Theoretical Particle Physics at Hadron Colliders: An Introduction

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Logic of these lectures

Two possible approaches

- Exciting possible new physics: the ideas and concepts behind the models, the structure of their predictions
- Critical issues without which LHC physics cannot be understood: what the expts can and can't do, what the Standard Model (SM) does and doesn't do, and what it makes experimentally possible and impossible.

It is easier to find the first in other lectures on-line and in books. Will provide some references.

But the key to understanding the first is understanding the second, and it is much harder to find summarized in the literature. For this reason I'm taking the second route.

Also, given 5 lectures and wanting to cover many topics, will be short on technicalities, especially ones that are easy to find elsewhere

Ok, here we go...

Lecture 1

The Standard Model (SM) of Particle Physics

- Three gauge groups, $SU(3)_C \times SU(2)_W \times U(1)_Y$ with
  - 8 gluons, 3 weak bosons W^a_μ and 1 hypercharge boson X_μ.
  - (Running) coupling constants g_3, g_2, g_1.
  - QCD confinement scale $\Lambda \sim 300$ MeV, Electroweak breaking (EWSB) scale $v = 246$ GeV
  - Three generations of matter, each containing these 2-component left-handed Weyl fermions:
    - a lepton ℓ_i and neutrino ν_i doublet of $SU(2)$
    - an antilepton $\bar{\ell}_i$ singlet of $SU(2)$
    - an up-type and down-type quark doublet u_i, d_i of $SU(2)$
    - an up-type antiquark $\bar{u}_i$ and a down-type antiquark $\bar{d}_i$, singlets of $SU(2)$
  - A "Higgs sector", consisting of at least one $SU(2)$ doublet scalar (fundamental or composite), to break $SU(2)_W \times U(1)_Y \to U(1)_{EM}$ and give mass to the fermions.

	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$
$L_i = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
$\bar{\ell}_i$	$\mathbf{1}$	$\mathbf{1}$	-1
$Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
$\bar{u}_i$	$\bar{3}$	$\mathbf{1}$	-$\frac{2}{3}$
$\bar{d}_i$	$\bar{3}$	$\mathbf{1}$	$\frac{1}{3}$
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$

$(i = 1, 2, 3)$

Electroweak Symmetry Breaking:

- Photon massless: Electric Charge $Q = T^3 + Y$ (where T_3 is third component of weak-isospin $SU(2)_W$ and Y is hypercharge); for instance
  - for electron $T^3 = -\frac{1}{2}$ and $Y = -\frac{1}{2}$
  - for up quark $T^3 = +\frac{1}{2}$ and $Y = \frac{1}{6}$
  - for up antiquark $T^3 = 0$ and $Y = -\frac{2}{3}$
- W, Z bosons massive (Weak force becomes weak at long distances)

via Higgs mechanism

$$\mathcal{L} = (D_\mu H)^\dagger D^\mu H - V(H^\dagger H)$$

where, defining $\mathbf{1}$ and τ^a as 2 by 2 unit and Pauli matrices,

$$D_\mu = \partial_\mu \mathbf{1} - \frac{i}{2}(g_1 X_\mu \mathbf{1} + g_2 W^a_\mu \tau^a)$$

If the potential $V(H^\dagger H)$ has a minimum away from zero – for instance, if

$$V(H^\dagger H) = -\mu^2 H^\dagger H + \frac{1}{2}\lambda(H^\dagger H)^2$$

with $\mu^2 > 0$, and thus a minimum at $H^\dagger H = \frac{1}{2}v^2$, where $v^2 = \mu^2/\lambda$, for instance

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

(and any $SU(2) \times U(1)$ rotation of this example) then we can redefine fluctuations of H as follows

$$H \equiv \langle H \rangle + \delta H = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} \mathcal{G}^+ \\ \frac{1}{\sqrt{2}}(h^0 + i\mathcal{G}^0) \end{pmatrix}$$

Then

$$D_\mu H \to \frac{iv}{2\sqrt{2}}\left[(g_1 X_\mu + g_2 W_\mu^3)\begin{pmatrix} 0 \\ 1 \end{pmatrix} + g_2(W_\mu^1 + iW_\mu^2)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] + D_\mu(\delta H)$$

and so $(D_\mu H)^\dagger D^\mu H$ contains the terms

$$M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu$$

where

$$M_W = \frac{1}{2}g_2 v \ , \ M_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2}\,v$$

$$W_\mu^+ = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2) \ , \ Z_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_1 X_\mu + g_2 W_\mu^3)$$

These terms make the Ws and Z massive, leaving the photon

$$A_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_2 X_\mu - g_1 W_\mu^3)$$

massless.

