QCD is everywhere at LHC

Critical special features of QCD:

- $N_c = 3$ colors
- Strong coupling scale $\Lambda_{QCD} \equiv \Lambda \sim 300$ MeV
- $N_f = 6 \quad : \quad N_f^{light} = 3$ with $m < \Lambda$; $N_f^{heavy} = 3$ with $m \gg \Lambda$

QCD FACT 1: asymptotically free in the ultraviolet, "confining" in the infrared.

This is not general!

More generally,

- Beta function $\beta_{\alpha_s} \propto -(\frac{11}{3}N_c - \frac{2}{3}N_f)$ negative at all scales $\mu \gg \Lambda$, unless N_f is quite large.
- If running coupling $\alpha_s N_c \ll 1$ and N_f is not too large, then for $\mu \gg \Lambda$

$$\alpha_s(\mu)N_c \approx \frac{2\pi}{(\frac{11}{3} - \frac{2}{3}\frac{N_f}{N_c})\log(\Lambda/\mu)}$$

(Strictly correct only if N_f did not change with μ. This detail is not critical here.)

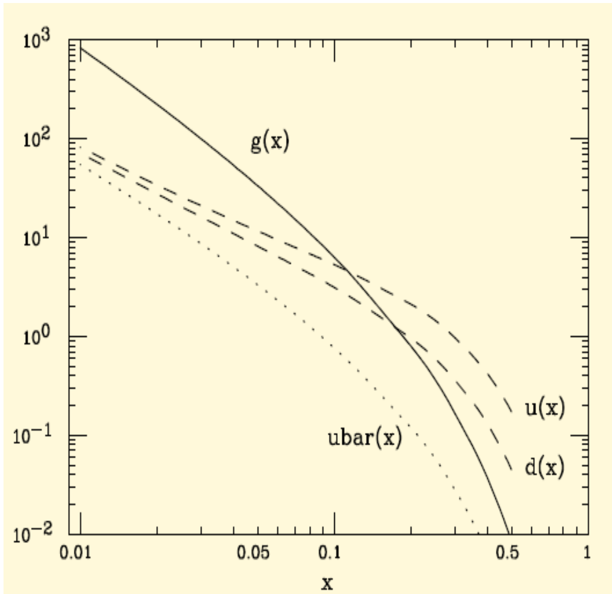
- Perturbation theory is good approximation when $\alpha_s N_c \ll 1$.
- If N_f is too large then $\beta > 0$.
- Otherwise, as $\mu \to 0$, α_s grows large.
- But *confinement does not logically follow.* In fact, if N_f is large but $\beta < 0$ higher-loop corrections may cause $\beta(\alpha_s) \to 0$ at a finite α_s. *In QCD, N_f is too small and this does not happen.*
- Conversely, confining theories need not be asymmptotically free; even if $\alpha_s \to \infty$ as $\mu \to 0$, may be, as $\mu \to \infty$, that $\alpha_s \to$ nonzero constant.

Thus, coexistence of UV freedom and IR slavery not automatic. $N_f \ll \frac{11}{2}N_c$ is responsible.

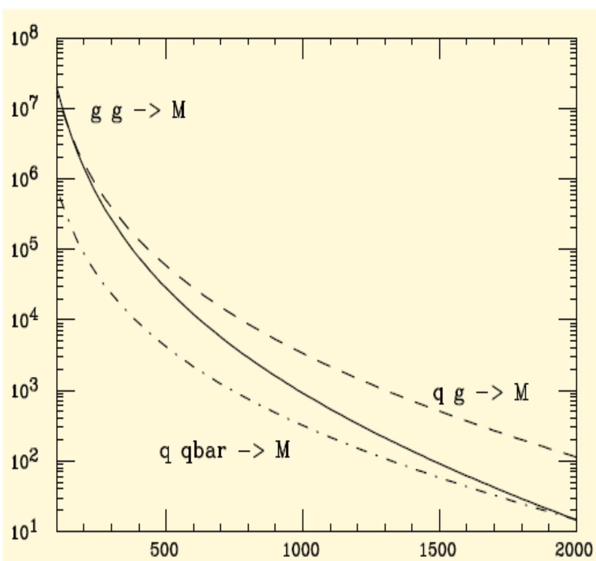
EXERCISE: Look up the formula for the beta function of a QCD-like theory at two loops. Show that there is a zero of the beta function when $0 < 11N_c - 2N_f \ll N_c$. Bonus question, if you know about the large-N_c expansion: show that for large N_c this is stable against higher loop corrections. [Hint: what value does $\alpha_s N_c$ take at the beta-function zero?]

QCD FACT 2. "Factorization": Partons inside hadrons can be described by convolving process-independent (universal!) parton distribution functions (pdfs) with process-dependent short-distance ($\hat{s}, \hat{t}, \hat{u} \gg \Lambda^2$) perturbative amplitudes.

- Clearly without $\alpha_s N_c \ll 1$ at $\mu \gg \Lambda$ this would be false, and without confinement, irrelevant.
- Highly nontrivial proof does exist in this special case; other cases lack proof.



Parton distribution functions



Parton-Parton Luminosities ; the horizontal axis is the parton-parton center-of-mass energy “M”, or sqrt(s-hat). Note gg crosses q-qbar at 2 TeV, and that qg is always large.

1

Parton Distribution Functions and Parton Luminosities

Figures 2a and 2b show some parton distribution functions (pdfs) and parton luminosities. [Caution: don't get too attached to these until we've reached the end of the course.]

Observe in figure 2a (where $f_u(x)$ is called $u(x)$, etc.)

- gluons dominate at $x < .1$
- u, d quarks dominate at $x > .1$
- $u \sim 2d \gg \bar{u}$ for $x \sim .3$.
- $u \sim d \sim \bar{u}$ for $x \ll .1$

Note: E_b= 7 TeV $\rightarrow x = .1$ at $E = 700$ GeV, .01 at $E = 70$ GeV.

Figures 3-10 show more of the pdfs, *sometimes weighted by x or x^2 to make the graphs readable – look carefully at the captions*

- $\bar{d}, \bar{u}, \bar{s}$ all pretty similar at all x.
- $s = \bar{s}, c = \bar{c}$ to good approximation at all x.
- $\bar{q} < g$ at all x.
- $u, d, s, c, \bar{u}, \bar{d}, \bar{s}, \bar{c}$ all pretty similar at very small x.

