String Theory and the LHC

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The LHC program is finally underway. Can string theory have any impact on our understanding of phenomena which we may observe?

- Supersymmetry?
- Warping?
- Technicolor?
- Just one lonely higgs?

In these three lectures, I will focus mainly on supersymmetry, for reasons of time and because here I know how to make the most concrete statements and models.

Virtues

1. Hierarchy Problem
2. Unification
3. Dark matter
4. Presence in string theory (often)

The last item is one of the reasons for this focus: supersymmetry is an important crutch in our understanding of string theory; most of what we claim to understand is based on study of supersymmetric vacua (configurations).

Hierarchy: Two Aspects

1. Cancelation of quadratic divergences
2. Non-renormalization theorems (holomorphy of gauge couplings and superpotential): if supersymmetry unbroken classically, unbroken to all orders of perturbation theory, but can be broken beyond: exponentially large hierarchies.

But reasons for skepticism:

1. Little hierarchy
2. Unification: why generic in string theory?
3. Hierarchy: landscape (light higgs anthropic?)

Reasons for (renewed) optimism:

1. The study of metastable susy breaking (ISS) has opened rich possibilities for model building; no longer the complexity of earlier models for dynamical supersymmetry breaking.
2. Supersymmetry, even in a landscape, can account for hierarchies, as in traditional thinking about naturalness ($e^{-\frac{8\pi^2}{g^2}}$)
3. Supersymmetry, in a landscape, accounts for stability – i.e. the very existence of (metastable) states.

Plan of the Lectures

- Lecture 1: Low Energy Supersymmetry: Supersymmetry and its (metastable) breaking, supersymmetry and particle physics - the MSSM.
- Lecture 2: Microscopic models of supersymmetry breaking and its Mediation
- Lecture 3: What might we mean by a string phenomenology.

Lecture 1. Low Scale Supersymmetry and Its Breaking. Outline

1. Some features of $N = 1$ Supersymmetry
2. Metastable vs. stable supersymmetry breaking; Simple Models
3. The MSSM
4. Gauge Mediated models (continuing into lecture 2)

Supersymmetry Review

I am going to assume some familiarity with supersymmetry, but it is perhaps worth reviewing some basic features of $N = 1$ theories. First, without gravity. Two types of supermultiplets (superfields) describe particles of spins up to 1:

1. Chiral fields: consist of a complex scalar and a Weyl fermion

$$\Phi(\theta) = \phi(\theta) + \sqrt{2}\theta\psi + \theta^2 F. \tag{1}$$

2. Vector fields: contain a gauge boson and a Weyl fermion (gaugino). Described by a real superfield, V; for our discussion enough to consider the gauge covariant, spin-1/2 chiral field, W_α

$$W_\alpha = \lambda_\alpha + \theta_\beta[\delta_\alpha^\beta D + (\sigma^{\mu\nu})_\alpha^\beta F_{\mu\nu} \ldots \tag{2}$$

F and D are auxiliary fields; they play an important role as they are order parameters for supersymmetry breaking.

At the level of terms with two derivatives, a supersymmetric lagrangian is specified by three functions:

1. The superpotential, $W(\Phi_i)$, a *holomorphic* function of the chiral fields.
2. The Kahler potential, $K(\Phi_i, \Phi_i^\dagger)$
3. The gauge coupling functions, $f_a(\Phi_i)$, again holomorphic functions of the fields (one such function for each gauge group.

The lagrangian density has the form, in superspace:

$$\int d^4\theta K(\Phi, \Phi^\dagger) + \int d^2\theta \left(f_a(\Phi) W_{\alpha a}^2 + W(\Phi) \right) \tag{3}$$

Renormalizable Interactions

In this case, $K = \sum \Phi_i \Phi_i^\dagger$, $f_a = -\frac{1}{4g_a^2}$ and W is at most cubic. I will leave the details of the component lagrangian for textbooks and focus here on the scalar potential:

$$V = \sum F_i^2 + \frac{1}{2} \sum D_a^2. \tag{4}$$

$$F_i = \frac{\partial W}{\partial \Phi_i} \qquad D^a = \phi_i^2 T^a \phi_i. \tag{5}$$

Classically, supersymmetry is unbroken if $F_i = D_a = 0 \; \forall \; i, a$; conversely, it is broken if not. In this case, there is a Goldstone fermion,

$$G \propto F_i \psi_i + D_a \lambda_a. \tag{6}$$

In supersymmetry, a class of symmetries known as *R*-symmetries play a prominent role. Such symmetries can be continuous or discrete. Their defining property is that they transform the supercurrents,

$$Q_\alpha \rightarrow e^{i\alpha} Q_\alpha \quad Q^*_\alpha \rightarrow e^{-i\alpha} Q^*_\alpha. \tag{7}$$

Necessarily, the superpotential transforms as $W \rightarrow e^{2i\alpha}$ under this symmetry.

For the question of supersymmetry breaking, these symmetries play a crucial role, embodied in a theorem of Nelson and Seiberg:

In order that a generic lagrangian (one with all terms allowed by symmetries) to break supersymmetry, the theory must possess an *R* symmetry. This theorem is easily proven by examining the equations $\frac{\partial W}{\partial \Phi_i} = 0$, and recalling that they are holomorphic.

But, instead, some examples:

Varieties of R symmetric lagrangians

In general, W has R charge 2. Suppose fields, X_i, $i = 1, \dots N$ with $R = 2$, ϕ_a, $a = 1, \dots M$, with R charge 0. Then the superpotential has the form:

$$W = \sum_{i=1}^{N} X_i f_i(\phi_a). \tag{8}$$

Suppose, first, that $N = M$. The equations $\frac{\partial W}{\partial \Phi_i} = 0$ are solved if:

$$f_i = 0; \quad X_i = 0. \tag{9}$$

(R unbroken, $\langle W \rangle = 0$.) The first set are N holomorphic equations for N unknowns, and generically have solutions. Supersymmetry is unbroken; there are a discrete set of supersymmetric ground states; there are generically no massless states in these vacua.
(Again, R unbroken, $\langle W \rangle = 0$.)