Exercise: Verify these formulas, and reexpress them in terms of $\sin\theta = g_1/\sqrt{g_1^2 + g_2^2}$, $\cos\theta = g_2/\sqrt{g_1^2 + g_2^2}$*, etc. From the masses of the* W *and* Z*, calculate* $\tan\theta = g_1/g_2$*. Which of the two couplings is weaker? What happens in the limit* $g_1 \to 0$*?*

Loose end: massless (massive) spin-one bosons have 2 (3) degrees of freedom. Where did the 3rd degree of freedom for W, Z come from?

Answer: $\mathcal{G}^\pm$ and $\mathcal{G}^0$, the would-be "Nambu-Goldstone bosons" (NGBs) of δH.

- Without the gauge interactions, the potential has a classical symmetry which is broken by $\langle H \rangle$, and this would give 3 NGBs $\mathcal{G}^\pm, \mathcal{G}^0$.
- With the gauge interactions the symmetry is a fake, and there aren't NGBs;
- but the degrees of freedom for the Higgs are still real. They get absorbed by the terms in $(D_\mu \langle H \rangle^\dagger) D_\mu \delta H$ that are of the form $(W_\mu^- v)(\partial_\mu \mathcal{G}^-)$, etc.
- Through these mixing terms the would-be NGB's become the **longitudinal polarization modes** of the massive W, Z bosons.

Loose end on the loose end: what happens to h^0?

This is the physical Higgs boson of the standard model. Notice that it has unusual interactions.

- Like any charged scalar, it has $hhVV$ interactions with gauge bosons

$$(D_\mu H)^\dagger D^\mu H \to \frac{1}{4} g_2 W^-_\mu h^0 W^{+\mu} h^0$$

- Unlike charged scalars, it does NOT have an hhV interaction.
- *Unlike any other type of scalar*, it DOES have a tree-level hVV interaction, proportional to its vev v.

$$(D_\mu H)^\dagger D^\mu H \to g_2 h^0 (M_W W^+_\mu W^{-\mu} + \frac{1}{2} M_Z Z^0_\mu Z^{0\mu})$$

  Exercise: CHECK THIS FORMULA! These interactions are a direct sign of electroweak symmetry breaking!
- There are also $h^0(\mathcal{G}^\pm W^\mp + \mathcal{G}^0 Z^0$ interactions.
- There is **no** tree-level $h^0 A_\mu A^\mu$ interaction.
- Small (but important) loop-induced interactions, of the form $h^0 F_{\mu\nu} F^{\mu\nu}$, are present for both photons and gluons.

The Higgs also gives mass to standard model fermions, **none of which can have mass until the Higgs has a vev.** Any mass terms for SM fermions without electroweak symmetry breaking would violate gauge invariance!

Lepton masses come from "Yukawa couplings":

$$\mathcal{L} = \sum_{i,k=1}^{3} y^{(\ell)}_{ik} (L_i \cdot H^\dagger) \bar{\ell}_k = \sum_{i,k=1}^{3} y^{(\ell)}_{ik} (\nu_i H^- + \ell_i H^{0*}) \bar{\ell}_k \to \sum_{i,k=1}^{3} m^{(\ell)}_{ik} \ell_i \bar{\ell}_k$$

where

$$m^{(\ell)}_{ik} = \frac{1}{\sqrt{2}} y_{ik} v$$

Thus the masses of the leptons are proportional to their coupling strength to the Higgs boson; more precisely the masses are related the eigenvalues of the Yukawa couplings.

Similarly for down-type and up-type quark masses, which come from $(Q_i \cdot H^\dagger)\bar{d}_k = (u_i H^- - d_i H^{0*})\bar{d}_k$ and $(Q_i \times H)\bar{u}_k = (u_i H^0 - d_i H^+)\bar{u}_k$ terms.