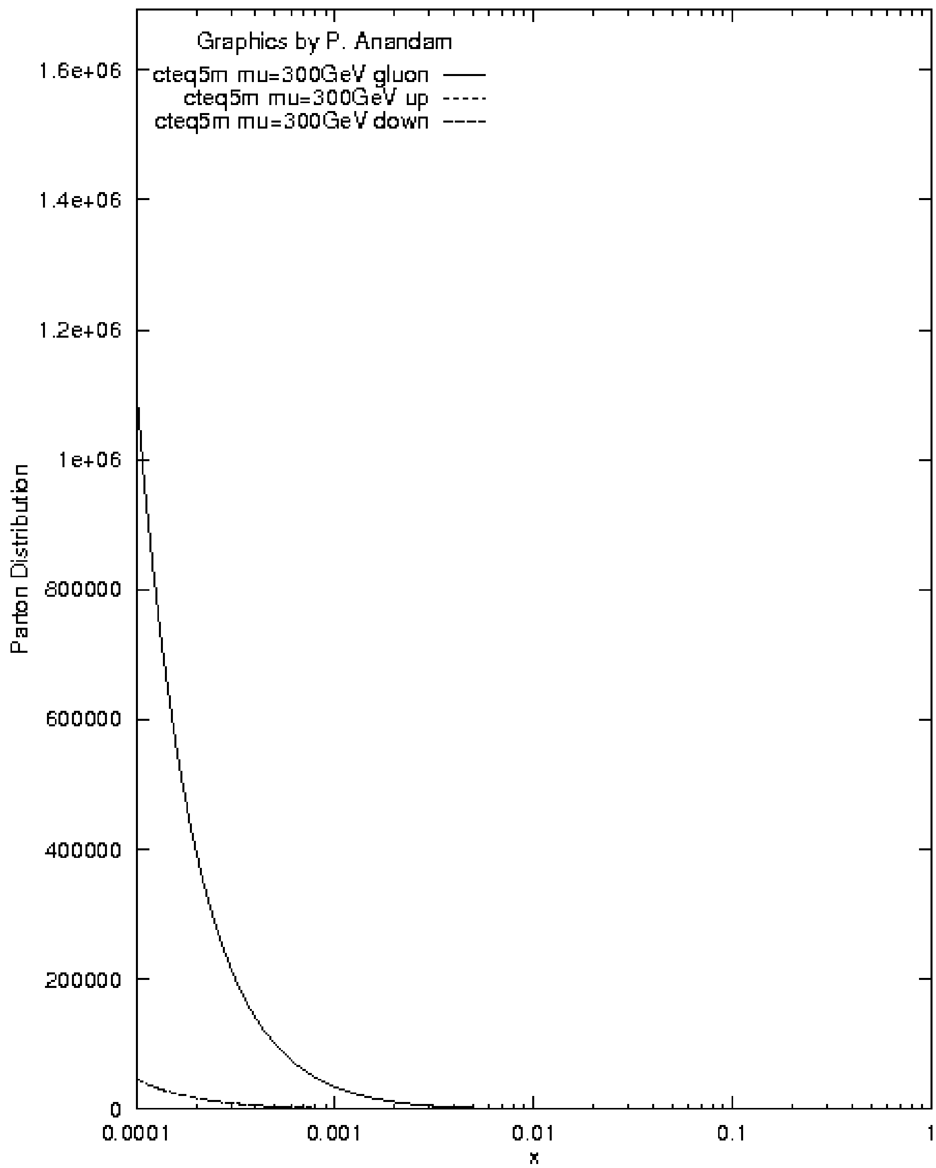
Interpretation: The proton has

- valence quarks $u_v = u - \bar{u}$, $d_v = d - \bar{d}$.
- $u_v, d_v \rightarrow 0$ as $x \rightarrow 0$.
- most nonvalence partons are gluons, which dominate at small x and do not go to zero as $x \rightarrow 0$.
- sea of quark-antiquark pairs $\bar{u}, u_s = \bar{u}, \bar{d}, d_s = \bar{d}$, etc.,

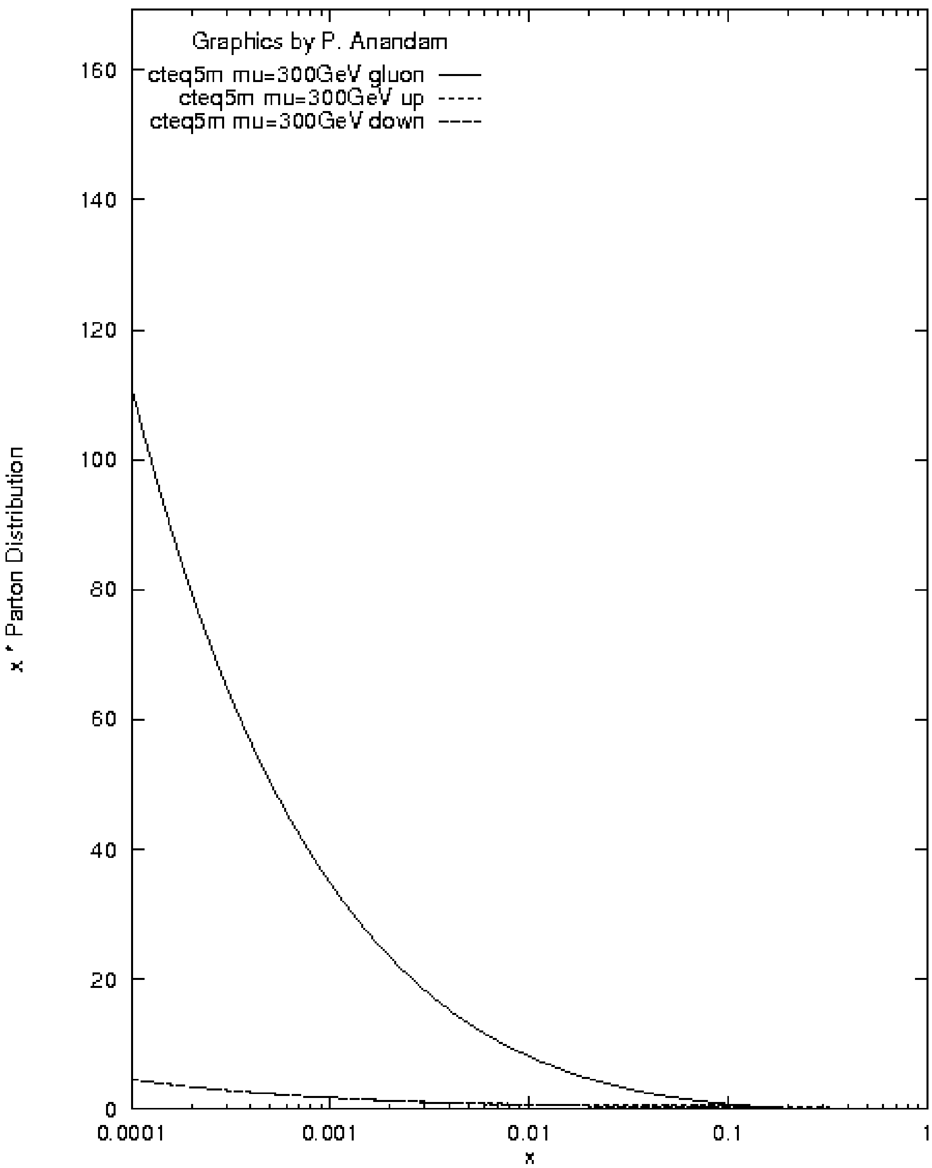
Furthermore

- Dominantly the gluons are emitted from the valence quarks or from each other, grow fast at small x.
- The sea quarks and antiquarks are emitted by the gluons, grow with the gluons at small x but can never catch up.

Notice for $x > .01$, f_g always falls much faster than $1/x$, $f_{u,d}$ falls just a bit faster, until $x \sim .3$ when both drop off faster.