Next suppose that $N < M$. Then the equations $f_i = 0$ involve more equations than unknowns; they generally have an $M - N$ dimensional space of solutions, known as a moduli space. In perturbation theory, as a consequence of non-renormalization theorems, this degeneracy is not lifted. There are massless particles associated with these moduli (it costs no energy to change the values of certain fields).

If $N > M$, the equations $F_i = 0$ in general do not have solutions; supersymmetry is broken. These are the O'Raifeartaigh models. Now the equations $\frac{\partial W}{\partial \phi_i} = 0$ do not determine the X_i's, and classically, there are, again, moduli. Quantum mechanically, however, this degeneracy is lifted.

Aside 1

The non-renormalization theorems.
Quite generally, supersymmetric theories have the property that, if supersymmetry is not broken at tree level, then to all orders of perturbation theory, there are no corrections to the superpotential and to the gauge coupling functions. These theorems were originally proven by examining detailed properties of Feynman diagrams, but they can be understood far more simply.
E.g. consider the $N = 1, M = 1$ case and restrict attention to renormalizable theories.

$$W = X\left(\frac{m}{2}\phi^2 + \frac{\lambda}{3}\phi^3\right). \tag{10}$$

Here I have chosen not to include a linear term in ϕ; any such term can be absorbed in a redefinition of ϕ.

Now the R symmetry restricts any corrections to W to the form $Xf(\phi)$. Suppose that $\lambda = 0$. Then the theory has a larger symmetry, under which X and ϕ transform with phases $e^{2i\omega}$ and $e^{-i\omega}$ respectively. We can think of λ as itself the expectation value of a chiral field with charge 2 under this symmetry. Any correction to λ necessarily has the form

$$\delta f = \lambda^n c_n \tag{11}$$

but this does not respect the symmetry. Arguments of this type apply more generally.

Metastable Supersymmetry Breaking

Intriligator, Shih and Seiberg: example of metastable supersymmetry breaking in a surprising setting: vectorlike supersymmetric QCD. At a broader level, brought the realization that metastable supersymmetry breaking is a generic phenomenon. Consider the Nelson-Seiberg theorem, which asserts that, to be generic, supersymmetry breaking requires a global, continuous R symmetry. We expect that such symmetries are, at best, accidental low energy consequences of other features of some more microscopic theory.

This is illustrated by the simplest O'Raifeartaigh model:

$$W = \lambda X(A^2 - \mu^2) + mYA. \tag{12}$$

R symmetry with

$$R_Z = R_Y = 2; R_A = 0; X(\theta) \rightarrow e^{2i\alpha} X(e^{-i\alpha}\theta), \text{ etc..} \tag{13}$$

(Also Z_2 symmetry, $Y \rightarrow -Y, A \rightarrow -A$ forbids YA^2). SUSY broken; equations:

$$\frac{\partial W}{\partial X} = \frac{\partial W}{\partial Y} = 0 \tag{14}$$

are not compatible.
Features: If $m^2 > \mu^2$, $\langle A \rangle = 0 = \langle Y \rangle$; X undetermined.

Potential for X at one loop (Coleman-Weinberg); $\langle X \rangle = 0$. X lighter than other fields (by a loop factor). Scalar components – light pseudomodulus. Spinor is Goldstino.

$$\langle F_X \rangle = \lambda \mu^2 \tag{15}$$

is the decay constant of the Goldstino.

Aside 2. The Coleman-Weinberg Potential

Basic idea of Coleman Weinberg calculations is simple. Calculate masses of particles as functions of the pseudomodulus. From these, compute the vacuum energy:

$$\sum(-1)^F \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_i^2} \tag{16}$$

$$= \sum(-1)^f \left(\Lambda^4 + m_i^2\Lambda^2 + \frac{1}{(16\pi^2)} m_i^4 \ln(m_i)^4\right).$$

The first two terms vanish because of features of supersymmetry. The last must be evaluated, when supersymmetry is broken.

One finds (Shih) that if all fields have R charge 0 or 2, then the R symmetry is unbroken. Shih constructed models for which this is not the case. One of the simplest:

$$W = X_2(\phi_1\phi_{-1} - \mu^2) + m_1\phi_1\phi_1 + m_2\phi_3\phi_{-1}. \tag{17}$$

Continuous Symmetry from a Discrete Symmetry

The continuous symmetry of the OR model might arise as an accidental consequence of a discrete, Z_N R symmetry.
E.g.

$$X \to e^{\frac{4\pi i}{N}} X; \;\; Y \to e^{\frac{4\pi i}{N}} Y \tag{18}$$

corresponding to $\alpha = \frac{2\pi}{N}$ in eqn. 13 above.
Suppose, for example, $N = 5$. The discrete symmetry now allows couplings such as

$$\delta\mathcal{L} = \frac{1}{M^3}\left(aX^6 + bY^6 + cX^4Y^2 + dX^2Y^4 + \dots\right). \tag{19}$$

Note that $W \to e^{\frac{4\pi i}{N}} W$.
The theory now has N supersymmetric minima, with

$$X \sim \left(\mu^2 M^3\right)^{1/5} \alpha^k \tag{20}$$

where $\alpha = e^{\frac{2\pi i}{5}}$, $k = 1, \dots, 5$.

Metastability

Need a separate lecture to discuss tunneling in quantum field theory (some remarks in Banks' lectures). Suffice it to say that in models such as those introduced above, the metastable supersymmetric state can be extremely long lived. In particular, the system has to tunnel a “long way" (compared with characteristic energy scales) to reach the “true" vacuum. Thinking (correctly) by analogy to WKB, the amplitude is exponentially suppressed by a (large) power of the ratio of these scales.

MSSM:A supersymmetric generalization of the SM.

