large radius CFTs, actually succeed.

It seems to me that this is a place where progress can be made in assessing the reliability of the effective field theory approach to the String Landscape. There is an apparent conflict between the vast landscape of SUSY violating large radius AdS duals promised by the construction of approximate effective potentials, and our inability to construct even one example of the same from a reliable starting point. Perhaps the most controlled setting for studying this problem is that of AdS_3 models. The effective potential approach to these is quite similar to that for AdS_4, but in 2 dimensions we have a much richer arsenal of tools for studying CFTs without recourse to perturbation theory. This area is relatively unexplored and might repay the attention of young researchers.

I've deliberately avoided discussing the procedure of "uplifting the AdS solutions to meta-stable dS solutions by adding anti-D3-branes". This purports to be a small perturbation of the existing solutions, but it is manifestly not. No one knows how to describe the observables of meta-stable dS states, but it is clear that they have nothing to do with conformal field theories living on the boundary of a $3+1$ dimensional AdS space. The procedure of adding anti-branes is perfectly sensible when we are talking about a brane configuration of non-compact codimension 3 or more, embedded in a string model in asymptotically flat space-time. It may also be valid in co-dimension 2. For co-dimension zero the back reaction of branes on the geometry is simply not a small perturbation. If we recall that even a small change in the c.c. changes the high energy spectrum of the theory, we see immediately that one cannot play perturbative or low energy effective field theory games in this situation. We will discuss a possible theory of meta-stable dS spaces below.

4.1 Tunneling in gravitational theories

The key paper on gravitational tunneling is that of Coleman and De Lucia [10]. I urge every serious student of this subject to study that paper carefully and completely. The study of tunneling in general quantum systems is the study of *instantons*: Euclidean solutions of the classical equations of motion *with appropriate boundary conditions.* In QFT in Minkowski space, the boundary condition is that the scalar fields must rapidly approach their values at some meta-stable minimum of the scalar potential, as the radius goes to infinity. The classical solution is $O(d)$ symmetric in d Euclidean space-time dimensions, and defines a finite "critical bubble". The bubble wall is generically fuzzy, and is defined by saying that the field is closer to the meta-stable minimum than some small parameter ϵ. The derivative of the scalars vanishes at the center of the bubble, and this allows us to analytically continue the bubble geometry to the interior of a forward light cone in Minkowski space. The Euclidean solution provides initial conditions for the propagation of the scalar field inside this light cone. It is easy to see that as one proceeds forward on homogeneous slices of constant negative curvature the scalars smoothly approach their values at the absolute minimum of the potential. One says that *the false vacuum has decayed into the true vacuum.* We will continue to use the terms true and false minimum in the gravitational case even though we have emphasized that the concept of vacuum state does not make any sense in quantum gravity. We will also see that not all instantons describe decay.

In finite temperature field theory, this prescription is modified. The Euclidean time dimension is compactified on a circle, and one searches for periodic Euclidean solutions. The solutions no longer achieve the false minimum, and they describe the decay of a meta-stable thermal ensemble, through a combination of quantum tunneling and thermal hopping over the barrier.

Coleman and De Lucia generalized this prescription to include the dynamics of the gravitational field. Their presentation is oriented towards situations where the gravitational effects are "a small perturbation" of the flat space theory, but they discovered that in many cases this claim is untenable, and the gravitational effects are large. We will not make such a restriction, but it's important to emphasize that CDL discovered examples of every phenomenon we will discuss, within the confines of their restricted approximations. One of the most important features of the CDL analysis is the way in which the nature of gravitational tunneling depends on the cosmological constants at the true and false minima. We will present this as evidence that the nature of the actual quantum theory is in fact quite different in the case of zero, positive and negative c.c. .

People often ask me why I place so much confidence in the CDL calculations, since I am always warning that too much reliance on the field theory approximation is dangerous. Indeed, in the proposals I will present below the metric of space-time is not a fluctuating quantum variable, but is instead determined by a rigid set of kinematic constraints on the quantum theory. I believe a reasonable analogy is presented by the Wilson loop variables of large N gauge theory. In the planar limit, the Wilson loop expectation value satisfies a classical field equation in loop space [24] and the $1/N$ expansion can be viewed as a sort of Feynman diagram (string loop) expansion around this classical equation. However, for finite N the Wilson loop operators are not independent canonical variables, and the Hilbert space of the perturbation expansion is too big. The true quantum variables are the gauge potentials in some physical gauge. Nonetheless, Euclidean solutions of the equations for Wilson loops can be used to find tunneling corrections to the $1/N$ expansion. However, the real reason for paying attention to the CDL results is that they can all be related to more fundamental concepts in the theory of QG; concepts like the holographic principle and the AdS/CFT correspondence. We will now proceed to classify gravitational tunneling events according to initial and final values of the c.c. .

4.2 No tunneling to or from AdS space

One of the most annoying aspects of this subject is the tendency of many speakers to talk about tunneling to AdS space. Perhaps the most important point in the CDL paper is the demonstration that this NEVER occurs, except in the thin wall approximation. To understand the result we write the CDL equations for the gravitational field coupled to a scalar via the Lagrangian

$$\mathcal{L} = \sqrt{-g}[R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)]$$

. We work in four dimensions, with a single field, for simplicity, but our conclusions are general. Given a scalar potential

$$V(\phi) = \mu^4 v(\phi/M),$$

the natural space-time scale for motion is $L = \frac{M}{\mu^2}$. If we make a Weyl transformation to dimensionless field variables (we use conventions where coordinates are dimensionless and the metric tensor has dimensions of squared length), and write an $O(4)$ symmetric ansatz:

$$ds^2 = L^2(dz^2 + \rho^2(z)d\Omega^2),$$

$$\frac{\phi}{M} = x(z),$$

where z is a dimensionless radial coordinate and ρ is the dimensionless metric coefficient, then we get Euclidean field equations

$$(\rho')^2 = 1 + \epsilon^2\rho^2[\frac{1}{2}(x')^2 - v(x)].$$

$$x'' + 3\frac{\rho'}{\rho}x' = \frac{dv}{dx}.$$

$\epsilon = \frac{M}{\sqrt{3}m_P}$, where m_P is the reduced Planck mass 2×10^{18} GeV. Note that although μ does not appear explicitly in these equations, it must be less than m_P for the semi-classical approximation to be valid (how much less is a matter of conjecture). Note also that the quantity in square brackets in the first equation is what would have been the "conserved energy" of the second equation in the absence of the friction term.

By convention, the center of the bubble is at $z = 0$ and in the vicinity of this point $\rho = z$. The solution is non-singular only if $x'(0) = 0$. The boundary condition at the upper end of the z interval depends on the c.c. in the false minimum. Our present considerations are independent of that boundary condition. To analytically continue the solution to Lorentzian signature we take $z = it$ and use the Euclidean solution at $z = 0$ as an initial condition for the Lorentzian evolution. The initial conditions are $\dot{x}(0) = 0$, $\rho(0) = 0$ and $x(0)$ a fixed value determined by the boundary conditions at the other end. It must be in the basin of attraction of the true minimum.

The Lorentzian equations are

$$\dot{\rho}^2 = 1 + \epsilon^2 \rho^2 [\frac{1}{2}\dot{\phi}^2 + v].$$

$$\ddot{x} + 3\frac{\dot{\rho}}{\rho}\dot{x} + \frac{dv}{dx} = 0.$$

These equations have an AdS solution in which x is equal to the true minimum of v for all time, and $\rho = \sin(\sqrt{\Lambda}t)$. However, the solution determined by the instanton does not approach this solution, which is unstable to infinitesimal perturbations which are homogeneous and isotropic[16]. Indeed, since the Euclidean solution completely fixes the initial conditions for Lorentzian evolution, $\dot{x}$ will not go to zero as $\rho \to 0$. The kinetic energy of x goes to infinity, because the universe is contracting and we have Hubble anti-friction. x will not stay near the true minimum, but will explore its whole potential surface. This singularity will be reached in a time of order $\frac{M}{\mu^2 \epsilon} \sim \frac{m_P}{\mu^2}$. In a typical particle physics model μ is unlikely to be smaller than a few hundred MeV, so this time is shorter than 10^{-5} sec. For future reference we note that, according to the holographic principle, this implies that an observer trapped in this region can access an entropy that is at most $\sim (\frac{M_P}{\mu})^4 < 10^{80}$, only $\sim 10^{60}$ of which can be in the form of matter and radiation. The actual matter/radiation entropy of our universe is $\sim 10^{80}$. The reader who is confused by these numbers, will be able to go back and check them after we discuss the holographic principle.

The converse of this result is also true: a quantum AdS space cannot decay by tunneling. This follows from the AdS/CFT correspondence. The exact mathematical formulation of CFT requires one to have only unitary highest weight representations of the conformal group in the Hilbert space. It follows that the global Hamiltonian $K^0 + P^0$ is bounded from below. But the Lorentzian continuation of an instanton is always a zero energy solution in which the positive and growing kinetic energy of the expanding bubble is balanced by

[16] AdS spaces are sometimes stable to small perturbations which fall rapidly at infinity. These are the normalizable fluctuations of the AdS/CFT correspondence. The homogeneous isotropic solutions relevant for instanton physics are not normalizable. Generally they have no extension outside the FRW coordinate patch, as a consequence of the singularity we are discussing.

the increasingly negative potential energy of its interior. It always corresponds to a system which is unbounded from below.

This general argument is exemplified in a beautiful paper by Hertog and Horowitz [25]. These authors found an instanton solution, which seemed to indicate a non-perturbative decay of the supersymmetric(!) $AdS_4 \times S^7$ solution of 11 dimensional SUGRA. Upon closer examination, they found that although the perturbation fell off at infinity, it was not a normalizable solution, corresponding to a state in the CFT. Rather, it corresponded to a perturbation of the CFT by a marginal operator that was unbounded from below.

The correct way to interpret these facts is to say that if we look at a classical bulk Lagrangian, which has an AdS solution, as well as an instanton which behaves like a normalizable perturbation of this solution at Euclidean infinity, then we will have proven, in the classical approximation, that the AdS Hamiltonian of this system is unbounded from below, and cannot have a CFT dual. It is likely that such a solution is not part of any sensible quantum theory of gravity. Indeed, there is an interesting sidelight on this situation, which already indicates that something serious is wrong with the interpretation of this instanton as a decay of the original AdS space-time.

In ordinary quantum field theory, excitations around the false vacuum are meta-stable only up to some finite energy. If we make the energy density larger than the barrier height the system is simply unstable. Similarly, the thermal ensemble is meta-stable only up to some finite temperature. In quantum gravity in large radius AdS space, we can explore the thermal ensemble by looking at AdS-Schwarzschild solutions of the field equations, which are normalizable and have positive energy of arbitrarily large size. These solutions do not have classical instabilities, indicating that the vacuum decay paradigm of non-gravitational QFT is breaking down.

We can gain more insight into this when we realize that the expanding bubble of the Lorentzian instanton *does not penetrate the interior of a black hole*. The bubble expands only at the speed of light, while the interior geometry expands away from the bubble superluminally. A solution whose initial conditions consist of a space-like separated pair of a black hole and a nucleated critical bubble, has two causally separated future asymptotic regions, both of them space-like singularities. Multiple black holes in the initial state will lead to multiple causally disconnected future regions. Furthermore, single the bubble nucleation probability is *exponentially small* as $\frac{m_P}{\mu}$ goes to infinity, it is easy to see that the black holes can have exponentially larger entropy than the entropy accessible within the bubble. These semi-classical considerations suggest very strongly that there is no sensible quantum mechanical interpretation of AdS solutions that have genuine instanton instabilities. Certainly the interpretation of the instanton as a decay of the original AdS "state" into the system in the interior of the CDL bubble, is completely untenable. This analysis goes through in precisely the same way for CDL "unstable" asymptotically flat space-times, although the existence of Hawking instabilities of black holes in that case, poses further complications.

Our conclusion is that AdS solutions of bulk gravitational field equations never arise as the result of CDL decays, and do not decay in a way that resembles the vacuum decay of a non-gravitational QFT. Some of the solutions are stable, and may well belong to a real theory of QG, which would be defined by a CFT dual. The unstable ones surely belong to a very peculiar quantum theory, if they have any meaning at all. There is thus strong evidence from CDL tunneling, complementing that from the AdS/CFT correspondence, that

AdS solutions of gravitational field equations form little isolated models of QG, which have nothing to do with a larger landscape.

4.3 Gravitational tunneling to and from zero c.c. states

In asymptotically flat space-time, the asymptotic symmetry algebra is the Poincare group. If we do not insist on supersymmetry[17] there is no general argument that the Hamiltonian is bounded from below. However, there is a classical theorem [26] which shows that asymptotically flat solutions of certain Lagrangians do have classically positive energy.