Graphics by P. Anandam
cteq5m mu=300GeV gluon
cteq5m mu=300GeV up
cteq5m mu=300GeV down
1.6e+06
1.4e+06
1.2e+06
1e+06
800000
600000
400000
200000
0
Parton Distribution
0.0001
0.001
0.01
0.1
1
x


Graphics by P. Anandam
cteq5m mu=300GeV gluon
cteq5m mu=300GeV up
cteq5m mu=300GeV down
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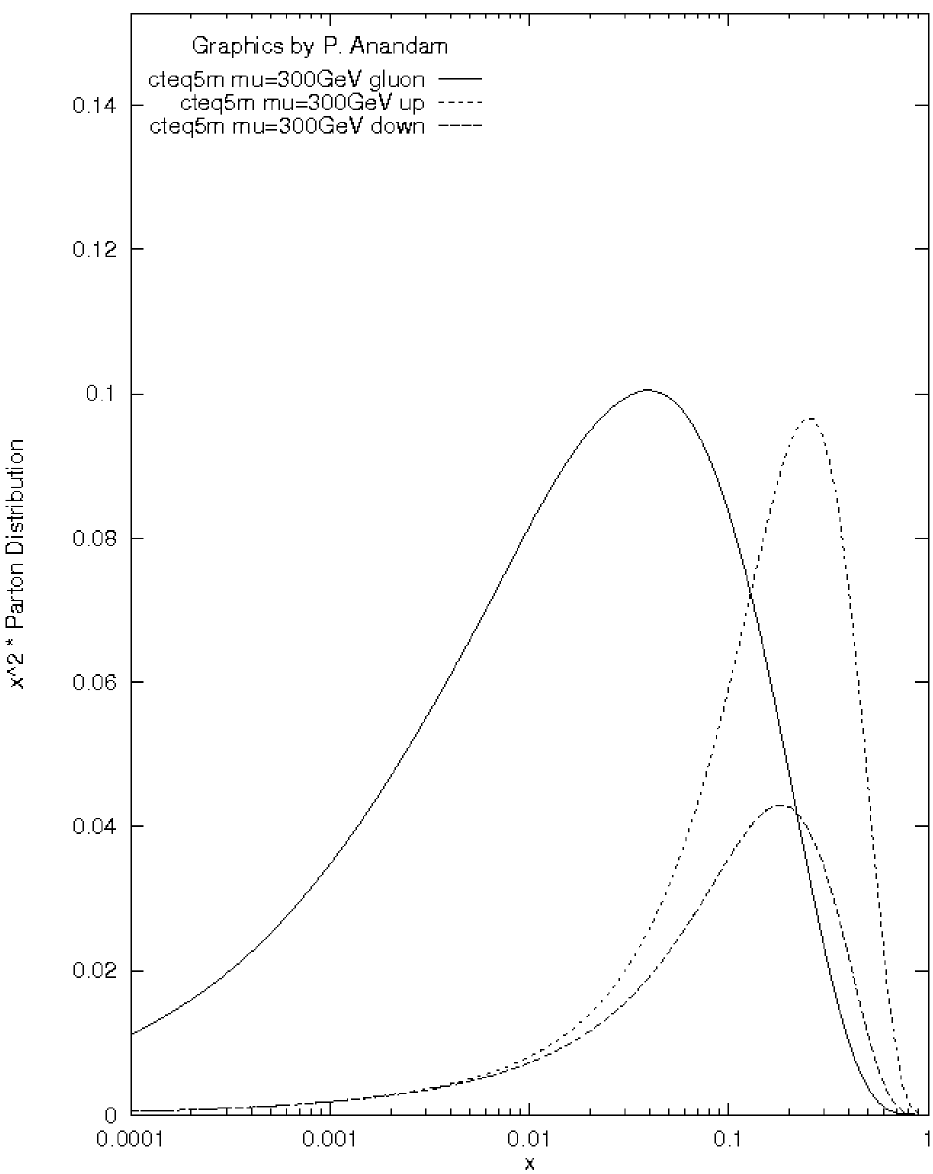
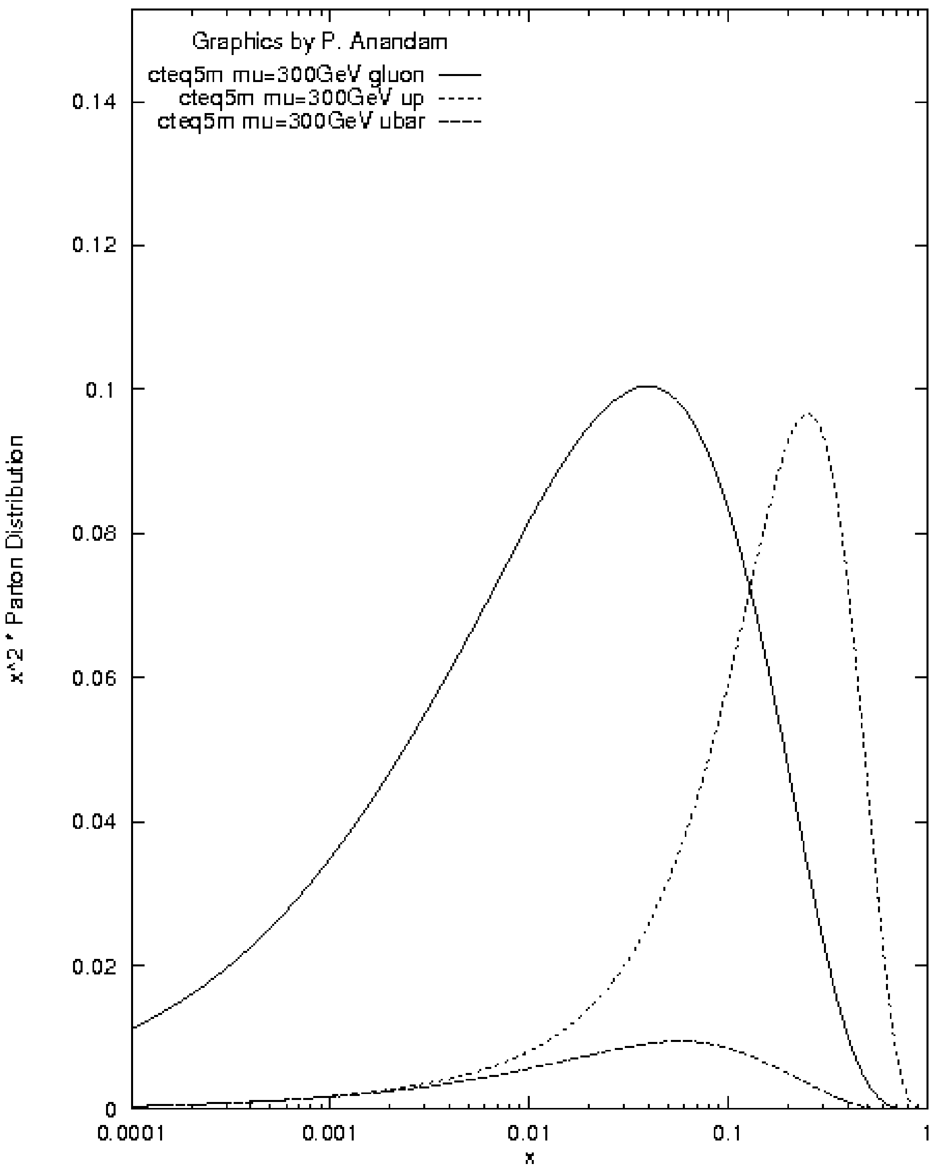
140
120
100
80
60
40
20
0
0.0001
0.001
0.01
0.1
1
x
x * Parton Distribution


Graphics by P. Anandam
cteq5m mu=300GeV gluon
cteq5m mu=300GeV up
cteq5m mu=300GeV down
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.0001
0.001
0.01
0.1
1
x
x^2 * Parton Distribution