The masses of all these particles are known, and in the standard model the top quark couples most strongly to the Higgs boson. The up and down quarks and the electron barely couple at all.

	$\bar{u}_i$	$\bar{d}_i$	$\bar{\ell}_i$
1st generation	~ 0.003	~ 0.006	0.000511
2nd generation	~ 1.2	~ 0.01	.1057
3rd generation	$\sim$172	~ 4.3	1.776

Neutrino masses are different. They come from a dimension-five (non-renormalizable) interaction $\frac{f_{ij}}{M_0}(L_i \times H)(H \times L_j)$. Here we define M_0 so that the f's are not larger than about 1. Presumably there is new physics at or below the scale M_0. The neutrino masses have not yet been measured; only some differences of masses-squared are known at present, and there is an upper bound on the sum of the masses.

What do we know? and what do we suspect?

The existence of the vev v is already certain. There is no other way to give masses to W and Z bosons, or the fermions, that would be consistent with experiment.

The existence of the particle H is less certain.

- There might be several $SU(2)$-doublet Higgs bosons H_r, $r = 1, 2, \ldots, n$, with $v = \sqrt{\sum \langle H_r \rangle^2}$, which would lead to $4n-3$ physical Higgs scalars, $2n-1$ of them real and neutral, and n each of charge $+1$ and -1.
- There could also be additional $SU(2) \times U(1)$ singlet Higgs bosons, which would give additional neutral scalars.
- There might be no physical Higgs boson at all.

But we know this: the SM **without the Higgs boson and with nothing else** is *impossible.*

Perturbative Unitarity Problem
Why? The SM, treated in perturbation theory, violates unitarity. $W^+W^- \to W^+W^-$, $W^+W^- \to Z^0Z^0$, $W^+W^- \to t\bar{t}$, and many other processes grow violently with energy above a few hundred GeV if the Higgs diagram is dropped.

There are several possible solutions to this problem.

- Unitarity is restored through the presence of a Higgs particle (or multiple Higgs particles) which through their coupling to masses give contributions that cancel off the energy growth. These particles must have mass near or below 1 TeV.
- Unitarity is restored through the presence of other classes of particles (such as a tower of W, Z, t copies) which through carefully adjusted couplings give contributions that cancel off the energy growth. The lightest of these particles must have mass near or below 1 TeV.
- Perturbative field theory calculations become invalid at or around 1 TeV, because the interactions of heavy particles — of W and Z bosons and at least t quarks — become strong, due to new dynamics (new forces and particles) at or below that scale.
- Quantum field theory itself breaks down at around 1 TeV.

I should note these are not all mutually exclusive.

Let me emphasize: **The unitarity problem of the Higgs-less standard model is an extremely serious problem.** It must be solved. And this is why a TeV-scale collider is needed, to explore a regime where the particles and forces we know so far form an inconsistent whole. No matter what such a machine finds around or below the 1 TeV range, it is "new physics", where the theory of the known particles makes no prediction.

Hierarchy Problem
There is a **second** problem, known as the hierachy problem. The problem is the following. In any quantum field theory, we expect dimensional analysis to hold unless there is a very good reason. Why?

Because even if you tried to adjust the classical (or better, the bare) interactions of a quantum field theory to be smaller than you would expect from dimensional analysis, quantum corrections would generally raise the effective size of the interaction that you measure to be what you expected from dimensional analysis.

Said another way, *quantum corrections generally respect dimensional analysis.* When they don't, there's always a good reason.

We don't see this much in the SM, because all the interactions *except* the Higgs mass have a good reason to violate dimensional analysis.

In particular, the gauge and Yukawa couplings are all dimensionless, and so expected to be of order 1. However the electron Yukawa coupling y^e is 10^{-5}. Is that reasonable?

If y^e were zero, there would be a new symmetry (a chiral symmetry rotating the e^+ field).

Therefore quantum corrections must satisfy $\delta y_e \propto y_e$, so if y_e is small quantum corrections leave it small.

So how large would we expect v to be?

We know $v^2 = -\mu^2/\lambda$ in the standard model. To get this result, we assumed perturbation theory (and thus a classical analysis) was a good approximation. This implies λ is at most 3 or so.
So $|\mu^2|$ cannot be larger than about (1 TeV)^2.

