The simplest observations

How do we know we made a W boson?

Easier: How do we know we made a Z boson? Let's consider $pp \to Z + \cdots \to e^+e^- \ldots$.

Trigger on events with 2 "electrons", require two isolated moderate-p_T tracks with

$$m_{e^+e^-} = \sqrt{(p_+ + p_-)^2} \approx m_Z$$

"≈" because $\Gamma_Z \approx 2.5$ GeV.

$m_{e^+e^-}$ is a strict Lorentz-invariant quantity ⇒ nice peak no matter what $p_{Z,T}$ is.

More general e^+e^- events from γ^*, Z^* *these interfere!*

(* means "generally off-shell")

Look for di-electron events and plot invariant mass: peak at m_Z and at zero.

Peak at 0 impacted by fakes at low p_T, trigger which requires moderate p_T, cuts;

but can be explored when both e^+ and e^- have high p_T and small $m_{e^+e^-}$.

Again: How do we know we made a W boson?

Look for event with $e^\pm$ or $\mu^\pm$ and nothing else.

Well, not nothing...

Expect lots of tracks (50-100?) and H_T (50-200 GeV?) from the underlying event.

- Trigger on event with electron with $p_T >?$ and $|\eta| <?$
- From these events, select events with high-p_T electron and no other high-p_T tracks
- Plot $p_{e,T}$

If W carries no p_T then $\vec{\not{p}}_T \approx -\vec{p}_{e,T}$.

If the W decayed isotropically (not quite true) expect simple $p_{e,T}$ distribution:

$$\frac{d\hat{\sigma}}{dp_T} = \frac{d\hat{\sigma}}{d\cos\theta_{pcom}} \frac{d\cos\theta_{pcom}}{dp_T} \propto \frac{d\cos\theta_{pcom}}{d\sin\theta_{pcom}} = \tan\theta_{pcom} = \frac{p_T}{\sqrt{m_W^2 - 4p_T^2}}$$

"Jacobian peak" at $p_T = m_W/2$

But the peak is somewhat smeared out.

- $p_{e,T}$ is not perfectly measured (small effect)
- $\Gamma_W \sim 2$ GeV

- q or $\bar{q}$ radiates low-p_T ("soft") gluon ("initial state radiation" ["ISR"]) $\Rightarrow p_{W,T} \sim 10$ GeV.

A better variable is "Transverse Mass"

Think two-dimensionally: define $\tilde{p}_e = (p_{e,T}, \vec{p}_{e,T})$, etc.
($\tilde{p}_e$ a 2+1 dimensional three-vector with momentum $\vec{p}_{e,T}$, massless: $\tilde{p}_e^2 = 0$ in 2+1)

$$m_T^2 = (\tilde{p}_e + \tilde{|\not{p}_T|})^2 = 2(|p_{e,T}||\not{\vec{p}}_T| - \vec{p}_{e,T} \cdot \not{\vec{p}}_T) < m_W^2$$

where $\phi_{e\nu} \equiv \phi_e - \phi_\nu$.

- m_T manifestly invariant under z-boosts, so go to frame where $y_W = 0$.
- Since a 2+1-dimensional Lorentz-invariant, invariant (at first order) to sideways boosts.
- Thus if the W gets a kick sideways, $m_T \rightarrow m_T - \text{order}(v^2)$

Note insensitivity to sideways kick, and negative shift in m_T, implies the edge in the plot is more stable against nonzero $p_{W,T}$ *and* theoretical error!

Experimentally, cut away events with $p_T <?$ and $|\eta| >?$ to reduce the effect of the trigger, tracking inefficiencies, fakes, etc.

Data agrees with theory, but ... not so easy .... let's see why.

Let's ask some simple-looking questions about W production:

- How many SM $W \rightarrow e\nu$ events will be produced at LHC?
- What will be the p_T distribution of the electrons from SM W production at the LHC?
- How many of these events will be **observed** at LHC?

These questions are in increasing level of difficulty.

We can almost answer the first, though it requires calculating corrections to the tree-level matrix element *(which are large! $gq \rightarrow Wq$ is comparable to $q\bar{q} \rightarrow W$; why?!)*

The second requires us to combine the W production process with the decay distributions. The W has spin so the e^- distribution is not isotropic in the W restframe. And the W is not at rest in the lab frame.

If the W has only p_Z and no p_T, this does not affect the p_T of the e^-. *Why?*

But when the W recoils against a jet (see above) it picks up p_T also, and this affects mainly the high-p_T tail of the e^- distribution.

The third is very difficult, because it requires knowing and combining

- the geometric structure of the detector (in particular the η range)
- the p_T cut on the electron required by detector and trigger effects, and
- the kinematic distribution in p_T, η of the e^- from W decay
- the probability that the electron is isolated

These calculations cannot be done analytically! Even with exact analytic expressions for the e^- distribution, the integrals over η, ϕ, x_1, x_2 would have to be done with a computer.

Therefore, for LHC physics, we must have a computer programs for dealing with the kinematical distributions.

"Monte Carlo event generators"

Rather than do the integrals directly, these "Monte Carlo" programs work by producing simulated events with correct probability weights, and thus do the integrals by sampling.

This is powerful and practical because it makes it possible to account for additional effects (cracks in the detector, inefficiency in detecting electrons, scattering of the electron off detector material) and to change the kinematic restrictions (such as the p_T cut) without redoing the integrals from scratch.

Not Optional! To do LHC physics you **must**, for practical reasons, use these programs.

Famous ones are Pythia, HERWIG, SHERPA, MadGraph, ALPGEN, CompHEP, MC@NLO, etc.; they are different, yet overlap.