Graphics by P. Anandam
cteq5m mu=300GeV gluon
cteq5m mu=300GeV up
cteq5m mu=300GeV ubar
x^2 * Parton Distribution
0.14
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0.08
0.06
0.04
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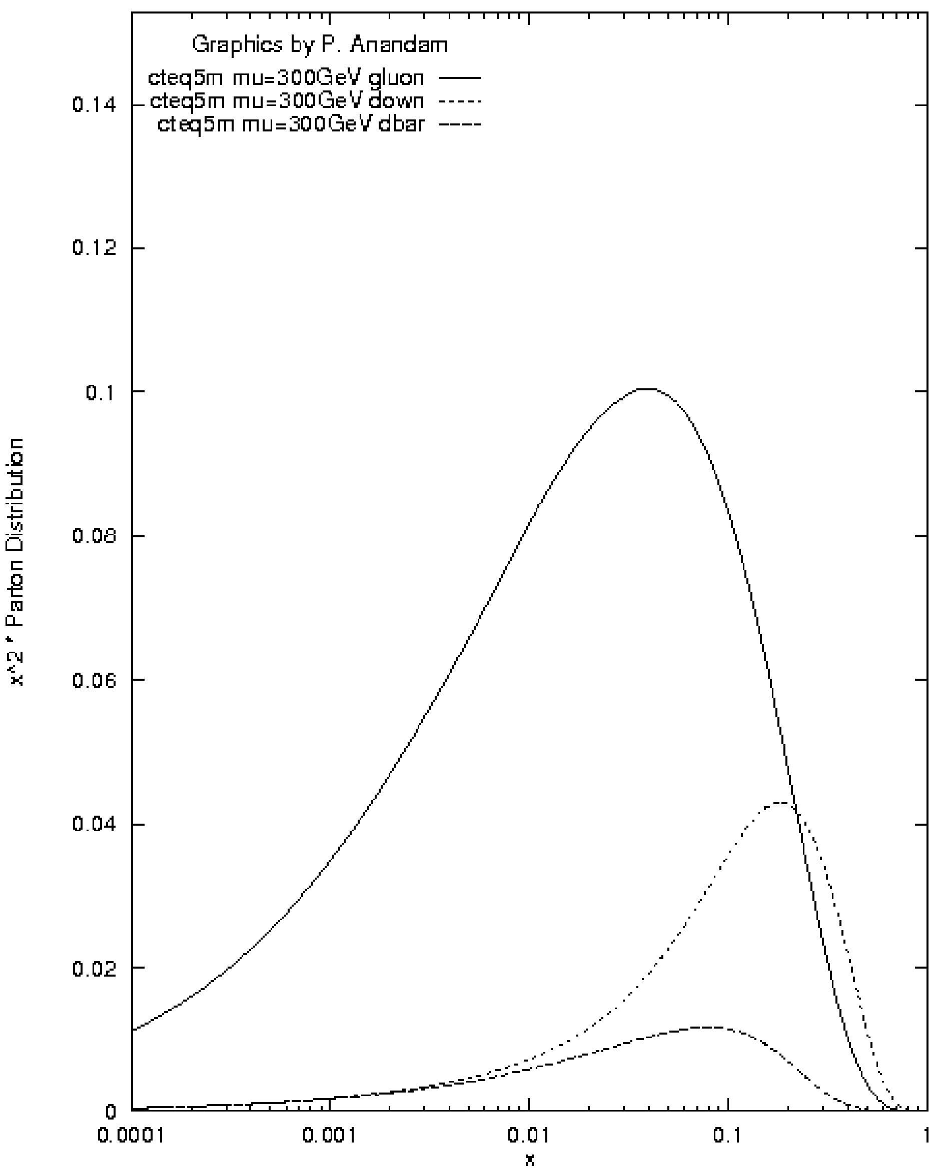
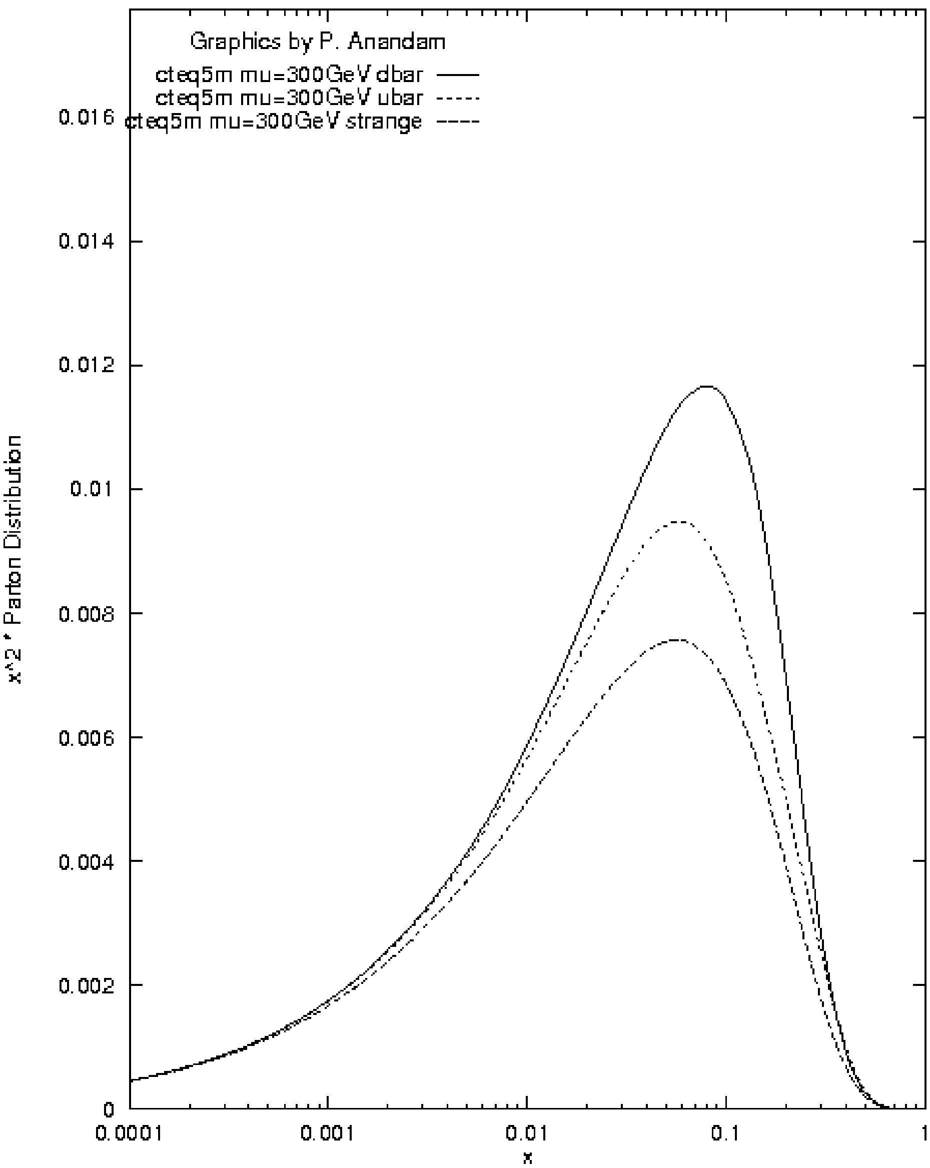
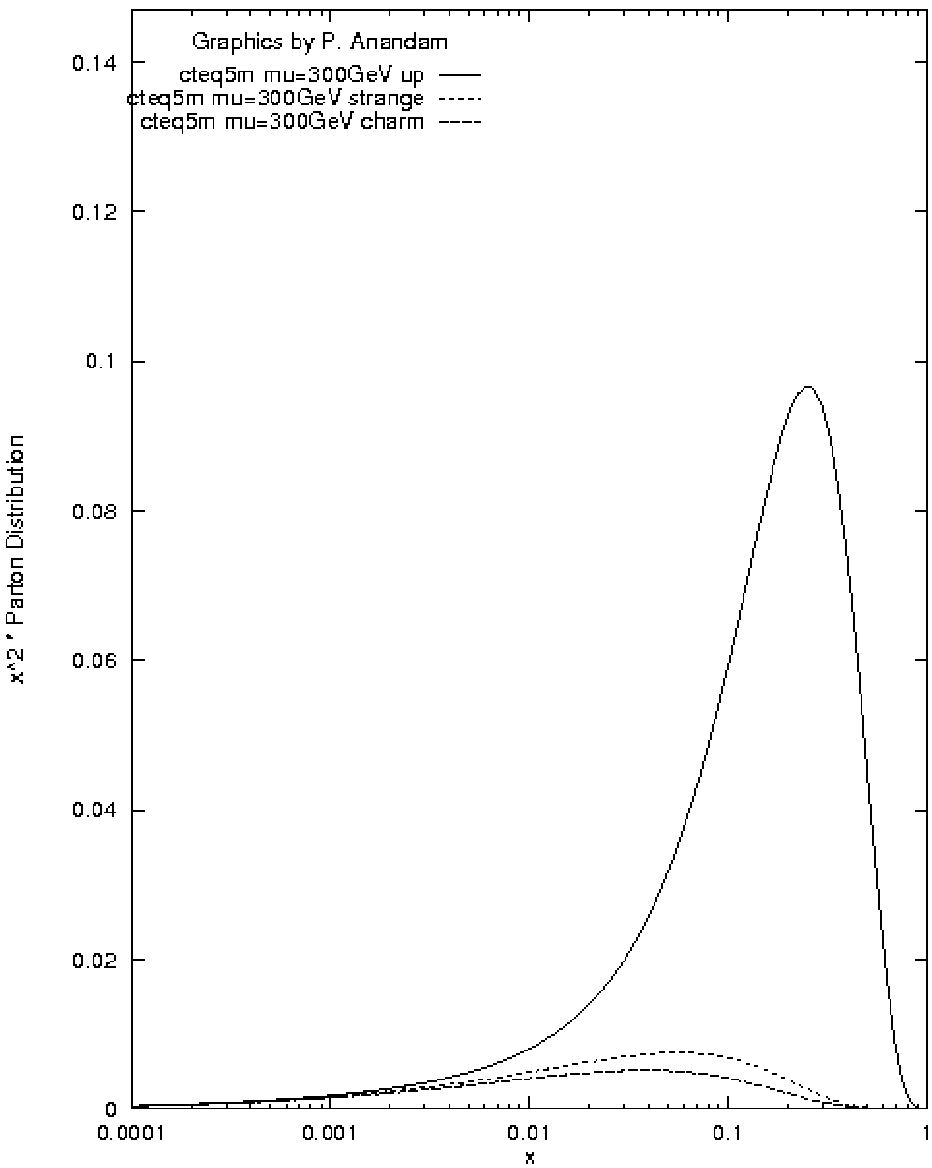
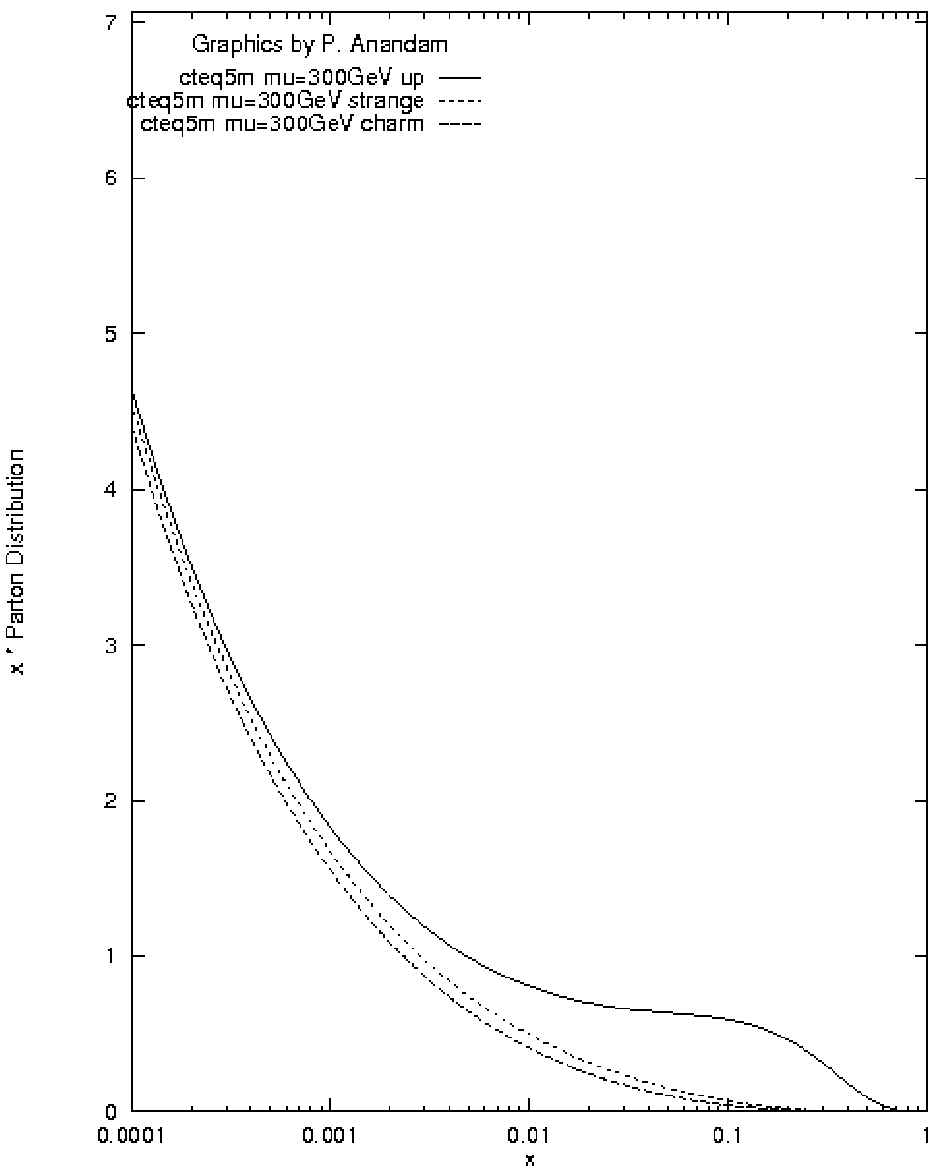
0.0001
0.001
0.01
0.1
1
x