But the natural value for $|\mu^2|$ would be the largest physical mass scale in particle physics: $\delta\mu^2 \propto M_{max}^2$. And (naively at least) M_{max} is at least as large as the Planck scale, 10^{18-19} GeV.

Therefore the natural value for v is either around the Planck scale (if $-\mu^2 < 0$) or zero (if $-\mu^2 > 0$).

But its actual value is 246 GeV. How is this possible?

This is the hierarchy problem. It is an unexplained failure of dimensional analysis. **The hierarchy problem of the standard model is a disturbing issue, but it is not an inconsistency of the theory.** It is merely an observation that there is something unnatural about the theory. (Indeed it is called a "naturalness" problem.) But this might just be an issue of perspective.

Many attempts are made to solve the hierarchy problem, by changing the theory to either (a) alter the dimensional analysis itself, or (b) create a reason why dimensional analysis is not satisfied by μ^2, or (c) change quantum field theory altogether. *All of these solutions produce new particles at the TeV scale.* These should also be detectable by a TeV-scale collider.

Both the serious perturbative unitarity problem and the disturbing hierarchy problem tell us that looking at the TeV scale is a good idea. There will definitely be new physics; there may be a LOT of new physics.

NOW: Why the LHC?

Max. beam energy 7 TeV ; Center of mass energy $\sqrt{s} = 14$ TeV

(I will talk about reduced energy of 7 or 10 TeV occasionally, but all numbers, unless otherwise specified, refer to 14 TeV collisions.)

Goals

- Explain electroweak symmetry breaking (EWSB) scale $v = \langle h \rangle = m_W / g_W = 246$ GeV =
- Find the Higgs boson(s) if present
- Find other particles or phenomena if present below few TeV
- Help resolve hierarchy problem
- Perhaps shed light on dark matter

These requirements determined:

- Accelerator Design
- Detector Design

as you will see understand through Jason Nielsen's lectures.

LHC a proton-proton collider. What is a proton?

True or false:

- There are three quarks in a proton
- Each quark carries on average 1/3 the momentum of the proton.
- There are only u and d quarks in a proton (not s, c, b, t)
- The quarks are confined inside the proton and can never escape it.

Actually all of these statements are false.

- A proton contains two u quarks, a d quark, and innumerable gluons and quark-antiquark pairs. (It still has the same quantum numbers as uud!)
- The pairs come in all flavors $u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}$ (t is exceedingly rare because of its large mass.)
- The gluons carry a substantial fraction of the proton's momentum.
- Quarks, if given a kick, can break free of their parent proton through quark-antiquark pair production, leading to at least one new hadron in the process: for example

$$(uudgggd\bar{d}) + \text{Energy} \rightarrow (uusgggd\bar{d}) + (\bar{s}d)$$

In some ways these are good things; otherwise the only collisions we would have are quark-quark collisions, and these are some of the worst ones for making new physics. $q\bar{q}$, qg, gg and even $b\bar{b}$ can all be better.

Also, if quarks were completely confined, then when we scattered quarks or produced new ones, we would never see that we had done so.

A Classic LHC Process:
collide two strongly interacting particles to make weakly-interacting ones

$$u\bar{d} \to W^+ \to e^+\nu_e$$

EXERCISE: Note that there are other initial states which are suitable for making a W^+. What are they?

Calculate the rate for this process?
First we need the initial u and d.

Parton Model:
Inside fast proton with momentum $P_\mu \approx (E, 0, 0, E)$,
can *sensibly* define the probability to find a u with momentum $p_\mu \approx x P_\mu$ $(0 < x < 1)$

Not obvious! Both an oversimplified model (will unsimplify later) and a highly nontrivial consequence of very special properties of QCD, not shared by all theories.

Define differential cross-section for $u\bar{d} \to W^+ \to e^+\nu_e$

$$d\hat{\sigma}_{u\bar{d}\to W\to e\nu}(p_u^\mu = x_1 P_1^\mu, p_{\bar{d}}^\mu = x_2 P_2^\mu, p_e^\mu, p_\nu^\mu)$$

We can calculate this (naively) using Feynman diagrams.