1. Gauge group $SU(3) \times SU(2) \times U(1)$; corresponding (12) vector multiplets.
2. Chiral field for each fermion of the SM: $Q_f, \bar{u}_f, \bar{d}_f, L_f, \bar{e}_f$.
3. Two Higgs doublets, H_U, H_D.
4. Superpotential contains a generalization of the Standard Model Yukawa couplings:

$$W_y = y_U H_U Q \bar{U} + y_D H_D Q \bar{D} + y_L H_D \bar{E}. \tag{21}$$

y_U and y_D are 3×3 matrices in the space of flavors.

Soft Breaking Parameters

Need also breaking of supersymmetry, potential for quarks and leptons. Introduce *explicit soft breakings*:

1. Soft mass terms for squarks, sleptons, and Higgs fields:
$$\mathcal{L}_{scalars} = Q^* m_Q^2 Q + \bar{U}^* m_U^2 \bar{U} + \bar{D}^* m_D^2 \bar{D} \tag{22}$$
$$+L^* m_L^2 L + \bar{E}^* m_E \bar{E}$$
$$+m_{H_U}^2 |H_U|^2 + m_{H_U}^2 |H_U|^2 + B_\mu H_U H_D + \text{c.c.}$$
m_Q^2, m_U^2, etc., are matrices in the space of flavors.
2. Cubic couplings of the scalars:
$$\mathcal{L}_{\mathcal{A}} = H_U Q\, A_U\, \bar{U} + H_D Q\, A_D\, \bar{D} \tag{23}$$
$$+H_D L\, A_E \bar{E} + \text{c.c.}$$
The matrices A_U, A_D, A_E are complex matrices
3. Mass terms for the U(1) (b), $SU(2)$ (w), and $SU(3)$ (λ) gauginos:
$$m_1 bb + m_2 ww + m_3 \lambda\lambda \tag{24}$$

Counting the Soft Breaking Parameters

1. $\phi\phi^*$ mass matrices are 3×3 Hermitian (45 parameters)
2. Cubic terms are described by 3 complex matrices (54 parameters
3. The soft Higgs mass terms add an additional 4 parameters.
4. The μ term adds two.
5. The gaugino masses add 6.

There appear to be 111 new parameters.

But Higgs sector of SM has two parameters.
In addition, the supersymmetric part of the MSSM lagrangian has symmetries which are broken by the general soft breaking terms (including μ among the soft breakings):

1. Two of three separate lepton numbers
2. A “Peccei-Quinn" symmetry, under which H_U and H_D rotate by the same phase, and the quarks and leptons transform suitably.
3. A continuous "R" symmetry, which we will explain in more detail below.

Redefining fields using these four transformations reduces the number of parameters to 105.
If supersymmetry is discovered, determining these parameters, and hopefully understanding them more microscopically, will be the main business of particle physics for some time. The phenomenology of these parameters has been the subject of extensive study; we will focus on a limited set of issues.

Constraints

Direct searches (LEP, Fermilab) severely constrain the spectrum. E.g. squark, gluino masses $>$ 100's of GeV, charginos of order 100 GeV. Spectrum must have special features to explain

1. Absence of Flavor Changing Neutral Currents (suppression of $K \leftrightarrow \bar{K}$, $D \leftrightarrow \bar{D}$ mixing; $B \rightarrow s + \gamma$, $\mu \rightarrow e + \gamma, \ldots$)
2. Suppression of CP violation (d_n; phases in $K\bar{K}$ mixing).

Might be accounted for if spectrum highly degenerate, CP violation in soft breaking suppressed.

The little hierarchy: perhaps the greatest challenge for Supersymmetry

Higgs mass and little hierarchy:
Biggest contribution to the Higgs mass from top quark loops. Two graphs; cancel if supersymmetry is unbroken. Result of simple computation is

$$\delta m_{H_U}^2 = -6\frac{y_t^2}{16\pi^2}\tilde{m}_t^2 \ln(\Lambda^2/\tilde{m}_t^2) \tag{25}$$

Even for modest values of the coupling, given the limits on squark masses, this can be substantial.

But another problem: $m_H > 114$ GeV. At tree level $m_H \leq m_Z$. Loop corrections involving top quark: can substantially correct Higgs quartic, and increase mass. But current limits typically require $\tilde{m}_t > 800$ GeV. Exacerbates tuning. Typically worse than 1 %.

$$\delta\lambda \sim 3\frac{y_t^4}{16\pi^2}\log(\tilde{m}_t^2/m_t^2). \tag{26}$$

Possible solution: additional physics, Higgs coupling corrected by dimension five term in superpotential or dimension six in Kahler potential.

$$\delta W = \frac{1}{M}H_U H_D H_U H_D \quad \delta K = Z^\dagger Z H_U^\dagger H_U H_U^\dagger H_U. \tag{27}$$

Microscopic Supersymmetry

Focus on two general approaches: Gravity mediation and gauge mediation.
In both cases, we need to know something about supergravity.
We expect supersymmetry to be a local symmetry. The most general lagrangian with terms up to two derivatives appears in Wess and Bagger, elsewhere; a good introduction also provided by Weinberg's textbook. Specified, again, by Kahler potential, superpotential, and gauge coupling functions,
For now, some features which will be important for model building, more general theoretical issues.

Supergravity

Some basic features:
Potential (units with $M_p = 1$):

$$V = e^K \left[D_i W g^{i\bar{i}} D_{\bar{i}} W^* - 3|W|^2 \right] \tag{28}$$

$D_i\phi$ is order parameter for susy breaking:

$$D_i W = \frac{\partial W}{\partial \phi_i} + \frac{\partial K}{\partial \phi_i} W. \tag{29}$$

If unbroken susy, space time is Minkowski (if $W = 0$), AdS ($W \neq 0$).
If flat space ($\langle V \rangle = 0$), then

$$m_{3/2} = \langle e^{K/2} W \rangle. \tag{30}$$

Mediating Supersymmetry Breaking

Generally assumed that supersymmetry is broken by dynamics of additional fields, and some weak coupling of these fields to those of the MSSM gives rise to soft breakings.
The classes of models called "gauge mediated" and “gravity mediated" are distinguished principally by the scale at which supersymmetry is broken. If terms in the supergravity lagrangian (more generally, higher dimension operators suppressed by M_p) are important at the weak (TeV) scale:

$$F_i = D_i W \approx (TeV) M_p \equiv M_{int}^2 \tag{31}$$

“gravity mediated" (more next lecture). If lower, “gauge mediated"; $F_i \approx \partial_i W$. In the latter case, usually renormalizable operators. Gauge interactions simplest; others ($=$ Yukawa couplings) are problematic.
In the low scale case, the soft breaking effects at low energies should be calculable, without requiring an ultraviolet completion; the intermediate scale case requires some theory like string theory.