Graphics by P. Anandam
cteq5m mu=300GeV gluon
cteq5m mu=300GeV down
cteq5m mu=300GeV dbar
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.0001
0.001
0.01
0.1
1
x
x^2 * Parton Distribution



Graphics by P. Anandam
cteq5m mu=300GeV dbar
cteq5m mu=300GeV ubar
cteq5m mu=300GeV strange
x^2 * Parton Distribution
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0.0001
0.001
0.01
0.1
1
x


Graphics by P. Anandam
cteq5m mu=300GeV up
cteq5m mu=300GeV strange
cteq5m mu=300GeV charm
x^2 * Parton Distribution
x
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.0001
0.001
0.01
0.1
1


Graphics by P. Anandam
cteq5m mu=300GeV up
cteq5m mu=300GeV strange
cteq5m mu=300GeV charm
x * Parton Distribution
x
0.0001
0.001
0.01
0.1
1
0
1
2
3
4
5
6
7

What is a parton luminosity?

We saw

$$d\sigma_{pp\to W^+ + \cdots \to e^+\nu_e}(P_1,P_2,p_e,p_\nu) = \int_0^1 dx_1 \int_0^1 dx_2\ f_u(x_1) f_{\bar d}(x_2)\ d\hat\sigma_{u\bar d\to W\to e\nu}(x_1P_1, x_2P_2, p_e, p_\nu)$$

plus some similar terms with other subscripts on f_i.