EXERCISE if time permits: Calculate the amplitude for the above process. Then square it, averaging over initial spins and colors, and summing over final spins, to get a simple expression which depends only on dot-products of the momenta. Assume the W boson is off-shell. Ignore all quark and lepton masses (but not the W mass!) Be careful to include the parity-violating couplings of the W

Then naively (and almost correctly!)

$$d\sigma_{pp\to W^+ + \dots \to e^+\nu_e + \dots}(P_1^\mu, P_2^\mu, p_e^\mu, p_\nu^\mu) =$$

$$= \int_0^1 dx_1 \int_0^1 dx_2 \; f_u(x_1) f_{\bar{d}}(x_2) \; d\hat{\sigma}_{u\bar{d}\to W\to e\nu}(x_1 P_1^\mu, x_2 P_2^\mu, p_e^\mu, p_\nu^\mu)$$

plus another term where u comes from proton number 2 and $\bar{d}$ from proton 1 *(plus terms for other initial quark-antiquark states, see the previous exercise.)*

Note the dots represent all the debris from the two disrupted protons (the **"Underlying Event"** *; we can't easily (and we're not going to try to) measure that debris precisely. At least our theories haven't suggested that new physics will be found there.* (Attention: Theory Bias!) *Will drop this notation below, but remember it is always there!*

In this and all classic processes, we take a partonic cross-section (which we calculate using Feynman diagrams), and convolve it with the parton distribution functions (which we measure in "deep inelastic scattering" and elsewhere) to obtain a proton-proton cross-section.

However, just because this process is **classic** does not mean it is **typical!**

Look at Figure 1, and be afraid.

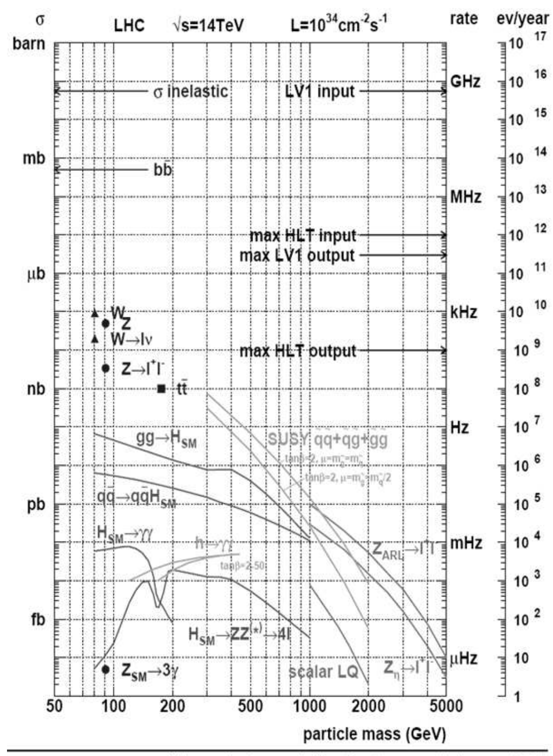

Figure 1: Production rates for various important processes at the LHC.

Some definitions, and interpretation of Figure 1.

Cross section: units of area – cm^2, or 1 barn $= 10^{-24}$ cm$^2 = (10^{-12}$ cm$)^2$ (mb, μb, nb, pb, fb)

- Radius of proton $= 10^{-13}$ cm:
- Geometric cross-section of proton $\pi r_p^2 \sim 30$ mb.
- $\sigma_{pp}^{total} \sim 80$ mb (slowly grows with energy [“Regge”, “Pomeron”])
- Weak scale: length 10^{-16} cm, or area 80 nb

And thus

$$\sigma_{pp\to W\to e\nu} \sim 20 \text{ nb} \sim 2\times 10^7 \text{ fb} \sim 10^{-7}\ \sigma_{pp}^{total}$$

BUT

$$\sigma_{pp\to h\to\gamma\gamma} \sim 10^2 \text{ fb} \sim 10^{-12}\ \sigma_{pp}^{total}$$

Lesson: Needle in a Haystack

Note light Higgs decays dominantly $h\to b\bar{b}$, why don't we look there?

$$\sigma_{pp\to h\to b\bar{b}} \sim 30 \text{ pb} = 10^{-7}\sigma_{pp\to b\bar{b}}$$

Self-Study:

Luminosity, Integrated Luminosity

- Number of Events / second = $\sigma\mathcal{L}$
- Number of Events over a period of time = $\sigma \int \mathcal{L} \; dt$

Units of Luminosity: $\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$: very rough estimate:

- 2010: $10^{30-31}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$
- 2011: $10^{31-32}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$
- 2013: $10^{32-33}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$
- 2016: $10^{34}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$
- 2019: $10^{35}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$

Units of Integrated Luminosity: cm^{-2} or pb^{-1} or fb^{-1} ("inverse picobarns")

How long is a year? 3×10^7 sec, but no accelerator runs, or funs efficiently, every day. Typically a good year of running is 10^7 sec.