The paper [27] clarified how the space of theories consisting of scalar fields coupled to gravity is divided up by the positive energy theorem. Consider a potential with classically stable Minkowski and AdS solutions and ask whether there is a static domain wall connecting the two solutions. For the AdS/AdS case, we saw that such domain walls with boundary conditions that correspond to normalizable solutions on both sides of the wall, are the holographic representation of RG flows between two fixed points. No such interpretation is possible here, because the analog of the UV fixed point is the Poincare invariant model, which is not a quantum field theory. The equations determining the domain wall are

$$\rho'^{\,2} = \epsilon^2 (\frac{1}{2} x'^{\,2} + v(x))$$

$$x'' + \frac{3\rho'}{\rho} x' + \frac{dv}{dx} = 0,$$

with boundary conditions

$$x(\pm\infty) = x_{\pm}.$$

$x_{\pm}$ are the false and true maxima of $v(x)$.

As is familiar from linear eigenvalue problems, this system does not have solutions for a generic potential. In the limit in which we model the domain wall as an infinitely thin brane with a given tension, there will only be one value of the tension for which the static solution exists [11]. For tensions below this value there is instead a solution which looks like the asymptotic limit of an expanding bubble wall, corresponding to CDL decay of the Minkowski background (but missing the instability of the previous subsection, for which one must go beyond the thin wall approximation). For tensions above this there is no interpretation of the solution as the limit of an object in the Minkowski background.

More generally, as in any eigenvalue problem, we can find a solution obeying both boundary conditions by tuning a single parameter in the potential. *Thus, the space of all potentials with a Minkowski solution of the field equations contains a co-dimension 1 submanifold, on which a static domain wall connecting Minkowski space to one particular AdS minimum exists, while all for all other AdS minima there are neither domain walls nor expanding bubble solutions.* For a given Minkowski minimum there will generally be only one domain wall,

[17]It is one of my contentions that we MUST insist on SUSY *i.e.* that every asymptotically flat model of QG is in fact Super Poincare invariant. However, we are exploring more general possibilities in this section, and our explorations lead to important insights for the program based on my conjecture.

though in supersymmetric situations there may be more [18]. This submanifold in the space of potentials is called *the Great Divide*. By perturbation theory one can show that above the Great Divide the Lagrangian has a positive energy theorem, while below it there are expanding bubble solutions and the ADM energy is unbounded from below.

In [27] we showed that by varying the parameter $\epsilon \sim M/m_P$ in potentials of the form $\mu^4 v(\phi/M)$ we crossed the Great Divide. For $\epsilon \ll 1$, the non-gravitational analysis, which indicates an instability is essentially correct. However, it should be noted that even in this regime, CDL showed that one is above the Great Divide if $|v(x_T)|$, the magnitude of the c.c. at the true minimum, is $\ll 1$. The Great Divide itself is located at $\epsilon = o(1)$ for generic functions $v(x)$. For those models below the Great Divide, the same issues arise as for unstable AdS spaces. Starting from a generic excited state of the Minkowksi solution, we end up with a future that contains multiple causally disconnected space-like singularities, most of whose entropy is contained in black holes. Here however we have to deal with the perturbative Hawking instability of black holes, which returns the degrees of freedom of the black hole to a region causally connected to the expanding bubble. Here we can encounter a paradox: The matter entropy outside the bubble is bigger than that measurable by any observer inside the bubble. One suspects that we are being too naive and neglecting back reaction of all of this matter on the bubble. A possible scenario is that collisions of the bubble wall with a sufficiently large matter density, converts the bubble into a black hole. Indeed, the bare expanding bubble solution has exactly zero energy in empty space. If it collects a finite surface energy density as it passes through a region filled with a uniform density of matter, then it will end up with a mass of order the square of its radius. For large enough radius the Schwarzschild radius of this distribution will be larger than the bubble radius. Thus, a resolution of the apparent paradox of a bubble sweeping up more entropy than any observer inside it can measure, may simply be that in attempting to swallow all of this entropy, the bubble forms a black hole around itself.

The bizarre conclusion of this story would be that, perhaps, below the Great Divide, empty flat space is unstable, but flat space with enough entropy in it nucleates a black hole around the expanding bubble. Of course, another possibility is that there are no actual theories of quantum gravity which contain such meta-stable flat space-time configurations. When we discuss the holographic space-time formalism, we will show that it suggests that all quantum theories of asymptotically flat space-time are exactly supersymmetric. If this is the case then they are automatically Above the Great Divide. This does not yet settle the question of the fate of the asymptotically dS universe, which we appear to inhabit.

4.4 CDL transitions from dS space

If we take a potential below the Great Divide and add a small positive constant to it, we do not make a significant change in the CDL transition rate. The entire story of the previous section replays with little change. Above the Great Divide the story is different. With mild assumptions, there is *always* a CDL instanton when a potential has a positive and negative

[18]In the AdS to AdS case, static domain walls correspond to relevant perturbations of the CFT for smaller absolute value of the c.c. , which point along RG flows to other fixed points. Such flows are non-generic unless both fixed points are supersymmetric.

minimum separated by a barrier[19]. However, the transition rate goes to zero like

$$e^{-\pi (RM_P)^2},$$

as R, the dS radius goes to infinity. This suppression by something of order the inverse of the exponential of the dS entropy, is what we would expect for a transition at infinite temperature for a system with a large finite number of states, into a very low entropy state. This is consistent with our previous remark that, according to the holographic principle, the maximal entropy observable in the negative c.c. Big Crunch is a microscopic number. We will see below that the interpretation of dS space as a system with a finite number of states, at infinite temperature, is consistent with all semi-classical evidence about dS space, including its finite Gibbons-Hawking temperature!

Transitions from one dS space to another are also consistent with this picture, and add an extra bit of evidence. Indeed, although we have not emphasized it above, the instantons for transitions out of dS space are compact manifolds, with positive scalar curvature, just like Euclidean dS itself. And like Euclidean dS space they have negative action. The probability interpretation of the instanton calculation comes by subtracting the dS action from the instanton action, which always gives a positive number. In the case of dS to dS transitions, we get two different probabilities, depending on which dS action we subtract. These are interpreted as the probabilities for the forward and reverse transitions

$$P_{1\to 2} = e^{-(S_I - S_1)},$$
$$P_{2\to 1} = e^{-(S_I - S_2)}.$$

The ratio of transition rates is thus

$$e^{-(S_1 - S_2)}.$$

It is a quite remarkable fact (analogous to a result about black holes first discovered by Gibbons and Hawking), that the dS action is exactly the negative of the dS entropy. This means that these transition rates satisfy the principle of detailed balance appropriate for a system with a finite number of states at infinite temperature. Unlike the case of dS transitions to a negative c.c. Crunch, this semi-classical calculation is under control in both directions. It seems perverse to attach any other meaning to it than what it seems to say: dS space is a system with a finite number of states. Its Hamiltonian is generic and the time evolution of a randomly chosen initial state will sweep out the entire Hilbert space. The dS space with larger c.c. is a low entropy configuration of this system and will be accessed only rarely, in direct proportion to the fraction of the total number of states corresponding to this configuration.

Note that this interpretation meshes perfectly with the one we have proposed for dS to Crunch transitions above the Great Divide. Note further that it does not agree with ANY interpretation of the same transition according to the theory of Eternal Inflation.

[19] The exceptions come for potentials in which the maximum is very flat. Analogies with ordinary quantum mechanics lead us to expect a transition from the false minimum to the top of the barrier, which is more or less semi-classical, followed by large quantum fluctuations on the flat top. However, since the system includes gravity, we don't really know how to explore the regime of large quantum fluctuations. It is possible that potentials this flat are simply forbidden in real theories of QG [28] [29] [30].

As we take the smaller c.c. to zero, the transition rate to the higher c.c. state goes to zero. This makes sense in our interpretation, because the probability of finding a finite entropy subspace of states starting from a random search through an infinite dimensional Hilbert space, is zero. Note however that the limit of zero c.c. is a very subtle one. In the section on stable dS space, we will see that a lot of states must be discarded from the dS Hilbert space in order to describe the Hilbert space of the limiting Poincare invariant theory. The entropy of the latter scales as $(RM_P)^{3/2}$ as the dS radius goes to infinity, while the total entropy of the dS Hilbert space is $\pi(RM_P)^2$. We will see in the next subsection that the required limit for a tunneling solution whose target is a zero c.c. space-time is quite different. The interpretation of such solutions is intertwined with attempts to construct a theory of the String Landscape, and we turn to that problematic subject next.

4.5 Implications for the landscape

The implications of these results in semi-classical gravity for the idea of a string landscape are profound. Asymptotically flat and AdS models of quantum gravity *are not part of the landscape* and do not communicate with hypothetical landscape states by tunneling. Tunneling only makes sense for meta-stable dS points on an effective potential. These can tunnel to other dS points, to negative c.c. Big Crunches, and to zero c.c. states. None of the physics of these states is encoded in anything like the boundary correlators that string theory has taught us how to compute. *If the landscape exists, the very definition of its observables must be completely different from that of ordinary string theory.*

We have seen that tunneling to negative c.c. crunches falls into two categories. Above the Great Divide, we've provided a plausible quantum interpretation of the CDL tunneling probabilities, in terms of a quantum theory of stable dS space with a finite dimensional Hilbert space. Below the Great Divide, we've argued that these transitions are fraught with interpretational ambiguities. The true endpoint of CDL decay is not a quiescent true vacuum, nor even a single big crunch. The final state depends on which initial excited state of the dS or flat "false vacuum" one begins with. It typically has multiple crunching regions, with different pre-crunch internal geometries, which are causally disconnected from each other.

Nonetheless, many advocates of the landscape insist that any sensible meta-stable model of dS space must be below the Great Divide. The argument is somewhat philosophical, but depends crucially on the fundamental claim that the landscape solves the c.c. problem by invoking the anthropic principle. In order to be certain that this is true, one counts meta-stable landscape points, according to some criterion, and claims that the number is of order 10^{500} or greater. It is important that this number is *much larger* than the ratio between a Planck scale c.c. and the c.c. we observe. One then argues that if generic minima of the potential have a c.c. that is a sum of a such a large number of positive and negative Planck scale contributions[20], there will inevitably be some with c.c. of the value we observe. If anthropic arguments can show that a value bigger than this is incompatible with the existence of intelligent life forms, one has "explained" the small value of the c.c. Note that in order for this counting to work in a way that does not require close scrutiny of each and

[20] In the Bousso-Polchinski [31] version of this argument there is one large negative contribution and a large number of smaller positive ones. In KKLT, one argues for a set of AdS solutions, with enough free parameters to make the negative c.c. small, and then adds a small positive contribution.

every minimum of the potential, there must be MANY solutions with a value of the c.c. close to ours. There is no reason for other properties of the low energy world to be similar to those we see. So questions like what the low energy gauge group and representation content are, as well as the value of most low energy parameters, must also be answered anthropically.

I will not spend time here rehashing the futility or experimental implausibility of this claim, but rather emphasize the general picture of string theory that it implies: string theory has MANY solutions with small c.c. If we are not to regard our own world as simply an accidental consequence of the theory, then we must come up with some argument that makes our conditions more hospitable for observers than other possible meta-stable states. The advocates of these ideas are led to contemplate the question of whether we are *typical* members of the class of observers that the landscape predicts. The answer is that this can only be true if our current dS condition does not last too long.

We know that the universe we observe began in a state of much lower entropy than it has today. This is why we see the second law of thermodynamics in operation. The visible entropy of the universe is dominated by cosmic microwave background photons, and the total entropy by the supermassive black holes in the centers of galaxies. According to modern cosmology, this entropy was created in the post-inflationary history of the universe, through the decay of the inflaton field into radiation, and the gravitational collapse caused by the action of the fluctuations of this field on non-relativistic matter. In the landscape picture, the beginning of this cosmic history is a tunneling event from a higher c.c. meta-stable point, to our own basin of attraction. It is a very low entropy fluctuation.

If the current c.c. dominated era of the universe lasts too long, there is a much more efficient way to make observers than to have a fluctuation that recreates the entire history of the universe. Such a fluctuation, by the CDL calculation, has a probability of order $e^{-10^{123}} = e^{-A/4}$, where $A = 4\pi(RM_P)^2$. On the other hand, in the asymptotic future dS space, the probability to have a random fluctuation that creates a localized mass equal to that of a "single intelligent observer" is $e^{-2\pi R m_O}$ and the probability that that mass is in the state corresponding to a live intelligent observer is at least $e^{-\frac{A_O}{4}}$, where A_O is the horizon area of a black hole which could enclose the observer. These ridiculously tiny probabilities, are much larger than the probability of the fluctuation that started the universe off. So, either the landscape explanation of the origin of our universe is wrong, or we are far from typical observers, *or the dS state must decay long before all these typical observers can be formed.* This is only possible if our meta-stable dS state is below The Great Divide, which is the choice made by many landscape theorists. As we will see, this claim creates some tension with the only extant proposal for making a true theory of the string landscape.