Note that $d\hat\sigma$ depends on $\hat s, \hat t, \hat u$ but not on $\hat y$! (That is, the differential cross-section depends on the Lorentz-invariant kinematics of the scattering partons, but not on how fast the p.c.o.m. frame is moving in the lab frame.) So rewrite integral in terms of $\hat s$ and $\hat y$.

$$\int dx_1 \int dx_2 = \frac{1}{s}\int d\hat s \int d\hat y$$

and thus

$$d\sigma_{pp\to W^+ + \cdots \to e^+\nu_e}(P_1,P_2,p_e,p_\nu) = \int_0^s d(\hat s/s) d\hat\sigma_{u\bar d\to W\to e\nu}(\hat s,\hat t,\hat u) L_{u\bar d}(\hat s/s)$$

where

$$L_{u\bar d} = \int_{-\infty}^{\infty} d\hat y\ f_u\left(\sqrt{\frac{\hat s}{s}} e^{\hat y}\right) f_{\bar d}\left(\sqrt{\frac{\hat s}{s}} e^{-\hat y}\right)$$

is the $u\bar d$ parton luminosity, which

- depends only on $\hat s$ and
- tells us probability to find left-moving u and right-moving $\bar d$ with particular fixed $\hat s$

(Since we could take a u from the right-moving proton, the total $u\bar d$ parton luminosity is typically defined with a factor of 2, but this is conventional.)

EXERCISE: Derive the last three equations.

In figure 2b: parton luminosities, summed over quark flavor. Note

- qg is as big as gg for $\sqrt{\hat s} > 200$ GeV
- qg is 10 times larger than gg for $\sqrt{\hat s} \sim 2$ TeV.
- qg is always of order 10 times larger than $q\bar q$.
- $q\bar q$ and gg are equal for $\sqrt{\hat s} > 2$ TeV.
- Not shown: $ug \sim 2dg$ for moderate to large $\sqrt{\hat s}$.

You may have heard it said that "the LHC is a gluon-gluon collider". *False!* The $q\bar q$ and qg initial states are crucial to understanding the phenomenology.

From Figure 3 we see

- $qg, q\bar q$ parton lum. fall a bit slower than $\sqrt{\hat s}^{-4}$
- gg parton lum. falls $\sqrt{\hat s}^{-3}$–$\sqrt{\hat s}^{-5}$

Let's return to figure 1.

- Total cross section dominated by elastic scattering and by glancing blows that create additional hadrons.
  - These events typically only create hadrons with $p_T < 1$ GeV.
  - If we choose bunch crossings at random, and look at the ones where we see a little bit of energy in the detector, we are imposing the minimum amount of bias from the trigger... so these are called "min. bias" events.
- With some small probability, two partons hit a little more strongly and scatter with $p_T > 1$ GeV. Once p_T is large enough (strictly, once $\hat{s}, |\hat{t}|, |\hat{u}| > 1$ GeV^2) then perturbation theory applies.
- Most likely: parton i scatters off parton j in t channel... long-distance force, classical scattering, small angle.
- Next: partons i and j annihilate in s channel and make partons i' and j', quantum scattering.
  - $gg \to b\bar{b} \quad \Rightarrow \quad \sigma_{pp\to b\bar{b}} \sim 300\ \mu$b.
- Probability to make partons with given p_T in any given process drops roughly like p_T^{-6} or more
  - $L_{ij} \sim (\sqrt{\hat{s}})^{-3}$–$(\sqrt{\hat{s}})^{-4}$
  - $d\sigma(\hat{s}) \sim 1/\hat{s}$
  - so everything is scaling like $(\hat{s})^{-5}$ or faster
  - typical p_T in high energy process often scales with $\sqrt{\hat{s}}$.
- To make W, Z need
  - Weak rather than strong interaction
  - Antiquark (rare)
  - Need accidentally-color-neutral initial state
- To make $t\bar{t}$ (mostly $gg \to t\bar{t}$) need
  - Strong interaction
  - Gluons only (not rare!)
  - this partially compensates for the larger mass but still $t\bar{t}$ 100 times smaller than W.
- Note SUSY lines extrapolated would fall a bit above $t\bar{t}$ due to production of squarks *and* gluinos.
- Higgs etc. later.

Caution: often we cannot detect most of the events corresponding to a single process, so the actual number of usable events may be much smaller than shown.

QCD FACT 3: **Light Hadrons — QCD has pions and Kaons which are lighter than the confinement scale, and all other mesons decay down to them.**

- $m_\pi < \Lambda$, $m_K \sim \Lambda$ because of "chiral symmetry breaking"; 3 light quarks and antiquarks $\Rightarrow$ $SU(3) \times SU(3)$ global symmetry, explicitly broken (by m_q) and spontaneously broken (by $\langle q\bar{q}\rangle \sim \Lambda^3$). PNG Bosons: $m_\pi \sim \sqrt{m_q \Lambda}$.
- If $N_f^{light} = 0$ there are no global symmetries and no light pions; very different spectrum, many stable hadrons ("glueballs", "Pure Yang-Mills Theory")
- For $N_f^{light} = 1$ and N_c small, similar to $N_f = 0$.

EXERCISE: Consider a QCD-like theory with $N_c = 3$ and $N_f^{light} = 2$ **massless** *quarks. What is the spectrum of the pion-like states and the nucleon-like states? How does it change if $N_c = 4$? How does it change if $N_f^{light} = 4$?*

Hadrons

Though in principle there could be others, in practice there are three types of bound states of quarks, antiquarks and gluons observed

- Mesons (quantum numbers of a quark and an antiquark)
- Baryons (quantum numbers of three quarks)
- Antibaryons

Baryon number conserved (to an excellent approximation), so lightest baryon/antibaryon stable.

Among these, from lowest mass to highest mass, are

- metastable hadrons: decay via electroweak interactions, lifetimes $\tau \gg \Lambda^{-1}$
- unstable hadrons (resonances), decaying via strong interactions to other hadrons, with lifetimes τ somewhat less than Λ^{-1}
- highly unstable states that can barely be defined, with $\tau \sim m^{-1}$.

The number of hadrons is infinite; but most are in the last category.

Other types of hadrons (such as "glueballs" and other "exotics" are presumably all in the last category.

We may divide them differently into those which, when boosted by a factor of 10-100, travel a distance

- $\gamma c\tau < 100\ \mu$m ("prompt"); not observed, only inferred
- $\gamma c\tau \gg 1$m (effectively "stable"); traverse the detector
- In between, certain hadrons may decay at a detectably displaced vertex.

Note that a π_0 is

- long-lived (metastable) from the point of view of the strong interactions
- short-lived (prompt) from the point of view of the detector

Some Experimentally Important Hadrons of QCD Source: pdg.lbl.gov

Not an exhaustive list! [A well-educated theorist can explain or estimate every entry]