So $\int \mathcal{L}\, dt$ for a year $\approx 10^7$ sec $\times \mathcal{L}$
Very rough estimate:

- 2010: 0.1 fb^{-1} at 7 TeV
- 2011: 1 fb^{-1} at 7 TeV
- 2012: shutdown for repairs
- 2013: $1 - 10\ \mathrm{fb}^{-1}$
- 2014-2015: $10 - 20\ \mathrm{fb}^{-1}$
- 2016-2018: $50 - 100\ \mathrm{fb}^{-1}$
- 2019: 1000 fb^{-1}

$\sigma \int \mathcal{L}\, dt$ has no units: it gives the number of events for a given process
Example: $\sigma_{pp\to W\to e\nu} \approx \sigma_{pp\to W}\mathrm{Br}(W \to e\nu) \sim 20$ nb $= 2 \times 10^7$ fb at 14 TeV (5 nb at 7 TeV)
(note shorthand dropping charges and bars when unambiguous)

- How many $W \to e\nu$ in 2010-11? 5×10^6.
- How many $W \to e\nu$ in 2014? $2 - 4 \times 10^8$.

EXERCISE: List the decay modes of the W boson. Estimate (theoretically) the "branching ratio" for $W^+ \to e^+\nu$ (i.e., what fraction of W^+ decays go to positron and neutrino?) [Hint: either ignore quark mixing, or (if you know how) account for it – you will get essentially the same answer. Why?] How many Ws will be produced in 2014?

Critical problem:

10^{12-16} events/year > all human computing power

Must reduce by 10^{4-6} to handle 10^3 TBytes/year $\sim 10^{9-10}$ events/year

$\Rightarrow$ *must throw away, real time, irretrievably,* 99.99(99)% *of collisions*

<u>Trigger</u>

- Keep only events that "look" interesting — to an automated system.
- Must throw out 99.9999% of hay and very little of needle.
- Need to be selective to enhance rare processes, reduce common ones
- *If we make bad choices and our automated system is badly or unwisely programmed, we will throw away the new physics.*

Theory plans a large role in guiding these choices (Attention: Theory Bias!) both through its predictions of background and its ideas of what signals of new physics will look like.
If theorists insufficiently imaginative, could miss the new physics!

Another Problem Beyond the Trigger

- 10^{9-10} events/year recorded
- Many new physics signatures arise $10 - 1000$/year
- 10^3 physicists

You're not going to find $1/10^6$ events by paging through event pictures!

Reconstruction and Analysis Software

Another automated system needs to do interpretation of data: initial ("reconstruction", automated) and detailed ("analysis", semi-automated)
this too introduces assumptions...

But if there's some process that looks different than what we typically expect, software might miss it, <u>even</u> in cases where the human eye might notice it. (Attention: Theory Bias)

Everything that can go wrong will go wrong — sometimes —

8–14 orders of magnitude between collision rate and new signal

Even things that happen in $1/10^6$ or even $1/10^{12}$ events can be significant challenges

- Background to $h \to \gamma\gamma$: fake isolated photons are as bad or worse than real ones

- Fake missing momentum typically dominates real missing momentum.
- Fake τs are very common
- "Beam Halo", "beam-gas collisions", secondary scattering.

Theory can barely predict anything reliably

- All processes are sensitive to treatment of strong interactions.
  - Perturbation theory is technically difficult and barely works.
  - Simple perturbative amplitudes need to be supplemented by resummation of infinite classes of Feynman graphs.
  - Nonperturbative processes are always present and sometimes are very important.
- Experiments simply can't measure what theorists can calculate easily
- Theorists can't calculate what experiments can measure easily.