The attempt to create a true theory of the landscape, analogous to our models of asymptotically flat or AdS spaces has been centered entirely in the Stanford-Berkeley group. The proposal is that the observables of the theory somehow reside in the causal diamond of a post-tunneling event into a zero c.c. region of the potential, which locally approaches one of the maximally supersymmetric flat space solutions of string theory. The original idea was to construct a sort of scattering theory in the Lorentzian space-time defined by the CDL instanton. It's indeed true that if we consider quantum field theory in such a space-time, one can define scattering states on the past and future boundaries. However, as I suggested in 2004, and was later proven rigorously by Bousso and Freivogel, generic scattering boundary conditions do not lead to small perturbations of the instanton geometry. This can be understood in a heuristic manner. The CDL geometry has a compact throat connecting its past and future regions. If we have an initial or final state with too large an entropy, it will create a black hole of radius larger than the throat. This leads to a space-filling space-like singularity, cutting the future off from the past.

The second proposal was to try to construct an analog of AdS/CFT where the CFT lives on the boundary of the negatively curved space-like slices of the CDL geometry. It is argued that the appropriate boundary conditions for this situation allow quantum fluctuations of the boundary geometry, so that the boundary CFT is coupled to quantum gravity. The hope is that this situation is well defined when the boundary is two dimensional, and leads to a boundary Liouville theory. Two dimensional boundaries are appropriate for 4 dimensional dS spaces, so this proposal relies on the folk theorem that there are no dS solutions of SUGRA above 4 dimensions.

I do not understand the details of this construction or the enthusiasm of its builders, so I will end this section with a list of questions that I think must be answered, if this approach is meaningful.

- What is the probability interpretation of the boundary field theory? Only some of the extant theories of fluctuating two geometries have a quantum mechanical interpretation. In those, the genus expansion is the *divergent* $1/N$ expansion (actually the double scaling limit) of a matrix quantum mechanics. In this context the genus expansion is said to converge. What are the probability amplitudes and what do they have to do with real world measurements? Is the theory quantum mechanics? What are the possible initial states?

- Most of the asymptotically SUSic regions of moduli space are decompactification limits, where the local asymptotically flat space-time has dimension higher than four. Why are only two dimensional boundaries relevant? One may want to argue that the theory has a two dimensional boundary for all finite FRW times, but the decompactifying dimensions should at least show up as an infinite number of low dimension operators. The formalism has so far restricted attention to massless bulk fields, but surely massive fields whose mass asymptotes to zero must be part of the picture?

- The construction is based on a particular instanton for decay of a particular meta-stable dS point into a particular locally flat geometry. How do all the other instantons fit into the picture? There must be some sort of monstrous duality in which the observables are actually independent of the choice of instanton geometry in the construction?

- Conversely, how does one pick out of the Liouville/CFT observables, the data relevant to our particular universe? This is of course a crucial step in trying to relate these ideas to the real world. Is there any relation between the answer to this question and the practices of those landscape enthusiasts who simply do effective field theory in a particular dS state? Is the answer to this problem computationally effective? That is, can one really hope to separate out the data corresponding to individual members of the 10^{500} strong ensemble?

- The construction purports to be a rigorous definition of what is meant by the phrase *eternal inflation*. What is its prescription for the solution of the *measure problem* in that context? (Some progress has been made on the answer to this question, but not enough to support phenomenological predictions).

- The transition from a dS space with small positive c.c. and one of the zero c.c. regions of the potential, is above the Great Divide. Supposedly one is saved from the problem of fluctuating intelligent observers by much more rapid decays into negative c.c. crunches. We are then left with the bizarre situation in which all of the rigorously defined data about our universe can only be measured in an extremely improbable history for the universe, one in which it lasts long enough for all sorts of fluctuated intelligences to exist.

I will not comment further on this proposal, except to mention that I personally find the challenges of Holographic Space-Time and Cosmological SUSY breaking much less daunting, and their connection to actual observations infinitely more direct. We turn next to an explanation of the Holographic space-time formalism.

5 Holographic space-time

Having devoted much verbiage to the description of what a theory of QG is *not* we are now ready to propose a general description of what it is. This framework is meant to subsume all of the well defined models we have discovered, which fall under the rubric string/M-theory. That claim has not yet been proven, and I will admit from the beginning that a fully dynamical implementation of the rules of *Holographic space-time* has not yet been found.

All well established models in the string/M-theory menagerie belong to one of two classes. The first corresponds to space-times in dimension $3 \leq d \leq 7$ with AdS asymptotics and an AdS curvature radius that can be taken parametrically large, in the sense that there is a closed set of boundary correlation functions, which can be calculated in a systematic expansion about the GKP/W [32] SUGRA limit. They all have exact AdS SUSY.

The observables in these models are correlation functions on a boundary of the form $R \times S^{d-1}$. In addition, there are many models of asymptotically flat space-time with dimension between 4[21] and 11. The only observable is the S-matrix. All of these models have exact super-Poincare invariance.

In addition there are models which can be viewed as describing certain infinite branes embedded in these spaces. In the AdS case, these are relevant perturbations of the CFT describing the original symmetric model[22] . These models need not be supersymmetric, but they are "supersymmetric in the majority of space-time". In the language of CFT, this means that the high energy, short distance behavior is dominated by a supersymmetric fixed

[21] In 4 dimensions we do not really have a complete theory of a gravitational S-matrix, because the analog of the Fadeev-Kulish [33] construction for electrodynamics has not been carried out.

[22] In calling these infinite branes, I am working in the Poincare patch of AdS space, which corresponds to the Hilbert space of CFT in Minkowski space. The corresponding solutions in global coordinates are localized at the center of a global coordinate system. There are also true brane solutions with AdS asymptotics, analogous to D-branes embedded in flat space-time.

point. Although there are many claims in the literature, there are no well established models with nonsupersymmetric fixed points at large curvature radius.

We want to construct a more local description of QG, which will reduce to these supersymmetric models in the infinite volume limit, but which will enable us to describe systems that do not fall into any of these categories, like cosmologies and the real world. In GR, local objects are never gauge invariant, so we should expect our description to be adapted to a certain coordinate system. Indeed, the fundamental postulates of the theory will contain in themselves an explanation for why local physics can never be gauge invariant in QG, a sort of quantum version of the principle of general covariance.

The basic principles of holographic space-time are simple to state:

- The Strong Holographic Principle (Banks-Fischler) - A causal diamond is the intersection of the interior of the backward light-cone of a point P with that of the forward light-cone of a point Q in the causal past of P. The boundary of a causal diamond is a null surface. When we foliate it with space-like $d-2$ surfaces, we find one of maximum area, called the holographic screen. According to the holographic principle, the quantum version of such a causal diamond is a Hilbert space whose dimension is $e^{\frac{A}{4L_P^2}}$, where A is the area of the holographic screen. This formula is asymptotic for large area. The proper quantum concept is the dimension of the Hilbert space, which is of course always an integer.

- Intersections of causal diamonds correspond to common tensor factors in the Hilbert spaces of two diamonds. Geometrically this defines the area of the maximal causal diamond which fits in the intersection. Thus we have

$$\mathcal{H}_1 = \mathcal{O}_{12} \otimes \mathcal{N}_1$$

$$\mathcal{H}_2 = \mathcal{O}_{12} \otimes \mathcal{N}_2.$$

  This encodes the causal structure of the space-time, if we have a rich enough collection of causal diamonds. We ensure this by beginning from a lattice, which encodes the topology of an infinite space-like slice (a Cauchy surface) of the manifold. For each lattice point $\mathbf{x}$ we have a sequence of Hilbert spaces $\mathcal{H}(n, \mathbf{x}) = \otimes \mathcal{P}^n$, where $\mathcal{P}$ is a finite dimensional space we will define below. Geometrically this represents a sequence of causal diamonds whose future tips have larger and larger proper time separation from the initial space-like slice. For a model of a Big Bang space-time we imagine the past tips to lie on the Big Bang hypersurface. This incorporates the idea that the particle horizon is very small near the singularity, but it is clear that nothing singular happens in the quantum theory. For a time-symmetric space-time we take the lattice to lie on a time-symmetric space-like slice, and the past and future tips of the diamonds lie an equal proper time before and after the time-symmetric slice.

- For nearest neighbor points on the lattice, at any n, we insist that the overlap Hilbert space is $\otimes \mathcal{P}^{n-1}$. We interpret these sequences of Hilbert spaces as the sequence of causal diamonds of time-like observers, which penetrate the chosen space-like slice at a given lattice point. The proper time interval between the tips of the nth diamond is a monotonically increasing function of n. Thus, two nearest neighbor sequences of Hilbert spaces, correspond to two time-like observers whose trajectories through space-time are almost identical. The overlaps between other points are constrained by two consistency conditions. Let $d(\mathbf{x}, \mathbf{y})$ denote the minimum number of lattice steps between two points. We require that the overlap not increase as we follow a path of increasing d, starting from $\mathbf{x}$, and that it decrease asymptotically as $d(\mathbf{x}, \mathbf{y})$ goes to infinity.

- The second consistency condition is dynamical. Let $N(\mathbf{x})$ be the maximal value of n at a given lattice point. We prescribe an infinite sequence of unitary operators $U_k(\mathbf{x})$,operating in the Hilbert space $\mathcal{H}(N(\mathbf{x}), \mathbf{x})$, with the property that for $k \leq N(\mathbf{x})$ $U_k = I_k(\mathbf{x}) \otimes O_k(\mathbf{x})$, where I_k is a unitary in $\mathcal{H}(k, \mathbf{x})$ while O_k operates in the tensor complement of this Hilbert space in $\mathcal{H}(N(\mathbf{x}), \mathbf{x})$. This sequence is interpreted as a sequence of *approximations to the S-matrix* in the time symmetric case, and a sequence of *cosmological evolution operators* in a Big Bang space-time. We then encounter the following set of fearsomely complicated consistency conditions. Consider the overlap Hilbert space $\mathcal{O}(m, \mathbf{x}; n, \mathbf{y})$. The individual time evolutions in $\mathcal{H}(N(\mathbf{x}), \mathbf{x})$ and $\mathcal{H}(N(\mathbf{y}), \mathbf{y})$, each prescribe a sequence of density matrices[23] on $\mathcal{O}(m, \mathbf{x}; n, \mathbf{y})$. These two sequences must be conjugate to each other by a sequence of unitary transformations. A collection of Hilbert spaces with prescribed overlaps, and evolution operators, satisfying all the consistency conditions, is our definition of a quantum space-time.

It's clear from this list, that any quantum space-time, which approximates a Lorentzian manifold when all causal diamonds have large area, will completely prescribe both the causal structure and the conformal factor of the emergent geometry. *We conclude that in this formulation of QG, space-time geometry is not a fluctuating quantum variable.* Given the results of [34] it is likely that any geometry that emerges from this framework will satisfy Einstein's equations with a stress tensor obeying the dominant energy condition. This is because the quantum system will obey the laws of thermodynamics, and those authors claim that this is enough to guarantee Einstein's equations, given the Bekenstein-Hawking connection between area and entropy. Indeed, if we imagine *defining* the stress energy tensor as the right hand side of Einstein's equations, then the only content of those equations is whatever energy conditions we impose. The holographic framework will certainly impose conditions sufficient to prove the area theorem.

This observation is completely in accord with our semi-classical conclusion that different asymptotic behaviors of space-time, even if they are solutions to the same set of low energy field equations, correspond to different models of QG. The holographic construction extends this principle to space-times whose boundaries are not simple conformal transforms of static geometries. One might object that the standard Feynman diagram construction of perturbative QG could not possibly be consistent with such a picture. This is not true. These expansions only describe particles, including gravitons, propagating in a fixed space-time background. Thus, to be consistent with them, one must only require that the quantum

[23]There is no reason for the state on the overlap to be pure. It is entangled with the other degrees of freedom in each causal diamond.

variables describe arbitrary scattering states of gravitons in asymptotically flat or AdS space-times (or any other example over which we claim to have good semi-classical control).

Our next task is to introduce just such variables.

5.1 SUSY and the holographic screens

Consider a "pixel" on a holographic screen. Naively, it's characterized by a null vector and a plane transverse to it, describing the orientation of this pixel in space-time. This is the information content of solutions of the Cartan-Penrose equation

$$\bar{\psi}\gamma^{\mu}\psi(\gamma_{\mu})^{\alpha}_{\beta}\psi^{\beta} = 0,$$

where ψ is a commuting Dirac spinor. Indeed, this equation implies that $n^{\mu} = \bar{\psi}\gamma^{\mu}\psi$ is a null vector, and that ψ itself is a transverse or null-plane spinor corresponding to this null vector. That is, if $\gamma^{\mu_1 \cdots \mu_k}$ are anti-symmetrized products of Dirac matrices with $k \geq 2$ then

$$\bar{\psi}\gamma^{\mu_1 \cdots \mu_k}\psi$$

are non-zero only for hyperplanes embedded in a particular $d-2$ plane transverse to n^{μ}. The spinor has only $2^{[\frac{d-2}{2}]}$ independent components. In eleven dimensions this is 16 real components, S_a.