Name	Mass(MeV)	Lifetime (sec)	Dominant Decay	"Flavor"
π^+	140	3×10^{-8}	$\mu^+\nu_\mu$	$u\bar{d}$
π^0	135	8×10^{-17}	$\gamma\gamma$	$u\bar{u}, d\bar{d}$
K^+	494	1×10^{-8}	$\mu^+\nu_\mu, \pi^+\pi^0$	$u\bar{s}$
K^0_S	498	9×10^{-11}	$\pi^+\pi^-, \pi^0\pi^0$	$d\bar{s}, s\bar{d}$
K^0_L	498	5×10^{-8}	$\pi\pi\pi, \pi\ell\nu$	$d\bar{s}, s\bar{d}$
η	548	6×10^{-19}	$\gamma\gamma$,$3\pi^0$	$u\bar{u}, d\bar{d}, s\bar{s}$
ρ^+	770	4×10^{-24}	$\pi^+\pi^0$	$u\bar{d}$
ρ^0	770	4×10^{-24}	$\pi^+\pi^-$	$u\bar{u}, d\bar{d}$
ω	782	8×10^{-23}	$\pi^+\pi^-\pi^0$	$u\bar{u}, d\bar{d}$
K^{*+}	892	1×10^{-23}	$K^+\pi^0, K^0\pi^+$	$u\bar{s}$
K^{*0}	896	1×10^{-23}	$K^+\pi^-, K^0\pi^0$	$d\bar{s}$
η'	958	3×10^{-21}	$\pi^+\pi^-\eta, \ldots$	$u\bar{u}, d\bar{d}, s\bar{s}$
p	938	$> 10^{42}$	–	uud
n	940	887	$pe^-\bar{\nu}$	udd
ϕ^0	1020	1×10^{-22}	$K^+K^-, K^0_LK^0_S$	$s\bar{s}$
Λ	1115	2×10^{-10}	$p\pi^-, n\pi^0$	uds
Σ^+	1189	8×10^{-11}	$p\pi^0, n\pi^+$	uus
Σ^0	1193	7×10^{-20}	$\Lambda\gamma$	uds
Ξ^0	1314	3×10^{-10}	$\Lambda\pi^0$	uss
Ξ^-	1321	2×10^{-10}	$\Lambda\pi^-$	dss
Ω^-	1672	8×10^{-11}	$\Lambda K^-, \Xi^0\pi^-$	sss
D^+	1869	1×10^{-12}	$K+\cdots$	$c\bar{d}$
D^0	1864	4×10^{-13}	$K+\cdots$	$c\bar{u}$
B^+	5279	2×10^{-12}	$D+\cdots$	$b\bar{u}$
B^0	5279	2×10^{-12}	$D+\cdots$	$b\bar{d}$

Note also that

- μ has mass 105 MeV, lifetime 2×10^{-6} sec, decays to $e\nu\bar{\nu}$
- τ has mass 1777 MeV, lifetime 3×10^{-13} sec, decays to
  - 17%: $e^-\nu\bar{\nu}$
  - 17%: $\mu^-\nu\bar{\nu}$
  - 50%: 1 charged hadron plus ≥ 0 neutral hadrons
  - 15%: 3 charged hadrons plus ≥ 0 neutral hadrons

Lifetime from detector perspective

Recall 10^{-12} sec $\times\, c \sim 0.3$ mm; time dilation effects important.
Beampipe radius ~ 3 cm.

Prompt	Displaced (<1 cm)	Displaced (>1 cm)	Typically Stable
$\pi_0, \eta, \eta', \rho, \omega, \phi, K^*$		K_S	$\pi^\pm, K^\pm, K_L^0$
Σ^0		$\Sigma^+, \Lambda, \Xi, \Omega$	p, n
B^*, D^*	$B, D, \Lambda_c, \Lambda_b$	very high-p_T B	
τ	very high-p_T τ		μ, e, γ

When charges are not given all particles with this name are included.

Caution: lifetimes are averages; boosts vary, and decays are stochastic

- commonly a particle will decay earlier
- rarely a particle (especially a high-energy one) will decay late

EXERCISE: Consider π^+ particles with $p_T = 140$ GeV and $\eta = 0$. [I do mean GeV, not MeV.] What fraction of them will decay at a distance of less than 1 meter?

EXERCISE: Consider the B^0 meson. What is its average travel distance if its $p_T = 10$ GeV? At what energy do typical B^0 mesons traveling perpendicular to the LHC beam exit the LHC beampipe before decaying?

Detection Method

Name	Track	ECAL	HCAL	μ system
γ	N	Y	N	N
$e^\pm$	Y	Y	N	N
$\mu^\pm$	Y	N	N	Y
$\pi^\pm, K^\pm, p, \bar{p}$	Y	(Y)	Y	N
$K^0, n, \bar{n}$	N	N	Y	N
$\pi_0 \to \gamma\gamma, \eta \to \gamma\gamma$	N	YY	N	N

We'll discuss τ, D, B below

Generally at the CMS and ATLAS, stable hadrons of same charge are not distinguished. *(This is not true at LHCb and ALICE)*

Now, why are the pions so light?

So light, in fact, that essentially everything in QCD (except protons and neutrons) decays to pions eventually...

Because they are psuedo-Nambu-Goldstone bosons [PNGBs!]

The equations are almost the same as for the Higgs boson.

In a world with massless quarks, one would simply replace with

	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	**1**	**2**	$\frac{1}{2}$

$$V(H^\dagger H) = -\mu^2 H^\dagger H + \tfrac{1}{2}\lambda(H^\dagger H)^2$$

	$SU(2)_L$	$SU(2)_R$
$\Sigma = \begin{pmatrix} (\bar{u}u) & (\bar{u}d) \\ (\bar{d}u) & (\bar{d}d) \end{pmatrix}$	**2**	**2**

$$V_0(\Sigma^\dagger\Sigma) = -A\mathrm{tr}\left(\Sigma^\dagger\Sigma\right) + B\mathrm{tr}\left(\Sigma^\dagger\Sigma\right)^2$$

The details of the potential, which is generated quantum mechanically by the strong QCD interactions, don't matter.

The point is that $\langle\Sigma\rangle = \chi\mathbf{1}$ in the QCD vacuum, breaking $SU(2)_L \times SU(2)_R \to SU(2)_V$ (V for vector, or D for diagonal).

The three broken generators lead to NGBs, which are massless pions.

But in the real world, masses for $\bar{u}u$ and $\bar{d}d$ break $SU(2) \times SU(2)$ explicitly down to up-number and down-number.

$$V = V_0 + m_u\bar{u}u + m_d\bar{d}d + c.c. = V_0 + \frac{1}{2}\mathrm{tr}\left(\mathcal{M}\Sigma^\dagger\right) + \text{c.c.}$$

where $\mathcal{M}$ is a diagonal matrix with nonzero entries m_u, m_d.

Famously, this means the NGBs become PNGBs, with $m_\pi^2 \propto m_u + m_d$ times a factor of order a few hundred MeV. *(Why is the $\pi^\pm$ a bit heavier?)*

The generalization of this story to $SU(3) \times SU(3)$ explains why the K's and η are light also.

Now why do we care about these hadrons?

They are all commonly seen at hadron colliders.

How are they produced in a proton-proton collision?

- In the underlying event, the protons fracture into many pieces, each of which forms into a hadron.
- When a quark, antiquark or gluon is kicked out of the proton, it creates a "jet" of (generally excited) hadrons, more or less collimated in angle, which again decay to metastable hadrons.

Note that most of these hadrons are in excited states and decay down via the strong interactions to multiple metastable hadrons.

Why do jets form? Naively

- Partons are confined (or "cloaked") so at least one hadron should emerge when a parton is kicked out of proton.
- Lots of energy $\Rightarrow$ no cost to make lots of hadrons.

But this does not explain why $p_{parton} \sim p_{jet}$, or why the hadrons emerge in a jet at all.

We will return to this later.

Consider $u\bar{d} \to W \to u\bar{s}$.