Minimal Gauge Mediation

Main premiss underlying gauge mediation: in the limit that the gauge couplings vanish, the hidden and visible sectors decouple. Simple model:

$$\langle X \rangle = x + \theta^2 F. \tag{32}$$

X coupled to a vector-like set of fields, transforming as 5 and $\bar{5}$ of $SU(5)$:

$$W = X(\lambda_\ell \bar{\ell}\ell + \lambda_q \bar{q} q). \tag{33}$$

For $F < X$, $\ell, \bar{\ell}, q, \bar{q}$ are massive, with supersymmetry breaking splittings of order F. The fermion masses are given by:

$$m_q = \lambda_q x \quad m_\ell = \ \lambda_\ell x \tag{34}$$

while the scalar splittings are

$$\Delta m_q^2 = \lambda_q F \qquad \Delta m_\ell^2 = \lambda_\ell F. \tag{35}$$

In such a model, masses for gauginos are generated at one loop; for scalars at two loops. The gaugino mass computation is quite simple. Even the two loop scalar masses turn out to be rather easy, as one is working at zero momentum. The latter calculation can be done quite efficiently using supergraph techniques; an elegant alternative uses background field arguments. The result for the gaugino masses is:

$$m_{\lambda_i} = \frac{\alpha_i}{\pi} \Lambda, \tag{36}$$

For the squark and slepton masses:

$$\widetilde{m}^2 = 2\Lambda^2[C_3 \left(\frac{\alpha_3}{4\pi}\right)^2 + C_2 \left(\frac{\alpha_2}{4\pi}\right)^2 \tag{37}$$

$$+\frac{5}{3}\left(\frac{Y}{2}\right)^2 \left(\frac{\alpha_1}{4\pi}\right)^2],$$

where $\Lambda = F_x/x$. $C_3 = 4/3$ for color triplets and zero for singlets, $C_2 = 3/4$ for weak doublets and zero for singlets.

Features of MGM

1. One parameter describes the masses of the three gauginos and the squarks and sleptons
2. Flavor-changing neutral currents are automatically suppressed; each of the matrices m_Q^2, etc., is automatically proportional to the unit matrix; the A terms are highly suppressed (they receive no one contributions before three loop order).
3. CP conservation is automatic
4. This model cannot generate a μ term; the term is protected by symmetries. Some further structure is necessary.

General Gauge Mediation

Much work has been devoted to understanding the properties of this simple model, but it is natural to ask: just how general are these features? It turns out that they are peculiar to our assumption of a single set of messengers and just one singlet responsible for supersymmetry breaking and R symmetry breaking. Meade, Seiberg and Shih have formulated the problem of gauge mediation in a general way, and dubbed this formulation *General Gauge Mediation* (GGM). They study the problem in terms of correlation functions of (gauge) supercurrents. Analyzing the restrictions imposed by Lorentz invariance and supersymmetry on these correlation functions, they find that the general gauge-mediated spectrum is described by three complex parameters and three real parameters. Won't have time to discuss all of the features here, but the spectrum can be significantly different than that of MGM. Still, masses functions only of gauge quantum numbers of the particles, flavor problems still mitigated.

Lecture 2. Microscopic Models of Supersymmetry Breaking. Outline

1. MGM and GGM (conclusions)
2. Intermediate Scale Supersymmetry Breaking (“Gravity Mediation")
3. Low Energy, Dynamical Supersymmetry Breaking: A connection to the Cosmological Constant
4. The importance of Discrete R Symmetries
5. Gaugino condensation and its generalizations
6. Building models of Low Energy Dynamical Supersymmetry Breaking
7. Assessment
8. If time, a theorem about the superpotential

Intermediate Scale Supersymmetry Breaking ("Gravity Mediation")

Suppose, e.g., and O'Raifeartaigh like model breaks supersymmetry. W_0 chosen so that the cosmological constant vanishes.

$$F_Z \equiv D_Z W = \frac{\partial W}{\partial Z} + \frac{\partial K}{\partial Z} W \neq 0. \tag{38}$$

Leads to soft masses for squarks, sleptons; $\int d^2\theta Z W_\alpha^2$ coupling gives gaugino masses
Problem:

$$\delta K = \frac{\gamma^{\bar{i}j}}{M^2} Z^\dagger Z \phi_i^\dagger \phi_j \tag{39}$$

yields flavor-dependent masses for squarks and sleptons. Other potential difficulties include cosmological problems, such as gravitino overproduction, moduli problems.

While I won't consider string constructions per se, I will focus on an important connection with gravity: the cosmological constant. I will not be attempting to provide a new explanation, but rather simply asking about the features of the low energy lagrangian in a world with approximate SUSY and small Λ.

With supersymmetry, an inevitable connection of *low* energy physics and gravity

$$\langle |W|^2 \rangle = 3\langle |F|^2 \rangle M_p^2 + \text{ tiny}. \tag{40}$$

So not just F small, but also W. Why?

1. Some sort of accident? E.g. KKLT assume tuning of W relative to F (presumably anthropically).
2. R symmetries can account for small W (Banks). We we will see, $< W >$ can be correlated naturally with the scale of supersymmetry breaking. *Scale of R symmetry breaking: set by cosmological constant.*

Suggests a role for R symmetries. In string theory (gravity theory): discrete symmetries. Such symmetries are interesting from several points of view:

1. Cosmological constant
2. Give rise to approximate continuous R symmetries at low energies which can account for supersymmetry breaking (Nelson-Seiberg).
3. Account for small, dimensionful parameters.
4. Suppression of proton decay and other rare processes.