The holographic principle implies that the Hilbert space of a pixel should be finite dimensional, so the only operator algebra we can write down for the S_a, consistent with transverse rotation invariance, is

$$[S_a(n), S_b(n)]_+ = \delta_{ab}.$$

n is a label for the pixel, which we will discuss in a moment. This algebra is the same (up to normalization) as that of a massless superparticle with fixed momentum in 11 dimensions. The smallest representation is the 11D SUGRA multiplet, and all the others correspond to particles that, according to the Coleman-Mandula theorem, cannot have an S-matrix different from 1. If we think about different pixels, they should have independent degrees of freedom, and we would normally ask that the corresponding operators commute. However each of the individual pixel algebras has an automorphism $S_a(n) \rightarrow (-1)^{F(n)} S_a(n)$, which we treat as the Z_2 gauge symmetry called $(-1)^F$. We can use this to choose a gauge where spinors corresponding to different pixels anti-commute

$$[S_a(m), S_b(n)]_+ = \delta_{ab}\delta_{mn}.$$

The spin-statistics connection familiar from local field theory is thus built in to the holographic formalism.

Now let us think about the notion of pixel. The holographic principle again requires that a finite area holoscreen should have a finite number of pixels, to each of which we assign a copy of the single pixel algebra. The naive notion of pixel can be thought of as a way to approximate the algebra of functions on the holographic screen by the algebra of characteristic functions of a finite cover of the screen by open sets. This opens the door to more general approximations of the algebra of functions by finite dimensional algebras that are not necessarily commutative. This has numerous advantages. For example, in the case relevant to the real world, a two dimensional holographic screen with $SO(3)$ rotation invariance, we can use the fact that $SU(2)$ has finite dimensional representations of every integer dimension to construct the so called fuzzy sphere. The algebra of $N \times N$ matrices inherits a natural action of $SU(2)$, which contains all integer spins between zero and $N-1$. It approximates the algebra of functions on the sphere by the usual finite sums of spherical harmonics. The specification of whether we get smooth, continuous, measurable or square integrable functions is encoded in the behavior of the expansion coefficients for large spin.

More generally, if the holographic screen has a Poisson structure, there is a well developed theory of deformation quantization, which, for compact manifolds, leads to a sequence of approximations to the algebra of smooth functions by finite dimensional matrix algebras. In general, this procedure has ambiguities; the analog of the usual ordering ambiguities in quantum mechanics. However, for Kahler manifolds there is much less ambiguity. The space of sections of a holomorphic line bundle over a Kahler manifold is finite dimensional and has a natural Hilbert space structure induced by the Kahler potential. If we take sequences of holomorphic line bundles with dimension going to infinity, we get natural fuzzy approximations to the manifold. Almost all of the manifolds that arise in string compactification are related to Kahler manifolds in some way. Calabi-Yau manifolds are an obvious example, and the Horava-Witten bundles of Calabi-Yau manifolds over an interval are another. It is not known whether general G_2 manifolds have a Poisson structure, but those which exhibit non-abelian gauge groups, are $K3$ fibrations over a sphere or lens space. A choice of Kahler form on the $K3$, combined with the unique $SO(3)$ invariant Poisson structure on S^3 or a lens space, defines a Poisson structure on the entire 7-fold.

Combining these ideas, we obtain a general prescription for compactification of holographic space-time. For compactifications to 4 dimensions we introduce variables satisfying the commutation relations

$$[(\psi^M)_i^A, (\psi^\dagger N)_B^j]_+ = \delta_i^j \delta_B^A Z^{MN} \quad i = 1 \ldots K, \quad A = 1 \ldots K+1.$$

The operators ψ and $\psi^\dagger$ are $K \times K+1$ and $K+1 \times K$ matrices, sections of the two spinor bundles over the fuzzy 2-sphere, the holographic screen for 4 dimensional space-time. The indices M, N can be thought of as either minimal spinor indices in 7 dimensions or $(2,0)$ or $(1,1)$ spinors in 6. We know that in string compactifications with 8 or more supercharges, these different interpretations morph into each other as we move around in moduli space. In the interior of moduli space, where we expect the real world to lie, it may be that no particular geometric description is picked out. To be more precise, M and N label a basis in the space of sections of the spinor bundle on the appropriate manifold, appropriately truncated. This gives us a possible new insight into string dualities. It is well known for example that the algebra of $N \times N$ matrices can actually be thought of as a fuzzy approximation to the space of functions on *any* Riemann surface. The topology and geometry of manifolds emerges from fuzzy geometry in the large N limit, by discarding different sets of matrices in the definition of the limiting algebra. In the interior of moduli space in string theory, where the string coupling is not weak, compact manifolds have volumes that are finite in Planck units and should therefore be thought of as finite pixelations. The dual geometry is obtained by taking a different large N limit.

The operators Z^{MN} are sums of $p - forms$ and we may think of them as measuring the charges of branes wrapped around cycles of manifolds. More precisely, each $p-form$ component of Z^{MN} will be a sum of terms, each of which has such an interpretation. Specifying the number of terms in this sum, for each p will tell us the number of independent $p - cycles$ in the manifold. In the string theory literature, the Z^{MN} are often called central charges in the SUSY algebra. However we know that there are interesting examples of singular manifolds, where their algebra is non-abelian, and this gives rise to Yang-Mills gauge potentials in the non-compact dimensions.

We have suppressed another set of matrix indices in the formula for the anti-commutation relations above. Our internal spinors and $p - forms$ are really sections of the corresponding bundles over some fuzzy approximation to the internal manifold. The enumeration of cycles in the previous paragraph is part of the structure of these bundles. The geometry and topology of the manifold are all encoded in the super-algebra of the generators $\psi, \psi^{\dagger}, Z$. The smallest representation of this super-algebra, for fixed i, A, is the pixel Hilbert space $\mathcal{P}$ referred to above.

An extremely interesting consequence of this method of compactification is that fuzzy manifolds differ from each other discretely. There are no moduli. This is a direct consequence of the holographic principle and has nothing to do with dynamical minimization of potentials. We have noted above that space-time geometry is part of the kinematical framework of holographic space-time. Our discussion of semi-classical gravity and the principle that different solutions of the same gravitational field equations can correspond to different quantum models, rather than different states of the same model, here finds its ultimate justification. Continuous moduli can emerge from the holographic framework when we take the dimension of the function algebra to infinity. There can be different ways to do this, and quantities which go to infinity simultaneously at fixed ratio, define continuous moduli of the limiting geometry.

For example, a fuzzy compactification of a Kahler manifold is provided by the algebra of matrices in the space of holomorphic sections of a line bundle over the manifold. The dimension of this space is fixed by the element of the Picard group, which characterizes the line bundle. These elements are labeled by quantized $U(1)$ fluxes threading two cycles of the manifold and (for ample bundles) the dimension goes to infinity along directions in the Picard group where the fluxes go to infinity. But there are many such directions if the manifold has many two cycles, and the ratios of fluxes through different cycles define continuous Kahler moduli of the limiting manifold.

Note that one cannot really take this kind of limit for a single pixel, or rather if one does so then one has taken the four dimensional Planck length to zero. This would define, at best, a free theory, analogous to free string theory, or at least an interacting subsector that decouples from gravity. The moduli problem of conventional string theory is a result of taking this sort of limit as the starting point of the theory, and then perturbing about it. This remark is even more striking in the context of the theory of stable de Sitter (dS) space that we present in the next section. It follows from the above remark, and the assumption that this theory has a finite number of quantum states, *that it has no moduli.* Furthermore, for a fixed value of the c.c., the volume of the internal manifold in Planck units is severely limited, and the limitation is related to the scale of SUSY breaking! We will deal with this in more detail below, but the essential point is that the dimension of the Hilbert space of the theory is $\pi(RM_P)^2 = K(K+1)\mathrm{lnD}$, where D is the dimension of $\mathcal{P}$, and R the dS radius. Using conventional Kaluza-Klein ideas, we find $\mathrm{lnD} = (M_P/M_D)^2 = (VM_D^{D-4})$. Here D is either 10 or 11, M_D is the D dimensional Planck mass and R the four dimensional dS radius. We will see that the parameter that controls the validity of any four dimensional effective field theory description is $K^{-1/2}$. Thus, a good field theory approximation, for fixed RM_P, requires VM_D^7 to be bounded.

The key restriction on compactifications in this framework is that the algebra of a single pixel should have a representation with precisely one graviton and gravitino in the $K \to \infty$ limit. The classification of such algebras is one of the two central goals of the holographic space-time program. The other is to find equations that determine the scattering matrix. By the way, our focus on four dimensional compactifications is motivated by the search for dS solutions of SUGRA. In the limit (ΛM_P^d) small, a quantum theory of dS space should produce a de Sitter solution of a SUGRA theory. The only known SUGRA Lagrangians that have such solutions, and which also correspond to true compactifications are Lagrangians with minimal SUSY in $d = 4$. Such Lagrangians can have many chiral multiplets, with a relatively unconstrained Kahler potential and superpotential, which can easily have dS minima.

6 The theory of stable dS space

The global geometry of dS space is described by the metric

$$ds^2 = -dt^2 + R^2 \cosh^2(t/R) d\Omega_3^2,$$

where $d\Omega_3^2$ is the metric on a unit 3 sphere. As in asymptotically flat or AdS spaces, we can obtain useful information about the quantum theory by investigating perturbations, which do not disturb the asymptotic behavior. Since most ways of foliating this geometry give compact spatial sections, the asymptotic regions to be considered are past and future infinity.

To get an idea of the constraints on such perturbations, consider the exercise of setting small masses m on each point of the sphere, *i.e.* making the "co-moving observers" physical. If we do this at global time T, and space the masses by the particle's Compton wavelength (since in a quantum theory, no particle can be localized more precisely than that), then at $t = 0$ the particle number density is

$$m^3 \cosh^3(T/R),$$

and the 00 component of the stress tensor is exponentially large if $T \gg R$. In other words, long before $t = 0$, the back reaction on the geometry of the test masses becomes important. In order to avoid this, we must make $m \sim \cosh^{-1}(T/R)$ at time T. This strongly suggests that, *if we want to preserve dS asympotics in the future, we must not try to fill the apparently huge volumes of space available in the past with matter.* Rigorous results along these lines have been obtained in [35] [36]. The conclusion of those studies is that if one inserts too much matter in the infinite past, then a singularity forms before $t = 0$. If the singularity can be confined within a marginally trapped surface of radius $< 3^{-\frac{1}{2}}R$, this can be viewed as a black hole excitation of dS space, but if not, the whole space-time experiences a Big Crunch and we are no longer within the class of asymptotically dS space-times.

It is much simpler to understand the finite entropy of dS space, and the arguments that this represents a Boltzmann counting of the total number of quantum states corresponding to the thermodynamic equilibrium state called "the dS vacuum", from the point of view of static coordinates, where

$$ds^2 = -d\tau^2 f(r) + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2,$$

and

$$f(r) = (1 - \frac{R_S}{r} - \frac{r^2}{R^2}).$$

The parameter $R_S d \equiv \frac{2M}{M_P^2}$ is the Schwarzschild radius of a Schwarzschild-de Sitter black hole, and R is the dS radius of curvature. Empty dS space corresponds to $M = 0$. These coordinates cover the maximal causal diamond of a time-like geodesic observer in dS space.

Only the τ translation plus $SO(3)$ rotation generators preserve the static coordinate patch. If we consider quantum field theory on the full dS manifold, then there is an action of the dS group on the field theory Hilbert space, and for free fields, a unique Gaussian state whose two point functions approach those of the Minkowski vacuum at short distances. It has been known for a long time that [37] that this is a thermo-field state for the thermal density matrix in the static patch[24], with temperature $T = \frac{1}{2\pi R}$. Alternatively, this is the state which is chosen by analytic continuation of Euclidean functional integrals on the 4-sphere.

QFT in this geometry actually has an infinite number of states at very low energy, where energy is defined as conjugate to the time τ, at $r = 0$. $f(r)$ vanishes near the horizon, $r = R$, so there is a red shift of finite near horizon frequencies to low frequencies at the origin. If one uses the boundary conditions imposed by the so called Bunch-Davies vacuum on the global dS manifold, one finds an infinite number of states of arbitrarily low energy. It is important to realize that this is exactly the same infinity encountered in global coordinates. At $\tau = 0$ the global geometry has only a finite size and all states are localized in the causal diamond (the other half of the global geometry is just a trick, the thermo-field double trick-for computing thermal averages in the causal diamond). As $\tau \to \infty$, nothing falls through the horizon. Rather things get pasted closer and closer to the horizon and they redshift.