Consequently

- Theory must be passed through extensive and not-entirely-reliable software ("Monte Carlo": event simulation, detector simulation) in order to make predictions for what experiments actually measure
- Theory that predicts SM backgrounds must be tested in the data to reduce uncertainties (and this is not always easy)
- Searches for new physics typically try to compare data with data, not data with theory.

If there's anything that could possibly make a theorist humble, it's a hadron collider!

Because of this complexity, and the limitations of theory, a critical part of understanding how to do physics at the LHC is to understand, in detail, how the experiments are done. So PAY ATTENTION to Jason Nielsen's lectures; you may find it challenging, but you will not regret it!

Lecture 2

Some Kinematical defns and essential formulas
(see particle data book pdg.lbl.gov, chapter "Kinematics")

Natural coordinates cylindrical around beampipe : θ polar angle, ϕ azimuthal angle

Pseudorapidity = a function of polar angle
($\theta = 0$ forward,π backward, $\pi/2$ central)

$$\eta \equiv -\log\tan(\theta/2) = \log\cot(\theta/2)$$

$$\sinh\eta = \cot\theta \ ; \ \cosh\eta = 1/\sin\theta \ ; \ \tanh\eta = \cos\theta \ ;$$

- $\eta \approx \pi/2 - \theta$ for $\theta \sim \pi/2$
- $\eta \approx \log(2/\theta)$ for $\theta \ll 1$
- $\eta \approx \log(2/[\pi - \theta])$ for $\pi - \theta \ll 1$

EXERCISE: What angles θ correspond to $\eta = 0, \pm 1, \pm 2, \pm 3, \pm 4$? Obtain these as numbers and sketch them on a diagram.

Define "distance" between two directions

$$\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$$

Rapidity = a function of E, p_z

$$y = \frac{1}{2}\log\frac{E + p_z}{E - p_z} = \tanh^{-1}(p_z/E) = \tanh^{-1} v_z$$

EXERCISE: Prove that under boost in z direction with velocity β, $y \to y + \tanh^{-1}\beta$ [thus demonstrating that Δy is Lorentz-invariant under boosts along the beam direction.]

EXERCISE: Prove that for a massless particle, $y = \eta$ [thus demonstrating that for massless or nearly massless particles moving in the detector, ΔR is Lorentz-invariant under boosts along the beam direction.]

These two facts, combined with the additional fact that since in general $x_1 \neq x_2$, **the center-of-mass frame of the parton-parton collision is not the lab frame but is instead in motion along the beampipe**, motivate the use of η and R rather than θ.

Naive Parton-Parton Kinematics

For particles with $m \ll E$,

$$(k_1 + k_2)^2 \approx 2k_1 \cdot k_2 = 2E_1E_2 - 2\vec{p}_1 \cdot \vec{p}_2 = 2E_1E_2(1 - \cos\theta_{12})$$

Mandelstam variables

proton momenta $P_1^\mu = E_b(1, 0, 0, 1)$, $P_2^\mu = E_b(1, 0, 0, -1)$

$s \equiv (P_1 + P_2)^2 = (2E_b)^2 = E_{com}^2 = 2m_p^2 + 2P_1 \cdot P_2 \approx 2P_1 \cdot P_2.$

parton momenta $p_i = x_iP_i$

$\hat{s} \equiv (x_1P_1 + x_2P_2)^2 = \hat{E}_{pcom}^2 \approx 2x_1P_1 \cdot x_2P_2 \approx x_1x_2s.$

Thus $\quad \hat{E}_{pcom}/E_{com} = \sqrt{x_1x_2}$

Compare lab frame with parton c.o.m. frame:

p.c.o.m frame	lab frame
$E^{tot} = E_{pcom} = \sqrt{\hat{s}} = \sqrt{x_1x_2}\sqrt{s}$	$E^{tot} = (x_1 + x_2)E_b = \frac{1}{2}(x_1 + x_2)\sqrt{s}$
$p_z^{tot} = 0$	$p_z^{tot} = (x_1 - x_2)E_b = \frac{1}{2}(x_1 - x_2)\sqrt{s}$
$\vec{p}_T^{\,tot} = 0$	$\vec{p}_T^{\,tot} = 0$
$\hat{v}_z = 0$	$\hat{v}_z = \frac{x_1 - x_2}{x_1 + x_2}$
$\hat{y} = 0$	$\hat{y} = \log\sqrt{\frac{x_1}{x_2}}$ $\;(\exp^{\hat{y}} = \sqrt{x_1/x_2})$
$\hat{\gamma} = 0$	$\hat{\gamma} = \frac{x_1 - x_2}{x_1 + x_2}$