Continuous R Symmetries from Discrete Symmetries

This is a brief recapitulation of points made earlier
OR model:

$$W = X_2(A_0^2 - f) + mA_0 Y_2 \tag{41}$$

(subscripts denote R charges). If, e.g., $|m^2| > |f|$, $F_X = f$.

Can arise as low energy limit of a model with a discrete R symmetry:

$$X_2 \to e^{\frac{2\pi i}{N}} X_2;\ \ Y_2 \to e^{\frac{2\pi i}{N}} Y_2;\ \ A_0 \to A_0. \tag{42}$$

Allows $\delta W = \frac{X^{N-n} Y^{n+1}}{M_p^{N-2}}$. N susy vacua far away. Approximate, accidental R symmetry. SUSY breaking metastable.

$\langle W \rangle$: Gaugino Condensation

W transforms under any R symmetry; an order parameter for R breaking.

Gaugino condensation: $\langle \lambda\lambda \rangle \equiv \langle W \rangle$ breaks discrete R without breaking supersymmetry.

Readily generalized (J. Kehayias, M.D.) to include order parameters of dimension one.

E.g. N_f flavors, N colors, $N_f < N$:

$$W = y S_{ff'} \bar{Q}_f Q'_f + \lambda \mathrm{Tr} S^3 \tag{43}$$

exhibits a $Z_{2(3N-N_f)}$ symmetry, spontaneously broken by $\langle S \rangle$; $\langle \bar{Q} Q \rangle$; $\langle W \rangle$.

The dynamics responsible for this breaking can be understood along the lines developed in Seiberg's lectures. Suppose, for example, that $\lambda \ll y$. Then we might guess that S will acquire a large vev, giving large masses to the quarks. In this case, one can integrate out the quarks, leaving a pure $SU(N)$ gauge theory, and the singlet S. The singlet superpotential follows by noting that the scale, Λ, of the low energy gauge theory depends on the masses of the quarks, which in turn depend on S. So

$$W(S) = \lambda S^3 + \langle \lambda\lambda \rangle_S. \tag{44}$$

$$\langle \lambda\lambda \rangle = \mu^3 e^{-3\frac{8\pi^2}{b_{LE} g^2(\mu)}} \tag{45}$$

$$= \mu^3 e^{-3\frac{8\pi^2}{g_{LE} g^2(M)} + 3\frac{b_0}{b_{LE}} \ln(\mu/M)}$$

$$b_0 = 3N - N_F;\ b_{LE} = 3N \tag{46}$$

So

$$\langle \lambda\lambda \rangle = M^{\frac{3N-N_f}{N}} e^{-\frac{8\pi^2}{Ng^2(M)}} \mu^{\frac{N_f}{N}}. \tag{47}$$

In our case, $\mu = yS$, so the effective superpotential has the form

$$W(S) = \lambda S^3 + (yS)^{N_f/N} \Lambda^{3-N_f/N}. \tag{48}$$

This has roots

$$S = \Lambda \left(\frac{y^{N_f/N}}{\lambda} \right)^{\frac{N}{3N-N_F}} \tag{49}$$

times a Z_{3N-N_F} phase.
Consistent with our original argument that S large for small λ.
Alternative descriptions of the dynamics in other ranges of coupling.

Gauge Mediation/Retrofitting

Retrofitted Models (Feng, Silverstein, M.D.): OR parameter f from coupling

$$X(A^2-\mu^2)+mAY\rightarrow \tag{50}$$

$$\frac{XW_\alpha^2}{M_p}+\gamma SAY.$$

Need $\langle W\rangle = fM_p = \Lambda^3$, $\langle S\rangle \sim \Lambda$, for example.

$$m^2 \gg \mu^2$$

SUSY breaking is metastable (supersymmetric vacuum far away).

Gauge Mediation and the Cosmological Constant

Gauge mediation: traditional objection: c.c. requires large constant in W, unrelated to anything else.
Retrofitted models: scales consistent with our requirements for canceling c.c. Makes retrofitting, or something like it, inevitable in gauge mediation.

Other small mass parameters: m, μ-term, arise from dynamical breaking of discrete R symmetry. E.g.

$$W_\mu = \frac{S^2}{M_p} H_U H_D. \tag{51}$$

Readily build realistic models of gauge mediation/dynamical supersymmetry breaking with all scales dynamical, no μ problem, and prediction of a large $\tan\beta$.

R Symmetry Breaking in Supergravity

Supergravity (moduli): $W = f\, M_p\, g(X/M_p)$. $X \ll M_p$ could give approximate R, along lines of Nelson Seiberg. But unclear how one can get a large enough W under these circumstances (suppressed by both R breaking and susy breaking?).
Alternatively, again, retrofit scales.
These sorts of questions motivate study of W itself as an order parameter for R-symmetry breaking.

A Bound on W

Theorem: In any theory with spontaneous breaking of a continuous R-symmetry and SUSY:

$$|\langle W \rangle| \leq \frac{1}{2}|F|f_a$$

where F is the Goldstino decay constant and f_a is the R-axion decay constant.
Here illustrate in simple models; can prove quite generally, even for strong coupling.

The bound in O'Raifeartaigh models

Consider a generic renormalizable OR model with an R-symmetry $\Phi_i \to e^{iq_i\xi}\Phi_i$.

$$K = \sum_i \Phi_i \bar{\Phi}_i, \quad W(\Phi_i) = f_i\Phi_i + m_{ij}\Phi_i\Phi_j + \lambda_{ijk}\Phi_i\Phi_j\Phi_k$$

If the theory breaks SUSY at $\phi_i^{(0)}$ then classically it has a pseudomoduli space parameterized by the goldstino superpartner[Rey; Shih, Komargodski].

$$G = \sum_i \left(\frac{\partial W}{\partial \phi_i}\right)\psi_i, \qquad \Phi = \sum_i \left(\frac{\partial W}{\partial \phi_i}\right)\delta\phi_i,$$

Wherever the R-symmetry is broken there is also a flat direction corresponding to the R-axion.