Within a causal diamond the infinity is analogous to the infinity of near horizon states of a black hole. And, as in the black hole case, there is a claim that the entropy of dS space is finite and equal to one quarter of the horizon area in Planck units. As with the black hole, we must think of this entropy as representing the maximally uncertain density matrix of the near horizon states, which means that the number of states is finite.

Quantum field theory in a fixed space-time background encourages us to think of dS space as having an infinite number of independent horizon volumes, which are causally disconnected from each other. The thermal entropy of a given horizon is interpreted as a finite entanglement entropy between causally disconnected states of this infinite system. This is supposed to explain the fact that the entropy depends only on the area. We have seen

[24]This is a direct generalization of Israel's discussion [38] of the Hartle-Hawking vacuum in the Kruskal manifold.

however that the myth of independent horizon volumes is untenable because of gravitational back reaction. Our global considerations suggest a total number of states for an eternal dS space, which is of order the exponential of the Gibbons-Hawking entropy.

I will first outline some general properties of a theory of global dS space, and then a more specific proposal, based on a cartoon of the pixel algebra described in the previous section. In my opinion the correct theory will require us to understand the list of consistent compactifications, which might be quite sparse. It is still within the realm of possibility that there is only *one* consistent answer, and that it describes the real world.

6.1 The two Hamiltonians of Wm. de Sitter

Our theory of dS space has two Hamiltonians. The first, H, has a random spectrum, distributed in an interval of order $T = \frac{1}{2\pi R}$. Starting from a random initial state, that Hamiltonian will generate expectation values for most operators, which quickly become identical to their thermal averages in the maximally uncertain density matrix. The number of states in the Hilbert space is of order $e^{\pi (RM_P)^2}$, and the average level spacing is $Te^{-\pi (RM_P)^2}$. There will be recurrences on time scales of order $Re^{\pi (RM_P)^2}$.

On time scales less than R, H evolution will not make much of a change in the state. We will postulate another Hamiltonian P_0, which is useful for describing some of the states of the system over these shorter time scales. P_0 will be the operator which approaches the Hamiltonian of a super-Poincare invariant system in the limit $RM_P \to \infty$. It will also be the appropriate operator to identify with approximate descriptions of the system in terms of quantum field theory in a background dS space[25].

In order to understand it, we must first understand which states of the system have such a field theoretic description. A local observer can see only a region of physical size R, so we must ask how many field theory like states can fit in such a region. The density of states of field theory in finite volume grows with energy and the entropy of field theory states in a region of linear size R is of order

$$(RM_c)^3,$$

where M_c is the UV cutoff. The energy of a typical state in this ensemble is

$$E \sim M_c^4 R^3.$$

These estimates are valid as long as the gravitational back reaction is small, a criterion which definitely fails once the Schwarzschild radius E/M_P^2 is of order R. Thus, we must have

$$M_c^4 R^2 < M_P^2,$$

which means that the entropy is of order $(RM_P)^{3/2}$, much less than the total dS entropy. Most of the localized states in the horizon volume are black holes whose radius scales like

[25] This is somewhat confusing since that Hamiltonian is usually associated with the static dS time coordinate. If we look at the action of the corresponding vector field at some interior point of the static observer's causal diamond, the static Hamiltonian converges to the Poincare Hamiltonian. However, they have very different actions on the cosmological horizon. One should identify H with the quantum operator that implements static time translation on the horizon, while P_0 is the corresponding action of the Poincare vector field.

the horizon volume, and these states do not have a field theoretic description in the horizon volume.

These estimates are valid for any low curvature region, and are similar to the deficit between the entropy of a star and that of a black hole of the same radius. In dS space however we can interpret the extra states as $(RM_P)^{1/2}$ copies of the field theoretic degrees of freedom in a single horizon volume. This allows us to understand the picture of an infinite number of horizon volumes predicted by QFT in curved space-time. As with a black hole, one should postulate a complementarity principle [39], according to which the global description, is a description of the same system as that in static coordinates. In the first the states are interpreted as being localized in different regions, while in the static coordinates the same set of states is seen as piled up at the horizon. The time evolution operators corresponding to the two descriptions do not commute with each other. In both the black hole and dS systems, the holographic principle provides an infrared cutoff on the number of states attributed to the system by QFT in curved space-time.

Recall that the Schwarzschild-de Sitter metric is

$$ds^2 = -dt^2 f(r) + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2,$$

where $f(r) = 1 - \frac{R_S}{r} - \frac{r^2}{R^2}$. The black hole mass parameter is given by $2M = M_P^2 R_S$. This metric has two horizons with

$$R^2 = R_+^2 + R_-^2 + R_+ R_-$$

and

$$R_S R^2 = R_+ R_- (R_+ + R_-) = R_+ R_- \sqrt{R^2 + R_+ R_-}.$$

Note that the total entropy of this configuration *decreases* as the black hole entropy $\pi(R_- M_P)^2$ *increases*. There is a maximal black hole mass at which the Schwarzschild and cosmological horizon radii coincide and equal $R_N = \frac{1}{\sqrt{3}} R$. The maximal black hole is called the Nariai black hole.

This entropy formula suggests a model of the system in which the Hilbert space has a finite number of states with logarithm $\pi(RM_P)^2$. Localized states are special low entropy configurations with an entropy deficit, for small R_S[26]

$$\Delta S = 2\pi R M.$$

If we interpret M as the eigenvalue of a Hamiltonian we will call P_0, this relation between the eigenvalue and entropy deficit indicates that the maximally uncertain density matrix is effectively a thermal distribution

$$\rho \propto e^{-2\pi R P_0},$$

for eigenvalues of P_0 much less than the Nariai black hole mass. As a consequence, the Poincare Hamiltonian, a generator acting on localized states in a single cosmological horizon

[26]For general M, the entropy deficit is $\Delta S = 2\pi R M (1 + \frac{\Delta S}{S})^{-\frac{1}{2}}$, which indicates that large black holes are present with somewhat larger than thermal probability.

of dS space, which converges to the the Hamiltonian of the super-Poincare invariant limiting theory when the c.c. goes to zero, can be written

$$P_0 = \sum E_n P_n .$$

The P_n are commuting orthogonal projection operators, with

$$\text{Tr } P_n = e^{\pi (R M_P)^2 - \delta S_n} .$$

$\Delta S_n = 2\pi E_n R$, when $E_n \ll M_P^2 R$, and near the maximal mass is given by the formula in the previous footnote.

We can summarize the previous few paragraphs by saying that the Bekenstein-Gibbons-Hawking formula for the entropy of black holes in dS space motivates a model for the quantum theory of dS space in which empty dS space is interpreted as the infinite temperature ensemble of a random Hamiltonian H bounded by something of order the dS temperature ($||H|| \leq cT$)[27]. This implies that localized black hole states are low entropy deformations of the vacuum, and gives a connection between the black hole mass parameter, which is the eigenvalue of another Hamiltonian P_0, and the entropy deficit of its eigen-spaces. This observation leads us to expect what we already know to be true: *the dS vacuum is a thermal state for quantum field theory with a unique temperature* $T = \frac{1}{2\pi R}$, and the present discussion can be viewed as an explanation of that fact from a more fundamental point of view. It is particularly satisfying that this explanation provides a rationale for the uniqueness of the dS temperature.

There is another piece of semi-classical evidence that this picture is valid. The Coleman-DeLucia formalism gives us an unambiguous calculation of the transition rates between two different dS spaces. As discussed above, the CDL formula implies that the ratio of the two transition rates is given by the *infinite temperature limit of the principle of detailed balance.* This is in perfect accord with our model of the dS vacuum as the infinite temperature ensemble in a Hilbert space of finite dimension. Similarly, Ginsparg and Perry [40] and Bousso and Hawking [41] have found instantons for the nucleation of black holes in dS space, and their results are completely consistent with the framework outlined above.

6.2 Towards a mathematical theory of stable dS space

It is my belief that the theory of dS space only makes sense in 4 dimensions. This follows from the basic principles I've enunciated, plus a knowledge of low energy effective field theory. The basic principle we use is that SUSY is restored as the c.c. goes to zero, with the gravitino mass going like $m_{3/2} = 10K\Lambda^{1/4}$. We will give two arguments for this behavior below. This formula implies that SUSY breaking must be describable in low energy field theory, which in turn implies that it must be spontaneous, since the gravitino mass and decay constant are much smaller than the Planck scale. Supergravity Lagrangians in 5 or more dimensions

[27] The bound on the Hamiltonian should be zero in the classical limit, consistent with the classical notion of a vacuum. This means it is of the form $Tf(T/M_P)$. Since the notion of localized observables in dS space only makes sense when $\frac{T}{M_P} \ll 1$, the linear approximation should be sufficient. So far I have not found any measurable quantity whose value depends on $f(0) \equiv c$.

do not have de Sitter solutions[28], while in four dimensions, models with chiral fields and appropriate super and Kahler potentials can have lots of dS solutions. This remains true for dimension less than four. However, the interesting physics of dS space is the localizable physics that is accessible to a local time-like observer. As we will see, this is described by an approximate S-matrix, which approaches that of a super-Poincare invariant model as $RM_P \to \infty$. In 2 and 3 space-time dimensions there can be no such limiting theory, so there is probably no useful model of low dimensional dS space either.

The notion of an approximate S-matrix can be formalized as follows. Consider a causal diamond in dS space whose holographic screen has an area $b\pi(RM_P)^2$, with $b < \frac{1}{2}$. Assume also that $(RM_P) \gg 1$. According to the general principles of holographic space-time there should be an *approximate scattering matrix* $S(b, R)$, which operates on the eigenstates of the Poincare Hamiltonian, relating two bases of eigenstates on the past and future boundaries of the diamond. We do not yet have a prescription for constructing S, but knowledge of effective field theory in dS space leads to the conclusion that this S-matrix becomes insensitive to the dS horizon as $R \to \infty$.

On an intuitive level this sounds obvious, but there is an important subtlety. We define the scattering matrix as the interaction picture evolution operator $U(T, -T)$ in an effective field theory in static coordinates. The time T is chosen such that the causal diamond of the geodesic observer at the origin, between $-T$ and T has holoscreen area $b\pi(RM_P)^2$. The intuitive argument that this S-matrix becomes independent of R as $RM_P \to \infty$ is that the maximal Gibbons-Hawking temperature encountered in that causal diamond is $\frac{(1-b)^{-\frac{1}{2}}}{2\pi R}$, which goes to zero in the limit. The local geometry also approaches Minkowski space. If we consider a configuration space Feynman diagram contributing to the S-matrix, then all parts of it within the causal diamond converge to their flat space values as the dS radius goes to infinity.

As we approach the horizon, field theory in static coordinates encounters an infinity. The coefficient of $d\tau^2$ vanishes, which means that the norm of the Killing vector field $\frac{\partial}{\partial\tau}$, goes to infinity. As a consequence, very high frequency modes of the field, localized near the horizon, are low energy states as viewed from the origin. As we approach the horizon, we appear to see an infinite number of modes, all of which "our friend at the origin" considers low energy.

A general relativist will attribute this to our insistence on using "bad coordinates". The message of the holographic principle is that the pileup of states near the horizon is just the the holographic image of all physical excitations which have fallen through the horizon in coordinate systems that are regular there. It also instructs us to cut off the infinity, so that the total entropy of these states is finite. The latter instruction cannot be understood in terms of quantum field theory, but must be built in to the quantum theory of dS space we are trying to construct. In the next section, we will describe how thinking about Feynman diagrams with internal lines that penetrate the horizon leads to a relation between the gravitino mass and the c.c. . We'll derive that relation from different considerations in this

[28]There are solutions of the form $dS \times K$, where K is a negatively curved manifold. If K is compact, there is no control over the amount of SUSY breaking, because there are large corrections to the classical bulk solutions, and both the dS and compact radii of curvature are naturally of order the cutoff. For the dS radius this is just the fine tuning of the c.c. in effective field theory, but the compact radius is an additional fine tuning. Some of the literature considers non-compact K, but throws away all but the constant mode on K. The meaning of these papers is completely obscure to me.

section.

According to our general formalism, all the states in dS space are accounted for in the irreducible representation of the pixel algebra

$$[(\psi^M)_i^A, (\psi^{\dagger\ N})_B^j = \delta_i^j \delta_B^A Z^{MN},$$

where M and N run over a basis of sections of the spinor bundle over the fuzzy compactification. For each pixel, the irrep has dimension D and we have

$$\pi(RM_P)^2 = K(K+1)\ln\, D.$$

In terms of Kaluza-Klein language, $\ln\, D = V$, the volume of the compact dimensions in higher dimensional Planck units. We also have the K-K relation $V = (M_P/M_D)^2$.