Note also that $\hat{\eta} = \pm\infty$ and that

$$x_1 = \sqrt{\frac{\hat{s}}{s}}e^{\hat{y}} \;;\; x_2 = \sqrt{\frac{\hat{s}}{s}}e^{-\hat{y}}$$

EXERCISE: Prove the formulas in the table and the formula above.

EXERCISE: Show that for fixed $\hat{s}$, $|\hat{y}| \leq \log\sqrt{\frac{\hat{s}}{s}}$.

If the process is $2 \to 2$ parton scattering $p_1, p_2 \to p_3, p_4$ define

$$\hat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2 \;;\; \hat{t} = (p_3 - p_1)^2 = (p_4 + p_2)^2 \;;\; \hat{u} = (p_4 - p_1)^2 = (p_3 + p_1)^2$$

(The equalities follow from momentum conservation.) Well-known that $\hat{s}+\hat{t}+\hat{u} = m_1^2+m_2^2+m_3^2+m_4^2$
Check this if you've never seen it before!

If the particles are all effectively massless then

$$\hat{t} = -2E_1E_3(1-\cos\theta_{13}) = -(\hat{s}/2)(1-\cos\theta_{pcom})$$

since $E_1 = E_2 = E_3 = E_4$ in p.c.o.m frame.

$$\hat{u} = -\hat{s} - \hat{t} = -2E_1E_4(1-\cos\theta_{14}) = -(\hat{s}/2)(1+\cos\theta_{pcom})$$

Note **transverse momentum** $\vec{p}_{i,T} \equiv (p_{i,x}, p_{i,y})$ is invariant under z-boosts; it is the same in the lab frame and in the p.c.o.m. frame.

In the p.c.o.m. frame

$$p_T = \frac{\sqrt{\hat{s}}}{2}\sin\theta_{pcom} = \frac{\sqrt{\hat{t}\hat{u}}}{2\hat{s}} \ ;$$

the latter expression is built from Lorentz invariants.

Summary of Lorentz invariants for "massless" observed particles
(γ, e, μ, most observed hadrons)

$$\hat{s}, \hat{t}, \hat{u} \ ; \phi, \Delta\eta \to \Delta R; p_T$$

Since p_T is invariant we may combine events whose particles have the same p_Ts but which have different $\hat{y}$.

These events may have the same $\hat{s}, \hat{t}, \hat{u}$; they have the same x_1x_2 but differ in x_1/x_2 and thus in $\hat{y}$ or $\hat{v}_z$ of the p.c.o.m frame as measured in the lab frame.

The massless particles from these events will appear at different η in the lab, but their $\Delta\eta$ and ΔR distances will be unchanged.

A couple of event-wide z-boost-invariant quantities

Measure of scale of visible p_T

$$H_T \equiv \sum_{\text{objects } i} |\vec{p}_{i,T}|$$

(which objects to include in the sum varies; read carefully)

Measure of invisible $\vec{p}_T$

$$\vec{\not{p}}_T \equiv -\sum_{\textbf{all}\text{ visible}} \vec{p}_{i,T}$$

This is the "missing transverse momentum"; $|\vec{\not{p}}_T|$ called the "missing energy" or "missing transverse energy" or "MET" or $\not{E}_T$. If the detector were perfectly hermetic then

$$\vec{\not{p}}_T \equiv +\sum_{\textbf{all invisible}} \vec{p}_{i,T}$$

Obviously must sum over everything visible, or everything invisible, to do correct accounting.

Question: why don't we look for missing p_z or missing E?

Answer: the scattered partons carried E and p_z but not $\vec{p}_T$, so the debris from the underlying event has $E < 14$ TeV and nonzero p_z, but no $\vec{p}_T$. Typically the hadrons in the debris have low p_T, but many have very high E, p_z. This means much of the debris remains inside or very near the beampipe. We cannot measure it there, so we cannot do an accounting of all the E or p_z to see if any is missing. We can only account for p_T.