Define two complex vectors $w_i = q_i \phi_i$ and $v_i^\dagger = \frac{\partial W}{\partial \phi_i}$
Since the superpotential has R-charge 2,

$$2\langle W\rangle = \sum_j q_j \phi_j \frac{\partial W(\phi_i)}{\partial \phi_j} = \langle v, w\rangle$$

On the pseudomoduli space we can write

$$|F|^2 = \sum_i \left(\frac{\partial W}{\partial \phi_i}\right)\left(\frac{\partial W}{\partial \phi_i}\right)^* = \langle v, v\rangle$$

Parameterizing $\phi_i(x) = \langle \phi_i(x)\rangle e^{i q_i a(x)}$ we obtain for the R-axion kinetic term:

$$\left(\sum_i |\phi_i(x)|^2 q_i^2\right)(\partial a)^2 \Rightarrow f_a^2 = \langle w, w\rangle$$

Then by the Cauchy-Schwarz inequality:

$$4|\langle W\rangle|^2 = |\langle v, w\rangle|^2 \leq \langle v, v\rangle\langle w, w\rangle = |F|^2 f_a^2$$

which is the bound to be established.

- The bound is saturated if $v \propto w$, in which case the R-axion is the Goldstino superpartner.
- Adding gauge interactions strengthens the bound because the D terms contribution to the potential makes $|F|^2$ larger.

Lecture 3. String Theory. Outline

1. What might it mean for string theory to make contact with nature – landscape as our only plausible setting (now).
2. The elephant in the room: The cosmological constant.
3. The Banks/Weinberg proposal. BP (string theory fluxes) as an implementation (details for Denef).
4. KKLT as a model. What serves as small parameter? Why is a small parameter important? Distributions of theories.
5. Supersymmetry in string theory and the landscape
6. KKLT as a realization of intermediate scale susy breaking.
7. A new look at susy breaking in the KKLT framework.
8. Landscape perspective on intermediate scale breaking.
9. Assessment
10. Discrete symmetries in string theory and the landscape
11. Strong CP and axions in string theory and the landscape (KKLT)
12. Assessment: string theory predictions(?)

String Theory and Nature

In Miriam Cvetic's lectures, you heard how string theory can come close to reproducing many features of the Standard Model: the gauge group, the number of generations, some features of Yukawa couplings. The constructions she described typically come with less desirable features, especially extra massless particles.

But these constructions raise obvious questions: why, say, intersecting branes, and not heterotic constructions, non-geometric models, F theory constructions... One answer is that perhaps we will find *the* model which describes everything, work out its consequences, and make other predictions.

Another is that we might find some *principle* which provides the answer, pointing to a unique string vacuum state. Finally, there is the point of view advocated by Banks at this school, that the different string “vacua" are actually different theories of quantum gravity; indeed, there are simply many different theoreies, just as there are many possible different field theories.

But there is at least one fact which points to a different possibility; this is is the cosmological constant.

The cosmological constant problem

Familiar to most of you. Illustrated by our earlier expression for the vacuum energy. Quartically divergent. Typically, in string models, if no supersymmetry, one obtains a result of the predicted order of magnitude, with a suitable cutoff (e.g. the string scale; first done by Rohm). Indeed, since there are typically moduli (and certainly moduli in any case where weak coupling computations make sense), this is the case.
So no evidence that string theory performs some magic with regards to this problem.

Banks, Weinberg: A proposal

Now I will use words that Tom told you one should never use, but it is to explain a brilliant idea which he put forward and Steven Weinberg fleshed out.

A possible way to understand small Λ. Suppose that the underlying theory has many, many vacuum states, with a more or less uniform distribution of c.c.'s (Bousso and Polchinski dubbed a discretuum). Suppose that the system makes transitions between these states, or in some other way samples all of the different states (e.g. they all exist more or less simultaneously). Now imagine a star trek type figure, traveling around this vast universe. (This requires all sorts of superluminal phenomenon, but we won't worry about this). Most universes will not be habitable; the c.c. will be of order, say, Planck scale. But sometimes the c.c. will be smaller, the universe will be comparatively flat. Under what conditions might this star trek character find intelligent life. Well, that's a tough question. As a proxy, Weinberg asked: under what circumstances, assuming all of the other laws of nature are the same, will this observer find galaxies. First, negative c.c. is really bad; if the resulting Hubble parameter is not much that a billion years or so, the universe will undergo a big

Bousso-Polchinski and KKLT

Bousso and Polchinski proposed that such a discretuum might arise in string theory as a result of fluxes. KKLT: 3-form fluxes in IIB theory. If χ types of fluxes, each taking L values, of order L^χ states. Can easily be enormous, large enough to give the sort of discretuum required. BP just supposed a stable *vacuum* or *state* for each flux choice, but not clear this is reasonable. E.g. not clear that moduli are fixed. KKLT provided a more detailed proposal, and a candidate for a small parameter which would allow exploration of some states.
Basics of KKLT (builds on much earlier work):

1. IIB Orientifold of CY, or F-theory on CY Four-Fold.
2. $h_{2,1}$ complex structure moduli, $h_{1,1}$ Kahler moduli, dilaton
3. Three form fluxes fix complex structure moduli, dilaton.
4. Additional branes required for matter, perhaps susy breaking.

Fixing the Kahler moduli

At this first stage, the Kahler moduli are undetermined. Low energy theory, after integrating out complex structure moduli, and assuming one Kahler modus, described by a *supersymmetric* effective action with

$$W = W_0 - Ae^{-\rho/b} \qquad K = -3\ln(\rho + \rho^\dagger) \tag{52}$$

Here

$$\rho = \rho + ia \tag{53}$$

where a is an axion, i.e. the theory respects a symmetry of discrete translations of a. Superpotential necessarily $e^{-c\rho}$. Here exponential arises, say, from gaugino condensation on branes.