Particle states localized within our causal diamond are described by considering the algebras of block diagonal matrices, with block sizes K_i, with $\sum K_i = K$ [42]. The spinor bundle over such an algebra is the direct sum of the set of $K_i \times K_i + 1$ matrices (and their $K_i + 1 \times K_i$ conjugates), each tensored with the internal spinor bundle. If, as $K \to \infty$, the representation space of the pixel algebra approaches a direct sum of supersymmetric particle state spaces, then the block diagonal construction, with $K_i \to \infty$ and $\frac{K_i}{K_j}$ fixed, approaches the Fock space of that collection of supermultiplets, with the correct Bose/Fermi gauge equivalence (particle statistics). We must of course include block decompositions with an arbitrary number of blocks. Indeed, a direct sum of algebras always has a permutation gauge symmetry, when we view it as constructed from block diagonal matrices.

If K is fixed and very large, only some of these block diagonal constructions really resemble particles. If K_i is too small, then the would-be particle will not be localizable on the holographic screen, whereas if K_i is too large there will not be any multi-particle states. The compromise, which maximizes the entropy, while still retaining particle-like kinematics, is to take each K_i of order $\sqrt{K}$. The total entropy in such states is of order $(RM_P)^{3/2}$, which is the same scaling we derived by heuristic consideration of particle states in dS space, which do not form black holes.

There are 3 important remarks to make about this construction.

- By considering off-diagonal bands in the block diagonalization of the algebra of $K \times K$ matrices[29], we see of order $\sqrt{K}$ identical copies of the highest entropy particle states. These may be considered *particle states in other horizon volumes* and we see how we can reproduce the claim of QFT in curved space-time, in the $K \to \infty$ limit[30]. However, thinking in terms of the static coordinates, all but one of these collections of particles should be lumped together into the states on a particular observer's holographic screen. There are of course of order $(RM_P)^2$ such states.

- The fixed ratios between the K_i should be interpreted as the ratios of magnitudes of the longitudinal momenta of the different particles. Those familiar with Matrix Theory will recognize this rule. We can motivate it by the following remarks. The conformal

[29]This means the ith upper off diagonal band, completed by the $K - i$th lower off diagonal.

[30]The transformations that map one off diagonal band into the next should be thought of as discrete analogs of the dS boosts, which change one static observer into another.

group of the two sphere is the spin-Lorentz group $SL(2, C)$ and the spinor bundle contains solutions of the conformal Killing spinor equation

$D_z s = \gamma_z s$, where z is a holomorphic coordinate on the sphere and γ_z is the pullback of the two dimensional Dirac matrices by the zweibein. The solutions of the conformal Killing spinor equation transform as a Dirac spinor q_α^a under $SL(2, C)$. The requirement that the representations of the pixel algebra are supermultiplets in the large K_i limit implies in particular that there are generators that converge to

$$S_a(\Omega_0) = S_a \delta(\Omega, \Omega_0),$$

where S_a are two component real spinors under $SO(2)$, which satisfy a Clifford algebra. These operators are a "basis" for the space of sections of the spinor bundle. They should be thought of as operator valued measures on the space of sections. When we integrate them against the conformal Killing spinors we get

$$Q_\alpha(\Omega_0) = \int S_a(\Omega_0, \Omega) q_\alpha^a(\Omega) = S_a q_\alpha^a(\Omega_0).$$

If

$$[S_a, S_b]_+ = p\delta_{ab},$$

then

$$[Q_\alpha, Q_\beta] = (\gamma^0 \gamma^\mu)_{\alpha\beta} P_\mu,$$
$$P_\mu = p(1, \Omega).$$

In deriving the continuous generators from the fuzzy sphere, the normalization p arises in the usual way. The discrete generators differ from the continuous one by an infinite normalization proportional to K_i, so the ratios of p_i are the ratios of K_i.

The precise super-particle spectrum that comes out in the limit depends on the details of the rest of the pixel algebra representation. The classification of pixel algebras whose limit gives rise to a super-particle spectrum containing the $N = 1$ SUGRA multiplet is the analog in this formalism of classifying all supersymmetric compactifications of string theory with minimal SUSY in 4 dimensions. However, if we keep the pixel algebra fixed and take $K \to \infty$, as is appropriate for a theory that is the limit of stable dS space, then we only obtain models with no moduli. Other supersymmetric models, which can be described in terms of perturbative string theory, come from more elaborate limits in which we take both K and the size of the pixel algebra to infinity at the same time, obtaining continuous moduli. These are not related to dS models.

The control parameter that governs the restoration of super-Poincare symmetry is the typical particle momentum, K_i, which scales like $\sqrt{K}$. Rotational symmetry is of course exact, while the Lorentz group is realized as the conformal group of the two sphere. The accuracy with which it can be represented is limited by the total number of spherical harmonics available, which scales like K On the other hand, we can expect the violation of the super-Poincare relation

$$[Q_\alpha, P_0] \sim K^{-\frac{1}{2}}.$$

For a theory with spontaneously broken SUSY, the superpartner of any state is that state plus one gravitino, so we get the estimate

$$m_{3/2} = K^{-\frac{1}{2}} M_P.$$

Taking into account the relation between the dS entropy, K and $\ln D$ we get

$$m_{3/2} = c(\ln D)^{1/4} \Lambda^{1/4} = 10 c \Lambda^{1/4}.$$

The last estimate incorporates Witten's idea [43] that the volume of extra dimensions is the explanation for the ratio of 100 between the reduced Planck scale and the unification scale. We might expect c to be of order 1 but we cannot say that we've accounted for all factors of 2π correctly. If c is of order 1 then we get a gravitino mass of order 10^{-2} eV and a gravitino decay constant $F \sim 30(\mathrm{TeV})^2$.

7 Implications for particle phenomenology

I'll begin this section with an alternative derivation between the gravitino mass and the cosmological constant, based on the notion of Feynman diagrams with internal lines going through the horizon. We want to consider a dS space with very large RM_P. Low energy physics is approximately the same as it is in the limiting super-Poincare invariant model. The latter is described by an $N = 1$ SUGRA Lagrangian, with a super-Poincare invariant vacuum. In order to ensure that the cosmological constant is self-consistently zero, we impose a discrete R symmetry on the low energy Lagrangian. We want to compute the leading correction to this supersymmetric Lagrangian, which leads to the SUSY violation we expect in dS space.

This is computed, as effective Lagrangians always are, in terms of Feynman diagrams, and the new effects of dS space obviously have to do with diagrams in which internal lines go out to the horizon. They cannot lead to explicit violation of SUSY, and renormalization of parameters in the effective Lagrangian will not violate SUSY. However, interactions with the horizon *can* violate R symmetry. If we consider a diagram whose external legs are localized near the origin, then lines going out to the horizon are extended over space-like intervals of geodesic length R. If we assume that the gravitino is the lightest R charged particle in the model, the leading R violating diagrams will have two gravitino lines leading out to the horizon and will have an exponential suppression $e^{-2m_{3/2}R}$. It does not make sense to neglect the gravitino mass in this formula, but the rest of the diagram is evaluated in the $\Lambda = 0$ theory. Recalling that the horizon has a huge number (infinite in the field theory approximation) of very low energy states, of order $e^{\pi(RM_P)^2}$, we can write the contribution of this diagram as

$$\delta\mathcal{L} e^{-2m_{3/2}R} \sum | < 3/2|V|s > |^2,$$

where V is the operator representing emission from and absorption of the gravitino by the horizon.

The horizon is a null surface and the massive gravitino can only propagate near it for proper time of order its Compton wavelength. As a quantum particle it does a random walk, and we take the proper time step to be the Planck scale. Thus, the area in Planck units that it covers is of order $\frac{M_P}{m_{3/2}}$, and we take this as an estimate of the logarithm of the number of states for which the matrix element is of order 1. The total contribution is thus of order

$$\delta\mathcal{L}e^{-2m_{3/2}R+b\frac{M_P}{m_{3/2}}}.$$

This formula can be self consistent only for one behavior of the vanishing gravitino mass in the $RM_P \to \infty$ limit. If we assume the gravitino mass goes to zero too rapidly, for example like the naive SUGRA prediction $m_{3/2} \sim \sqrt{\Lambda}/m_P$, then the formula predicts exponentially large corrections to the effective Lagrangian. If we assume it goes to zero too slowly the effective Lagrangian is exponentially small, which is inconsistent with the assumption. In effective field theory, it is this correction the the Lagrangian that is responsible for the gravitino mass. For self-consistency, the exponential dependence on R must cancel exactly

$$m_{3/2}\sqrt{\frac{bM_P}{2R}}.$$

This is the same scaling we found in the previous section, but we learn less about the coefficient.

We conclude that the low energy Lagrangian of stable dS space has the form

$$\mathcal{L}_0 + \mathcal{L}_{\Delta R}.$$

The full Lagrangian must predict a dS solution, and implement the relation between the gravitino mass and the c.c. An example of such a Lagrangian would be

$$\mathcal{L}_{\Delta R} = \int d^2\theta \; (W_0 + FG),$$

with G a single chiral superfield, the goldstino multiplet, which we assume is the only low energy matter field. $\mathcal{L}_0$ would have a discrete R symmetry, which forbade both of these terms. In order that there be no SUSY vacuum in low energy effective field theory, we have to assume that G has R charge 0. However, the demands of the underlying theory are not so strict. We could for example insist only that the R symmetry forbid terms up to cubic order in G and that the natural scale in $\mathcal{L}_0$ is just the Planck scale. Then there might be SUSY minima at $S \sim m_P$, but the Lagrangian could be above the Great Divide, and consistent with the underlying finite dimensional model for dS space.

While this model satisfies the basic consistency conditions, it is not our world. In the real world, we must couple the SUSY violating order parameter to standard model super-multiplets. In particular, gaugino masses would result from terms of the form

$$\int d^2\theta \; f_i(G/M) W_\alpha^{i\;2}$$

and would be given by

$$m^1_{1/2} = f_i^\prime(G/M)(F/M).$$

Since $F \sim 30(\mathrm{TeV})^2$, M cannot be larger than a few TeV if we are to obey the experimental bounds (there are factors of standard model fine structure constants in f_i) . This indicates that there must be a new strongly coupled gauge theory with confinement scale M, which contains fields transforming under the standard model. The Goldstino field G must be an elementary field with renormalizable couplings to the new gauge system, a composite field from that system, or a combination of both.

Given these couplings, squarks and sleptons will get mass via gauge mediation. We are forced, by the low maximal scale of SUSY violation, to consider a model of direct gauge mediation. Such models are notorious for having problems with coupling unification. One must have complete representations of the unified gauge group, with low multiplicities, which means that the hidden sector gauge group must be small and the representations of the new chiral matter of low dimension. So, for example, if the unified gauge group is $SU(5)$, we can, when two loop corrections are taken into account, tolerate at most 4 $5 + \bar{5}$ pairs in the hidden sector.

While I have not done a definitive survey, all examples I've studied of hidden sectors that satisfy these constraints contain light fields with standard model quantum numbers, which are ruled out by experiment. Simple unification appears incompatible with direct gauge mediation. One appears forced to utilize Glashow's *trinification* scheme, in which the standard model is embedded in

$$SU_1(3) times SU_2(3) \times SU_3(3) \rtimes Z_3,$$

where Z_3 cyclically permutes the three $SU(3)$ groups. The standard model chiral superfields are embedded in 3 copies of

$$(1, \bar{3}, 3) + (3, 1, \bar{3}) + (\bar{3}, 3, 1),$$

as the 15 states that transform chirally under the standard model. There is a nice embedding of this in E_6, but that would put us back in the forbidden realm of simple unification. More interesting is the way that this structure, *including the prediction of the number of generations*, arises from 3 D3-branes at the Z_3 orbifold in Type IIB string theory. We also note that the vector-like spectrum of this model contains 3 copies of the conventional SUSY Higgs fields. However, at least in the orbifold construction the implied structure of standard model Yukawa couplings comes out wrong.

We can add a hidden sector to trinification, without ruining standard model coupling unification, by postulating an $SU(N)$ gauge theory, with $N = 3, 4$ and chiral fields T_i and $\tilde{T}_i$ in the $(\bar{N}, 3_i) + (\bar{3}_i, N)$. These models have a pyramidal quiver diagram and are called the Pyramid Schemes [44]. At the level of the orbifold construction the new fields come from $D7$-branes and one can think of the model as an F-theory solution with an orbifold singularity in its base.

There is no room here to go into the intricate details of model building, but the Pyramid Schemes throw new light on the strong CP problem, the little hierarchy problem, the origin of the μ term in the MSSM, the nature of dark matter, *etc.*. They have a rich phenomenology and can easily be ruled out at the LHC. It is not clear whether the LHC energy is high enough to reveal the complete structure of these models.

What I would like to emphasize is that the theory of stable dS space we have adumbrated gives rather detailed predictions for Terascale physics. Thus, despite its rather abstract origins, and the incomplete nature of the theory of holographic space-time, we may hope in the near future for experimental input that could encourage us to continue to work on this set of ideas, or convince us to abandon them.