Figure 16a: from $Z \to q\bar{q}$, jets with energy ~ 45 GeV from colorless source

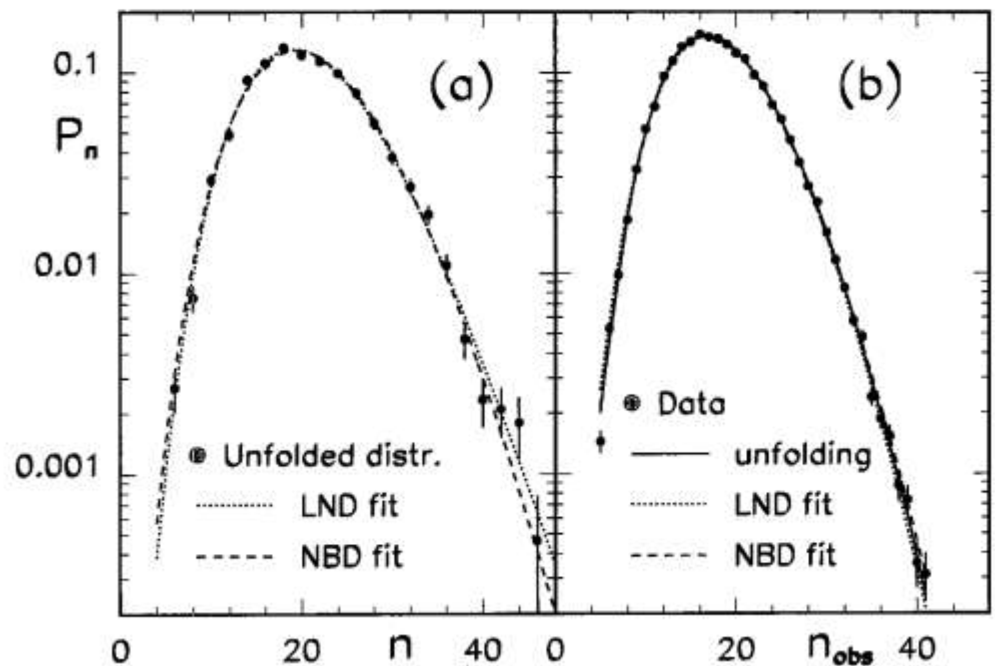
- have ~ 10 tracks (up to 20) $\Rightarrow \sim 15$ stable hadrons, mostly $\pi^{\pm}$.
- have large multiplicity fluctuations.

Figure 16b: Track multiplicity is gradually growing.

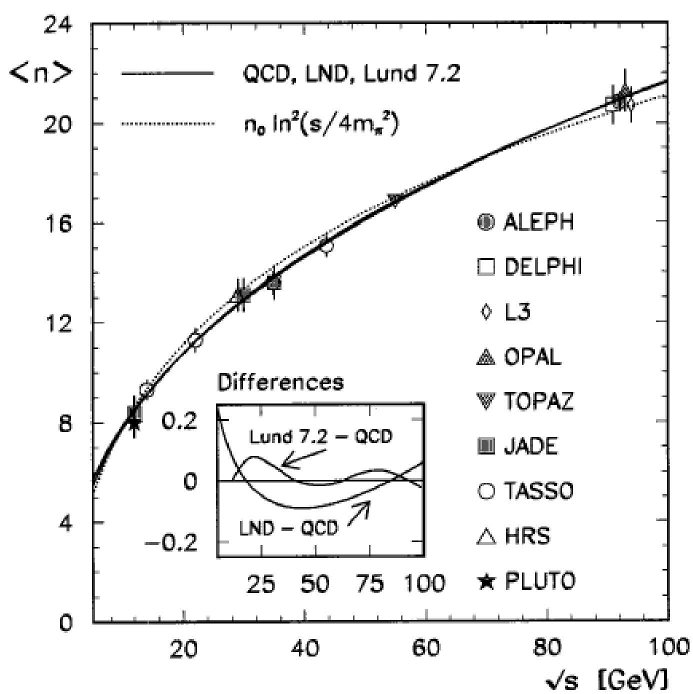
Many tracks are at low energy, or nearly overlap
$\Rightarrow$ easy to miss one.
$\Rightarrow$ total charge of g jet may be nonzero.
Cannot distinguish u, d, g jets by total hadron charge — the correlation is very weak.

So $u\bar{d} \to W \to u\bar{s}$ has background from $gg \to gg$ — Huge! Not likely to be observed.

g jets do have higher multiplicity on average (will see later)
but fluctuations around the average are large, so doesn't help much.



For 45 GeV jets in e+e- collisions, the probability for jets to have n charged particles. Ignore left-hand plot, it is prior to an experimental correction. Note about 2 percent of the jets have less than 8 charged particles; extrapolating down to 1 charged particle is delicate but of order 10^{-4}.



Average number of charged particles in e+e- → dijet events (the jets have energy equal to ½ of what is plotted on the horizontal axis.)

Lecture 3

QCD FACT 4: "Parton-Hadron duality":
Partons turn into relatively narrow jets of hadrons with $p^\mu_{jet} \sim p^\mu_{parton}$

This need not happen.

- Jet formation is a function of α_s.
  - Width of jet related to $\alpha_s N$ (*QCD jets are observed to be broad at small* μ*, narrow at large* μ*)*
  - If $\alpha_s N_c$ does not become UV-small, jets are too broad to observe as such.
- Hadronization process occurs with rate $\propto N_f^{light}/N_c$, or, if $N_f^{light} = 0$, $1/N_c^2$.
  - If this time is slow then momentum will be redistributed and $p_{jet} \neq p_{parton}$; jets likely broader.
  - If slow enough, jetty structure may be lost altogether through violent rearrangement of energy

Easy to understand N_f^{light} importance:

Quark-antiquark pair production rate

- scales with number of quarks of given mass
- is fast for $m_q \ll \Lambda$, since energy-density in confined chromo-electric field $\sim \Lambda^4$ *(dimensional analysis)*
- is exponentially suppressed for $m_q \gg \Lambda$ *(energy density too low for pair production)*

More subtle is $1/N_c$ dependence.

Heuristic argument: at strong coupling $\Rightarrow \alpha_s N_c \sim 1$.

Think of flux tube as chain of gluons with color and anticolor $c_i, \bar{c}_{i-1}$ connecting a quark of color c_1 and anticolor $\bar{c}_n$

$$(\bar{c}_n)(c_n\bar{c}_{n-1})\dots(c_4\bar{c}_3)(c_3\bar{c}_2)(c_2\bar{c}_1)(c_1)$$

I can always add a gluon at no cost:

$$(c_3\bar{c}_2) \to \sum_{c_0}(c_3\bar{c}_0)(c_0\bar{c}_2)$$

costs

$$\alpha_s \sum_{c_0} = \alpha_s N_c \sim 1$$

To fragment flux tube, split $c_3\bar{c}_2$ gluon: $g \to d\bar{d}$, where d has color c_3, $\bar{d}$ anticolor $\bar{c}_2$.

$$(\bar{c}_n)(c_n\bar{c}_{n-1})\dots(c_4\bar{c}_3)(c_3) \qquad (\bar{c}_2)(c_2\bar{c}_1)(c_1)$$

Now the flux tube is broken in two. The cost is

$$\sim \alpha_s \sim \frac{\alpha_s N_c}{N_c} \sim \frac{1}{N_c}$$

But I can do this with u or s quarks — so the rate for splitting by $g \to q\bar{q}$ goes as N_f^{light}/N_c.

If no light quarks, then to break the flux tube requires a non-planar graph. Non-planar graphs are suppressed by $1/N_c^2$... so hadronization is much slower in this case.

For example, this can happen through the exchange of a $(c_3\bar{c}_1)$ gluon between the first and third gluon from the right, leaving

$$(\bar{c}_n)(c_n\bar{c}_{n-1})\dots(c_4\bar{c}_1)(c_3\bar{c}_2)(c_2\bar{c}_3)(c_1) \to (\bar{c}_n)(c_n\bar{c}_{n-1})\dots(c_4\bar{c}_1)(c_1) \qquad (c_3\bar{c}_2)(c_2\bar{c}_3)$$

A closed loop has broken off from the flux tube. This required two vertices, but the color and anticolor of the gluon had to be precisely chosen, so the cost of this process is

$$\alpha_s^2 = \frac{(\alpha_s N_c)^2}{N_c^2} \sim \frac{1}{N_c^2}$$

Note the very big difference, for large N_c, between $N_f^{light} = 1$ and $N_f^{light} = 0$.

Now that we have some reason to believe that confinement in QCD may not badly impact jets, we will trust a perturbative treatment of jet formation... later.