KKLT (also Douglas, Denef): W_0 distributed uniformly as a complex variable. If many states, W_0 can be small. W_0 will serve as the small parameter (*note that if the number of states is finite, W_0 cannot be arbitrarily small, so there is not really a systematic expansion). For W_0 small, the superpotential has a supersymmetric stationary point:

$$D_\rho W \approx aA/be^{-\rho b} - \frac{3}{\rho + \rho^\dagger} W_0 = 0 \tag{54}$$

$$\rho = \rho_0 \approx -b \ln(|W_0|). \tag{55}$$

This nominally justifies a large ρ expansion, i.e. the α' expansion (Giddings, Kachru, Polchinski: mechanism to achieve weak string coupling).

Aside 3: How good is this expansion?

Corrections to the metric behave as $1/\rho F^2$ (compare one power of curvature in metric with three in flux terms; $\rho \sim 1/R^4$). Douglas, Denef: number of states of order $\chi^h_{2,1}$; sum of fluxes of order χ. Assuming W_0 naturally of order M_p, then smallest W_0 is

$$|W_0| > \frac{1}{\chi}^{h_{2,1}/2} \tag{56}$$

Flux terms of order χ (Douglas,Denef) If $\chi \sim h_{2,1}$, then the expansion is in powers of $1/\log(\chi)$. Probably not extremely small (ask Frederick). This estimate is very crude, but it is hard to see how things could be much better than this.

Supersymmetry Breaking in KKLT

So far, susy unbroken. By changing signs in superpotential (shifting a by π) we can find non-susy stationary points, but also AdS.

KKLT: assumed $\overline{D3}$ branes, break susy. Confusing since not clear how to describe in terms of a low energy, supersymmetric lagrangian (perhaps no four d description). In any case, can simply suppose there is another sector which spontaneously breaks susy, perhaps along lines of models we discussed here. What is important is that this extra sector gives a positive contribution to the cosmological constant (you can fill in Banks objections at this point).

So a plausible (but hardly rigorous) scenario to understand:

1. Large number of states
2. Fixing of moduli
3. Breaking of supersymmetry
4. Distribution of cc's, other parameters.

General Lessons, and Not

1. Is weak coupling important? At most, only to convince ourselves (to the extent we can) that such a landscape exists. E.g. with large W_0, one might still expect a uniform distribution of W_0 at low energies (Kachru).
2. $m_\rho^2 \sim \rho^2 m_{3/2}^2$; multiplet approximately supersymmetric (axion heavy). How general?
3. We used supersymmetry. Was this important?

What might we extract from the landscape?

Can't hope to find "the state". (Imagine problem of grad student given a vacuum with a bar code to study. Calculates Λ to third order, it's very small; get's very excited; to fourth, still small, to fifth, 40 years later to eighth– it's still small. Then, at ninth order; oh well, on to the next one.)

Want to ask questions about things like correlations – if such and such is true, is it often/always true that... Wati Taylor has studied such questions in intersecting brane models, looking at correlations between numbers of generations and other quantities, such as gauge groups. No good candidates.

But I would suggest that the most promising questions are those connected with questions of naturalness. This is precisely because the phenomena we are trying to explain seem at first sight unlikely.

1. The c.c. (already discussed)
2. The strong CP problem
3. Fine tuning of the electroweak scale.
4. Cosmological issues such as inflation.

There has been much work on the last of these, but it is very unclear what might be generic. I'll focus on the second and third.

WARNING: WE ARE ENTERING A ZONE OF (BIASED?) SPECULATION
***Needs a picture ***

Supersymmetry in the Landscape

At first sight, supersymmetry would seem special. Even if it is easier to explain hierarchies, we are talking about such large numbers of states that the number of non-susy states exhibiting huge hierarchies, could well overwhelm those in which the scale arises naturally (Douglas, Susskind).

Divide the landscape into branches:

1. States with no supersymmetry
2. States with approximate supersymmetry
3. States with approximate supersymmetry and discrete R symmetries.

One possible argument against the first branch and in favor of branches two and three: stability (now I will invoke a simple-minded version of Banks' concerns about "states"). In landscape, a typical state with small cc surrounded by vast number of states with large negative cc. What prevents decays to big crunch? In a typical state, no parameter which would account for smallness of decays. So, even though susy conditions special, this might favor susy states. Can play games in toy models to try and make quantitative, but probably can do no more than speculate at this stage.

On supersymmetric branch, why low energy susy favored? Here, conventional naturalness. Douglas/Denef: instances with uniform distribution of gauge couplings. If in significant fraction of typical states, dynamical susy breaking (seems plausible given our earlier discussions) then one might find most states with low electroweak scale have low susy breaking scale. Actually, one can do a simple analysis, using

1. Uniformity of W_0
2. Requirement of small cc.c.
3. Exponential variation of susy scale.
4. Fixed electroweak scale (anthropically)

One finds susy scales in fact logarithmically distributed.

R symmetries? Now W_0 exponentially distributed; perhaps correlated with susy breaking scale (as in retrofitting). Now very low scale susy breaking favored.

$$P(m_{3/2}) \propto \frac{1}{m_{3/2}^2} \tag{57}$$

*** **Check this. Consider deriving.*****

But R symmetry likely to be rare? Must set to zero all fluxes which transform; this is, for typical CY's, a finite fraction of the total – exponential suppression of the number of states.

But perhaps too naive. Consider tunneling again, now *to* the desired state. In toy models, decay to R symmetric states favored.

Axions: In KKLT, no light axion. Strong CP problem? Small θ not anthropically relevant. Much pessimism about this (Banks, Dine, Gorbatov; Donoghue).
But perhaps KKLT a little too simple as a model. Suppose multiple Kahler moduli, ρ_1, ρ_2, ρ_3, say.

$$W = W_0 + Ae^{-\frac{(n_1\rho_1 + n_2\rho_2 + n_3\rho_3)}{b}} \tag{58}$$

Now, “heavy" field,

$$\Phi = n_1\rho_1 + n_2\rho_2 + n_3\rho_3 \tag{59}$$

and two light fields. $\Phi \sim b\log(|W_0|)$. Low energy effective theory: Kahler potential for light fields, and constant superpotential. This theory can break susy, though needs additional fields or large Kahler potential modifications (plausible, given our statement about approximations). ρ_i all fixed, large. Light axions, if non-perturbative effects, like instantons,

$$\delta W \sim e^{-rhi_i} \tag{60}$$

(could even lead to “axiverse" of Dimopoulos et al; Raby, Acharya, Bobkov...; Dine).