8 Appendix: exercises on CDL Tunneling

In my lectures, I asked the students to work out some of the theory of gravitational tunneling for themselves, because there are so many erroneous notions in the community about the results of Coleman and De Luccia. Much of my second lecture was an extended "recitation section", in which I outlined the solution of these problems. There are also some exercises on black hole solutions.

1. Show that the metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2,$$

with

$$f(r) = 1 - c_d \frac{M}{r^{d-3} M_P^{d-2}} \pm \frac{r^2}{R^2},$$

solves the d space-time dimensional Einstein equations with cosmological constant Λ , where $R^{-1} = b_d \sqrt{\Lambda} / M_P^{d/2-1}$ is the Hubble scale associated with the c.c. . Work out the necessary constants for all d. The Einstein equations are

Show that positive c.c. corresponds to the choice of negative sign in $f(r)$. These are the Schwarzschild black hole solutions for all possible maximally symmetric background space times.

2. Show that for positive c.c. $f(r)$ has two zeroes, corresponding to the two positive roots of a cubic equation

$$(r - R_+)(r - R_-)(r + R_+ + R_-).$$

$R_\mp$ is the position of the black hole (cosmological) horizon. Show that both R_{pm} are $< R$ and the entropy deficit

$$\pi(R^2 - R_+^2 - R_-^2)M_P^2$$

is always positive and is approximately

$$\Delta S \simeq 2\pi R M$$

when $R_- << R$. Find the maximal black hole mass in de Sitter space and argue that it has the smallest total entropy. This little exercise shows that localized states are low entropy excitations of the dS vacuum, which we have argued should be modeled by an infinite temperature density matrix on a Hilbert space with a finite number of states.

3. The Coleman De Lucia (CDL) equations for gravitational tunneling are the equations for a scalar field coupled to Euclidean Einstein gravity, with $SO(4)$ symmetry. This is the Euclidean analog of FRW cosmology: a four dimensional space-time with a maximally symmetric 3 dimensional subspace. The equations are

$$\phi'' + 3\frac{\rho'}{\rho}\phi' = \frac{dU}{d\phi},$$

$$(\rho' 2 = 1 + \frac{\rho^2}{3m_P^2}(\frac{1}{2}\phi'^{\ 2} - U).$$

The metric is

$$ds^2 = dz^2 + \rho^2(z)d\Omega_3^2.$$

A.If the potential has the form

$$U = -\mu^4 v(x),$$

where $x = \phi/M$, and we make everything dimensionless using the space-time scale $z = \frac{M}{\mu^2}\tau$, show that the equations take the form

$$\ddot{x} + \frac{\dot{a}}{a}\dot{x} = -\frac{dv}{dx},$$

$$\dot{a} = 1 + \epsilon^2 a^2 (\frac{1}{2}\dot{x}^2 + v),$$

where $\epsilon^2 = \frac{M^2}{3m_P^2}$.

If we are tunneling *from* a solution with non-positive c.c.,then the Euclidean space-time is infinite. We insist that, at infinity, the solution approach the Flat or Hyperbolic space solution. By convention we set the field value at infinity to zero, and $v(0) = c$, so that the c.c. is $-c\mu^4$. The other maximum of v is called x_T. The solution for the metric at infinity is

$$a = \sinh(\epsilon c\tau)/\epsilon c \to \tau.$$

The last limit is $c \to 0$. The exact solution will also have a point where $a = 0$. This is the center of the vacuum bubble.

The equations (but not the metric) have the form of an FRW *EUniverse* with a scalar field. The "Big Bang" is the center of the bubble, which we conventionally call $\tau = 0$ and we must have $\dot{x} = 0$ there in order to have a regular solution. We must choose the value of $x(0)$ in order to satisfy the boundary conditions at infinity. We choose $\dot{a}(0) = 1$. This expanding Euniverse condition just says that we are following the Euclidean configuration to larger radius spheres. In Euclidean space, this analog Big Bang is not a singularity. Since $c \geq 0$, the real, Lorentzian signature c.c. is non-positive, but the analog EUniverse has non-negative c.c. at the maximum of v at $x = 0$. The equations correspond to motion under a complicated frictional force, plus the force derived from the potential v. Show that the EEnergy,

$$EE = \frac{1}{2}\dot{x}^2 + v,$$

will be monotonically decreasing as long as $\dot{a}$ remains positive, as will the speed.
B. In the non-gravitational case ($\epsilon = 0$) $\dot{a}$ is always positive. Then, it's clear that the friction term goes to zero at large t. Argue that there are values of $x(0)$, such that $x(t)$ will undershoot the maximum of v at $x = 0$. Argue that by starting close to x_T we can find solutions that overshoot $x = 0$. Argue that continuity implies there is a solution, which asymptotes to 0 at $\tau = \infty$. Argue that this analysis remains valid when $\epsilon \ll 1$, as long as v is of order one and $v(x_T) - v(0)$ is of order one.
C. When ϵ is of order 1 a new behavior sets in. The sign of $\dot{a}$ can change in the region where v is negative. If this happens, the solution never reaches infinite radius. The radius shrinks to zero and the $\dot{x}$ does not go to zero. Friction turns to anti-friction as the radius shrinks and the velocity actually goes to infinity. We can think of this as a Big Crunch of the EUniverse.

Such solutions exist when ϵ is very small, but they are confined to a small range of values of $x(0)$ near the minimum of v. The transition between overshooting and undershooting happens at a larger negative value of $x(0)$ so the instanton always exists. Argue that as ϵ is increased, the onset of Big Crunch solutions moves to more negative values of x. Eventually it crosses the transition point between overshoot and undershoot solutions, and no instanton exists.

D. When $c > 0$, even when the instanton exists, its interpretation is not that of an unstable bubble that can appear as a state in the AdS space. Argue this as follows: Near infinity the instanton $x(\tau)$ becomes small, and is well approximated by a solution of the equations for small fluctuations around Euclidean AdS space. Argue that, because of the boundary conditions on the instanton at $\tau = 0$ it is a linear combination of both the normalizable and non-normalizable solutions of the linearized equations. As you will learn in other lectures on the AdS/CFT correspondence, this means that it corresponds to adding an operator to the Hamiltonian. States in the model with the original Hamiltonian correspond to purely normalizable solutions at infinity. In all cases of the AdS/CFT correspondence where such instantons have been found, the operator that is added is unbounded from below.

E. The overshoot solutions are those for which $x(0)$ is near x_T. Thus, as ϵ is raised, the point at which instantons disappear is the point at which $x(0)$ is forced to x_T in order to avoid a crunch. However, this is no longer an instanton, because if we start a solution at x_T with zero velocity, it stays there. What happens instead is that the point recedes in geodesic distance, and the solution becomes infinite in both the $\tau = 0$ and $\tau = \infty$ limits. Show that the interpretation of this solution is as a static domain wall between two AdS regions (or an AdS and Minkowski region). Show more generally that the existence of such a static domain wall always requires the fine tuning of one parameter in the potential. We summarize this in the statement that the sub-manifold in the space of potentials, on which a static domain wall solution exists, *has co-dimension one.* This sub-manifold is called The Great Divide. On one side of the Great Divide instantons exist, while on the side we call Above the Great Divide, they don't exist. There is a connection between this and the positive energy theorem in General Relativity, which I will explain in the lectures.

F. Show that the Euclidean continuation of dS space is a 4 sphere, and that it has negative Euclidean action. In fact, in an echo of the Gibbons-Hawking [8] result for Euclidean black holes, the action is just equal to minus the entropy of dS space. Correspondingly, instantons for the "decay" of dS are compact 4 manifolds with negative Euclidean action. We make a probability formula that is ≤ 1 by subtracting the negative dS action[31] of the initial "decaying" state.

$$P_{12} = e^{-(S_I - S_{dS_1}}.$$

If the state 2 to which dS_1 "decays" is also a dS space, then we can form the reverse probability

$$P_{21} = e^{-(S_I - S_{dS_2})}.$$

This leads to

[31]The rule of subtracting off the action of the initial configuration is motivated by quantum field theory, where we can *prove* that this is the right thing to do.

$$\frac{P_{12}}{P_{21}} = e^{-\Delta\ Entropy}.$$

Argue that this is the infinite temperature form of the *principle of detailed balance*. Consider two finite collections of states, such that the transition amplitude for any state in collection 1 to any state in collection 2 is the same. Show that unitarity implies that there are reverse transitions, and that the probabilities for the two ensembles to decay into each other are related by the above equation. The CDL formula thus provides evidence for the picture expounded in the lectures, in which dS space is modeled as a system with a finite number of states. Notice that the instanton transition for the lower c.c. dS space, is here interpreted, not as an instability, but as a temporary sojourn of a large system in a very low entropy configuration, like the air in a room collecting in a little cube in the corner.

More controversial is the contention, also expounded in the lectures, that the same interpretation is valid *above The Great Divide* for dS "decays" into negative c.c. Big Crunches. The holographic principle shows that the latter are low entropy states, and we should expect rapid transitions back from them to the equilibrium dS configuration. These reverse transitions, cannot be modeled by instantons, because the initial configuration is not classical in any way.

9 Appendix: potentials in string theory

In tree level string theory, one can only add sources to the system if they correspond to vertex operators for asymptotic states of the system in a fixed space-time background. In asymptotically flat space, this means that one can only add constant sources, as in the definition of the field theoretic 1PI potential, for massless particles. They correspond to rather singular limits of genuine scattering amplitudes, but, *so long as the particle remains massless for all values of the source*, they seem sensible. The italicized phrase means that the analog of the effective potential can only be defined when it is exactly zero.

In non-supersymmetric string theory, even when there are no tachyons, the perturbation expansion is singular at one loop. Fischler and Susskind [20] argued that these singularities could be removed by changing the background space-time. This procedure leads to time dependent solutions, and general considerations show [?] that the time dependence is singular[32].

Fischler and Susskind tried to argue that their procedure gave a method for computing quantum corrections to the effective potential in string theory. They showed that there was a Lagrangian, at the appropriate order in string coupling, which reproduced the modified background solution they had found. Students who have studied the rest of these lectures, will know that such a demonstration says nothing whatever about the existence of other solutions of the same equations of motion, as bona fide theories of quantum gravity. This argument is independent of the question dealt with above, to the effect that the Fischler

[32]In a systematic F-S expansion, the time dependence appears linear but at large times this expansion breaks down. One can try to do a more exact solution of the low energy field equations, but this leads to singular cosmological solutions. There does not appear to be a way to make the F-S mechanism into a controlled expansion.

Susskind solution itself does not provide evidence for the existence of a model of quantum gravity based on their modified background.

Another attempt to define effective potentials in string theory tries to define a String Field Theory [45]. Open String Field Theory is an elegant construction, which reproduces tree-level open string amplitudes. However, at the loop level it is singular, because of the familiar fact that open string loops imply closed strings. *Any* regularization of that singularity forces us to introduce an independent closed string field. Closed String Field Theory is *not* a non-perturbative definition of theory. Its Lagrangian must be corrected at each order in perturbation theory, in order to reproduce the correct loop amplitudes. Furthermore, the series that defines the string field action is divergent. Much has been made of the fact that the open string field theory "contains closed strings automatically", and it's been proposed that this gives a non-perturbative definition of the theory. In fact, the appearance of closed strings is ambiguous and the relevant open string diagrams are singular. When one tries to regulate the singularities, one finds that one must introduce an independent closed string field, with the difficulties noted above.

The upshot of this is that there is no indication in any perturbative string theory calculation, that there is a beast like the mythical effective potential, whose minima classify different consistent theories of quantum gravity. Every non-perturbative definition of string theory leads to precisely the opposite conclusion, as we have sketched in the main lectures.

References

[1] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, "M theory as a matrix model: A conjecture," Phys. Rev. D **55**, 5112 (1997) [arXiv:hep-th/9610043].

[2] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[3] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109]. ;E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[4] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, "Large N field theories, string theory and gravity," Phys. Rept. **323**, 183 (2000) [arXiv:hep-th/9905111].

[5] J. D. Bekenstein, "Black Holes And The Second Law," Lett. Nuovo Cim. **4**, 737 (1972). ; J. D. Bekenstein, "Black Holes And The Second Law," Lett. Nuovo Cim. **4**, 737 (1972). ; S. W. Hawking, "Particle Creation By Black Holes," Commun. Math. Phys. **43**, 199 (1975) [Erratum-ibid. **46**, 206 (1976)].

[6] J. Polchinski, "S-matrices from AdS spacetime," arXiv:hep-th/9901076. ; L. Susskind, "Holography in the flat space limit," arXiv:hep-th/9901079. S. B. Giddings, The boundary S-matrix and the AdS to CFT dictionary, Phys. Rev. Lett. 83, 2707 (1999) [arXiv:hep-th/9903048]. ; M. Gary and S. B. Giddings, "The flat space S-matrix from

the AdS/CFT correspondence?," Phys. Rev. D **80**, 046008 (2009) [arXiv:0904.3544 [hep-th]]. ; M. Gary, S. B. Giddings and J. Penedones, "Local bulk S-matrix elements and CFT singularities," Phys. Rev. D **80**, 085005 (2009) [arXiv:0903.4437 [hep-th]].