So a coherent picture in which perhaps low energy supersymmetry is about to be found at LHC. But perhaps a house of cards. Many assumptions. Perhaps some other phenomena (warping, large extra dimensions, technicolor?) responsible for ew symmetry breaking. Or worse, just anthropic and we are about to find a single light Higgs. I hope I have outlined some questions and some possible approaches, but perhaps your young, fresh minds will come up with better ways of thinking about these questions. And perhaps, within a few years, we will have experimental verification of some of these ideas, or clues as to what physics lies Beyond the Standard Model.

Additional Slides Follow Elaborating Topics Above

While we won't review the analysis Meade et al in detail, it is easy to see, in simple weakly coupled models, how one can obtain a larger set of parameters. Take, for example, a model, as above, with messengers $q, \bar{q}, \ell, \bar{\ell}$, but replace the one singlet of the earlier model with a set of singlets, X_i. For the superpotential, take:

$$W = \lambda_i^q X_i \bar{q} q + \lambda_i^\ell X_i \bar{\ell} \ell. \tag{61}$$

Now, unlike the case of minimal gauge mediation, the ratio of the splittings in the multiplets to the average (i.e. fermion) masses is not the same for $q, \bar{q}$ and $\ell, \bar{\ell}$. For the fermion masses:

$$m_q = \sum \lambda_i^q x_i \quad m_\ell = \sum \lambda_i^\ell x_i \tag{62}$$

while the scalar splittings are

$$\Delta m_q^2 = \sum \lambda_i^q F_i \quad \Delta m_\ell^2 = \sum \lambda_i^\ell F_i. \tag{63}$$

In the case of MGM, the one loop contributions for fields carrying color were proportional to $\Delta m_q^2/m_q^2$, while those contributing to ℓ were proportional to $\Delta m_\ell^2/m_\ell^2$. One now finds, simply generalizing the previous computation, for the masses of the gauginos:

$$m_\lambda = \frac{\alpha_3}{4\pi}\Lambda_q \quad m_w = \frac{\alpha_2}{4\pi}\Lambda_\ell \tag{64}$$

$$m_b = \frac{\alpha_1}{4\pi}\left[2/3\Lambda_q + \Lambda_\ell\right].$$

where

$$\Lambda_q = \frac{\lambda_q^i F_i}{\lambda_q^j x_j} \quad \Lambda_\ell = \frac{\lambda_\ell^i F_i}{\lambda_\ell^j x_j} \tag{65}$$

(i and j summed).

Similarly, for the squark and slepton masses we have:

$$\widetilde{m}^2 = 2[C_3 \left(\frac{\alpha_3}{4\pi}\right)^2 \Lambda_q^2 + C_2 \left(\frac{\alpha_2}{4\pi}\right)^2 \Lambda_\ell^2 \tag{66}$$

$$+ \left(\frac{Y}{2}\right)^2 \left(\frac{\alpha_1}{4\pi}\right)^2 (\frac{2}{3}\Lambda_q^2 + \Lambda_\ell^2)]$$

At this point, it is easy to understand the parameter counting of Meade et al. We can write the general gauge-mediated spectrum in terms of three independent complex masses for the gauginos, and parameterize the general sfermion mass matrix as:

$$\widetilde{m}^2 = 2[C_3 \left(\frac{\alpha_3}{4\pi}\right)^2 \Lambda_{qcd}^2 + C_2 \left(\frac{\alpha_2}{4\pi}\right)^2 \Lambda_w^2 \tag{67}$$

$$+\left(\frac{Y}{2}\right)^2 \left(\frac{\alpha_1}{4\pi}\right)^2 \Lambda_b^2],$$

In the present case, there are two relations among these masses, which can be expressed as sum rules. But more generally, we have three independent complex parameters, the gaugino masses, and three additional real parameters.

Models with additional fields permit independent values for all of the parameters of GGM. In constructing examples, we will insist that the messengers fill complete multiplets of $SU(5)$, so as to preserve unification (one can legitimately ask why nature would be so concerned with achieving unification). For example, suppose one has a 10 and $\bar{1}0$ of messengers, and multiple singlets. The messengers can be denoted as $Q, \bar{Q}$, $U, \bar{U}$, and $E, \bar{E}$. One now has three independent parameters, which, by analogy to our previous example, can be denoted as $\Lambda_Q, \Lambda_U, \Lambda_E$.

The minimal, weak coupling theory which yields the full set of parameters of GGM consists of a 10 and $\bar{1}0$, and two $5, \bar{5}$ pairs. In this case, however, if the scale of supersymmetry breaking is low, the gauge couplings tend to get strong well below the unification scale. In addition, there is not an automatic messenger parity, so it is necessary to require additional structure in order to suppress the Fayet-Iliopoulos term for hypercharge. Finally, these models don't actually cover the full parameter space (though they have the maximal number of parameters); a strategy for doing this is described by Seiberg et al.

Simple Example of Superpotential Bound

$$W = \sum_{i=1}^{N_2} X_i f_i(\phi_a).$$

Here there are N_2 fields, X_i, of R charge 2, and N_0 fields, ϕ_a, of R charge zero. If $N_2 > N_0$, supersymmetry is broken, and there is a classical moduli space ($\frac{\partial W}{\partial \phi_a} = 0$ are N_0 equations for N_2 unknowns). On this moduli space

$$f_a = 2\sum |X_i|^2 \quad F = \left|\sum_i W_i\right|^2 \quad |\langle W \rangle|^2 = \left|\sum X_i W_i\right|^2$$

More intricate R assignments are more interesting (also needed (Shih) to obtain spontaneous R breaking on the pseudomoduli space).