[7] K. A. Khan and R. Penrose, "Scattering of two impulsive gravitational plane waves," Nature **229**, 185 (1971). ;T. Banks and W. Fischler, "A model for high energy scattering in quantum gravity," arXiv:hep-th/9906038. ;H. J. Matschull, "Black hole creation in 2+1-dimensions," Class. Quant. Grav. **16**, 1069 (1999) [arXiv:gr-qc/9809087]. ; D. M. Eardley and S. B. Giddings, "Classical black hole production in high-energy collisions," Phys. Rev. D **66**, 044011 (2002) [arXiv:gr-qc/0201034].

[8] G. W. Gibbons and S. W. Hawking, "Cosmological Event Horizons, Thermodynamics, And Particle Creation," Phys. Rev. D **15**, 2738 (1977).

[9] W. Fischler, Taking de Sitter seriously, Talk given at Role of Scaling Laws in Physics and Biology (Celebrating the 60th Birthday of Geoffrey West), Santa Fe, Dec. 2000; T. Banks, "Cosmological breaking of supersymmetry or little Lambda goes back to the future. II," arXiv:hep-th/0007146. ;T. Banks, "Cosmological breaking of supersymmetry?," Int. J. Mod. Phys. A **16**, 910 (2001).

[10] S. R. Coleman and F. De Luccia, "Gravitational Effects On And Of Vacuum Decay," Phys. Rev. D **21**, 3305 (1980).

[11] M. Cvetic, S. Griffies and H. H. Soleng, "Local and global gravitational aspects of domain wall space-times," Phys. Rev. D **48**, 2613 (1993) [arXiv:gr-qc/9306005]. ; M. Cvetic, S. Griffies and H. H. Soleng, "Nonextreme and ultraextreme domain walls and their global space-times," Phys. Rev. Lett. **71**, 670 (1993) [arXiv:hep-th/9212020]. ; M. Cvetic, S. Griffies and S. J. Rey, "Static domain walls in N=1 supergravity," Nucl. Phys. B **381**, 301 (1992) [arXiv:hep-th/9201007].

[12] T. Banks and M. Dine, "Dark energy in perturbative string cosmology," JHEP **0110**, 012 (2001) [arXiv:hep-th/0106276].

[13] L. Susskind, Unpublished, 1997; A. Sen, "D0 branes on T(n) and matrix theory," Adv. Theor. Math. Phys. **2**, 51 (1998) [arXiv:hep-th/9709220]. ; N. Seiberg, "Why is the matrix model correct?," Phys. Rev. Lett. **79**, 3577 (1997) [arXiv:hep-th/9710009].

[14] A. Giveon, D. Kutasov and N. Seiberg, "Comments on string theory on AdS(3)," Adv. Theor. Math. Phys. **2**, 733 (1998) [arXiv:hep-th/9806194].

[15] E. Brezin, V. A. Kazakov and A. B. Zamolodchikov, "Scaling violation in a field theory of closed strings in one physical dimension," Nucl. Phys. B **338**, 673 (1990). ; D. J. Gross and I. R. Klebanov, "One Dimensional String Theory on a Circle," Nucl. Phys. B **344**, 475 (1990). ; G. W. Moore, "Double scaled field theory at c = 1," Nucl. Phys. B **368**, 557 (1992). ; J. Polchinski, "Classical Limit Of (1+1)-Dimensional String Theory," Nucl. Phys. B **362**, 125 (1991) ;J. Polchinski, "On The Nonperturbative Consistency Of D = 2 String Theory," Phys. Rev. Lett. **74**, 638 (1995) [arXiv:hep-th/9409168].

[16] K. Dasgupta, G. Rajesh and S. Sethi, "M theory, orientifolds and G-flux," JHEP **9908**, 023 (1999) [arXiv:hep-th/9908088]. ; R. Blumenhagen, L. Goerlich, B. Kors and D. Lust, "Magnetic flux in toroidal type I compactifications," Fortsch. Phys. **49**, 591 (2001) [arXiv:hep-th/0010198].

; K. Becker, M. Becker, M. Haack and J. Louis, "Supersymmetry breaking and alpha'-corrections to flux induced potentials," JHEP **0206**, 060 (2002) [arXiv:hep-th/0204254]. ; P. K. Tripathy and S. P. Trivedi, "Compactification with flux on K3 and tori," JHEP **0303**, 028 (2003) [arXiv:hep-th/0301139].

; M. R. Douglas and S. Kachru, "Flux compactification," Rev. Mod. Phys. **79**, 733 (2007) [arXiv:hep-th/0610102], and references therein.

[17] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," Phys. Rev. D **66**, 106006 (2002) [arXiv:hep-th/0105097].

[18] D. Z. Freedman, G. W. Gibbons and P. C. West, "Ten Into Four Won't Go," Phys. Lett. B **124**, 491 (1983).

[19] C. Beasley, J. J. Heckman and C. Vafa, "GUTs and Exceptional Branes in F-theory - I," JHEP **0901**, 058 (2009) [arXiv:0802.3391 [hep-th]]. ; C. Beasley, J. J. Heckman and C. Vafa, "GUTs and Exceptional Branes in F-theory - II: Experimental Predictions," JHEP **0901**, 059 (2009) [arXiv:0806.0102 [hep-th]].

[20] W. Fischler and L. Susskind, "Dilaton Tadpoles, String Condensates And Scale Invariance," Phys. Lett. B **171**, 383 (1986). ; W. Fischler and L. Susskind, Phys. Lett. B **173**, 262 (1986).

[21] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, "Type IIA moduli stabilization," JHEP **0507**, 066 (2005) [arXiv:hep-th/0505160].

[22] T. Banks and K. van den Broek, "Massive IIA flux compactifications and U-dualities," JHEP **0703**, 068 (2007) [arXiv:hep-th/0611185].

[23] M. Dine, A. Morisse, A. Shomer and Z. Sun, "IIA moduli stabilization with badly broken supersymmetry," JHEP **0807**, 070 (2008) [arXiv:hep-th/0612189].

[24] Yu. M. Makeenko and A. A. Migdal, "Exact Equation For The Loop Average In Multicolor QCD," Phys. Lett. B **88**, 135 (1979) [Erratum-ibid. B **89**, 437 (1980)].

[25] T. Hertog and G. T. Horowitz, "Towards a big crunch dual," JHEP **0407**, 073 (2004) [arXiv:hep-th/0406134]. ; T. Hertog and G. T. Horowitz, "Holographic description of AdS cosmologies," JHEP **0504**, 005 (2005) [arXiv:hep-th/0503071].

[26] R. Schon and S. T. Yau, "On the Proof of the positive mass conjecture in general relativity," Commun. Math. Phys. **65**, 45 (1979). ; E. Witten, "A Simple Proof Of The Positive Energy Theorem," Commun. Math. Phys. **80**, 381 (1981).

[27] A. Aguirre, T. Banks and M. Johnson, "Regulating eternal inflation. II: The great divide," JHEP **0608**, 065 (2006) [arXiv:hep-th/0603107].

[28] T. Banks, M. Dine, P. J. Fox and E. Gorbatov, "On the possibility of large axion decay constants," JCAP **0306**, 001 (2003) [arXiv:hep-th/0303252].

[29] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, "The string landscape, black holes and gravity as the weakest force," JHEP **0706**, 060 (2007) [arXiv:hep-th/0601001].

[30] T. Banks, M. Johnson and A. Shomer, "A note on gauge theories coupled to gravity," JHEP **0609**, 049 (2006) [arXiv:hep-th/0606277].

[31] R. Bousso and J. Polchinski, "Quantization of four-form fluxes and dynamical neutralization of the cosmological constant," JHEP **0006**, 006 (2000) [arXiv:hep-th/0004134].

[32] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109]. E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[33] P.P. Kulish, L.D. Fadeev, Theor. Math. Phys. 4 (1970), p. 745.

[34] T. Jacobson, "Thermodynamics of space-time: The Einstein equation of state," Phys. Rev. Lett. 75, 1260 (1995); T. Padmanabhan, "The Holography of Gravity encoded in a relation between Entropy, Horizon area and Action for gravity," Gen. Rel. Grav. **34**, 2029 (2002) [arXiv:gr-qc/0205090]. ;T. Padmanabhan, "Is gravity an intrinsically quantum phenomenon? Dynamics of gravity from the entropy of spacetime and the principle of equivalence," Mod. Phys. Lett. A **17**, 1147 (2002) [arXiv:hep-th/0205278]. ;T. Padmanabhan, "Gravity from Spacetime Thermodynamics," Astrophys. Space Sci. **285**, 407 (2003) [arXiv:gr-qc/0209088]. ; T. Padmanabhan, "Gravitational entropy of static spacetimes and microscopic density of states," Class. Quant. Grav. **21**, 4485 (2004) [arXiv:gr-qc/0308070]. T. Padmanabhan, "Gravity and the thermodynamics of horizons," Phys. Rept. **406**, 49 (2005) [arXiv:gr-qc/0311036]. ;T. Padmanabhan, "Why gravity has no choice: Bulk spacetime dynamics is dictated by information entanglement across horizons," Gen. Rel. Grav. **35**, 2097 (2003). ;T. Padmanabhan, "Gravity: A new holographic perspective," Int. J. Mod. Phys. D **15**, 1659 (2006) [arXiv:gr-qc/0606061]. ;D. Kothawala, S. Sarkar and T. Padmanabhan, "Einstein's equations as a thermodynamic identity: The cases of stationary axisymmetric horizons and evolving spherically symmetric horizons," Phys. Lett. B **652**, 338 (2007) [arXiv:gr-qc/0701002]. ;T. Padmanabhan and A. Paranjape, "Entropy of Null Surfaces and Dynamics of Spacetime," Phys. Rev. D **75**, 064004 (2007) [arXiv:gr-qc/0701003]. ;T. Padmanabhan, "Gravity as an emergent phenomenon," Int. J. Mod. Phys. D **17**, 591 (2008). ;T. Padmanabhan, "Entropy density of spacetime and thermodynamic interpretation of field equations of gravity in any diffeomorphism invariant theory," arXiv:0903.1254 [hep-th]. ;T. Padmanabhan, "Equipartition of energy in the horizon degrees of freedom and the emergence of gravity," Mod. Phys. Lett. A **25**, 1129 (2010) [arXiv:0912.3165 [gr-qc]]. ;T. Padmanabhan, "Surface Density of Spacetime Degrees of Freedom from Equipartition Law in theories of Gravity," Phys. Rev. D **81**, 124040 (2010) [arXiv:1003.5665 [gr-qc]]. ;T. Padmanabhan, "Entropy density of space-time and gravity: A conceptual synthesis," Int. J. Mod. Phys. D **18**, 2189 (2009). ; S. Kolekar and T. Padmanabhan, "Holography in

Action," arXiv:1005.0619 [gr-qc]. "On the Origin of Gravity and the Laws of Newton," arXiv:1001.0785 [hep-th].

[35] G. Horowitz, *Unpublished*

[36] R. Bousso and B. Freivogel, "Asymptotic states of the bounce geometry," Phys. Rev. D **73**, 083507 (2006) [arXiv:hep-th/0511084].

[37] G. W. Gibbons and S. W. Hawking, "Cosmological Event Horizons, Thermodynamics, And Particle Creation," Phys. Rev. D **15**, 2738 (1977).

[38] W. Israel, "New Interpretation Of The Extended Schwarzschild Manifold," Phys. Rev. **143**, 1016 (1966).

[39] L. Susskind, L. Thorlacius and J. Uglum, "The Stretched Horizon And Black Hole Complementarity," Phys. Rev. D **48**, 3743 (1993) [arXiv:hep-th/9306069]. ; L. Susskind, "String theory and the principles of black hole complementarity," Phys. Rev. Lett. **71**, 2367 (1993) [arXiv:hep-th/9307168].

[40] P. H. Ginsparg and M. J. Perry, "Semiclassical Perdurance Of De Sitter Space," Nucl. Phys. B **222**, 245 (1983).

[41] R. Bousso and S. W. Hawking, Phys. Rev. D **54**, 6312 (1996) [arXiv:gr-qc/9606052].

[42] T. Banks, B. Fiol and A. Morisse, "Towards a quantum theory of de Sitter space," JHEP **0612**, 004 (2006) [arXiv:hep-th/0609062].

[43] E. Witten, "Strong Coupling Expansion Of Calabi-Yau Compactification," Nucl. Phys. B **471**, 135 (1996) [arXiv:hep-th/9602070].

[44] T. Banks and J. F. Fortin, "A Pyramid Scheme for Particle Physics," JHEP **0907**, 046 (2009) [arXiv:0901.3578 [hep-ph]].

[45]