Also these programs contain models of uncalculable or essentially uncalculable but experimentally relevant physical phenomena. We already need them for W/Z decay to leptons.

In particular, Pythia, HERWIG and SHERPA contain routines for producing a simulated version of the **Underlying Event** : the debris from the rest of the proton-proton collision which is experimentally important!

It produces dozens of low p_T tracks (pions mostly) and photons (mostly from π_0s).

These "soft" particles can affect the measured energies or general appearance of the "harder" leptons or jets we care about, and must be accounted for, at least statistically.

Also, these programs contain routines for estimating the likelihood of "**Initial State Radiation** ", a quark or gluon created in the collision prior to the formation of the W/Z.

Quarks or gluons, along with additional "**Final State Radiation** ", may also be produced in W and Z decays.

One of the most important things that Pythia, HERWIG and SHERPA do is contain a routine for **Jet Formation**: turning a quark or gluon into a jet, using the best available QCD theoretical techniques.

This requires three steps:

- Showering: semiclassical radiation of (mainly) gluons off the parent quark/gluon (jet formation)
- Hadronization: confinement of all the quarks and gluons in the shower into hadrons
- Decays: decay of all unstable hadrons to more stable hadrons or leptons

Most aspects of jets are created in the first stage, which is under very good theoretical control.

Pythia and HERWIG mainly contain routines for $2 \to 2$ scattering processes with $2 \to 1$ and a few $2 \to 3$ as well. Particle decays are simulated, with varying degrees of detail. Calculations are tree-level.

SHERPA, ALPGEN, MadGraph, CompHEP and others allow calculations of much more complex processes, including all tree-level Standard Model processes (in principle) and many new-physics processes (if the Feynman rules are entered by the user, now an automated procedure.)

1-loop-level event generation is coming!!! A huge advance in 1-loop calculations recently made; this will be fully automated in the next couple of years.

Comments on the third generation.

As the heaviest fermions, with Yukawa couplings ~ 1 (for t, and possibly for b and τ), the 3rd generation may have especially strong couplings to new phenomena. Not only the Higgs but many other new particles may decay mostly to this generation.

Quarks and charged leptons in the 2nd and 3rd generation decay by emitting a W boson.
The W is virtual, except for the top, which decays to a real W and a b quark.
For example, the tau may decay to a 3-body leptonic final state:

$$\tau^- \to \nu_\tau (W^-)^* \to \nu_\tau e^- \bar{\nu}_e$$

and the b quark can similarly decay to $c\mu^- \nu_\mu$.

In the standard model, all decays via Z boson or γ are absent at tree level and are generally small even when quantum effects are accounted for.

This absence of FCNC's (flavor changing neutral currents) in the SM and in the data is a very strong constraint on any type of new physics, which could give new sources of FCNCs.

Thus we do not observe $\tau^- \to \mu^- \gamma$ or $\tau \to e^- e^+ e^-$, and $b \to s\gamma$ and $b \to s\mu^+\mu^-$ are very rare (though observed.)

So almost all decays in the SM occur through flavor changing **charged** currents.

But even these are not simple. For example, we observe both $b \to c\mu\bar{\nu}$ and $b \to d\mu\bar{\nu}$. We also observe $s \to u$ and $c \to s$ and $c \to d$ transitions.

In fact in the SM all possible transitions via a W are allowed, with probabilities that are proportional to the squares of the entries in a **unitary** matrix, due to Cabibbo and to Kobayashi and Maskawa:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} .974 & .226 \pm .001 & .0036 \pm .0002 \\ .226 \pm .001 & .973 & .0415 \pm .0010 \\ .0087 \pm .0003 & .041 \pm .001 & .999 \end{pmatrix}$$

(There is a similar matrix in the lepton sector, but it has no experimental impact at the LHC, because ν_τ, ν_μ and ν_e are all equally invisible to the LHC detectors.)

Thus the branching fractions satisfy relations like

$$\frac{\mathrm{Br}(b \to u\mu\bar{\nu})}{\mathrm{Br}(b \to c\mu\bar{\nu})} = \frac{|V_{ub}|^2}{|V_{cb}|^2}$$

to a pretty good approximation. (Among other things, since $m_c \gg m_u$, the final-state kinematics are different in the numerator and demominator, so this relation isn't exact as written.)

We also have relations such as

$$\frac{\mathrm{Br}(b \to cd\bar{c})}{\mathrm{Br}(b \to cs\bar{c})} = \frac{|V_{cd}|^2}{|V_{cs}|^2}$$

to a moderately good approximation. (The more quarks appear in the decay, the more subtleties we have to account for from the fact that quarks are "confined" into hadrons.)

Because of the approximately diagonal structure of the CKM matrix, the most common quark-flavor decays are $t \to b$, $b \to c$, $c \to s$ and $s \to u$. We also have $d \to u$ in some cases, for example in neutron beta decay, but this is very slow.

From the decay diagram, it is clear that the decay of a particle decaying via a virtual W quark will be proportional to $|1/m_W^2|^2$. Then, by dimensional analysis, the width of quark q_i is proportional to

$$\Gamma \sim g_2^4 \frac{m_q^5}{m_W^4} |V_{ij}|^2$$

(times factors of order 2 and π etc., but we just need the overall behavior here) where V_{ij} is the largest CKM matrix element involving quarks q_j that are lighter than quark q_i (so that the decay is kinematically allowed.)

(For leptons the formula is the same, but all ν_i contribute equally so $|V_{ij}|^2$ is replaced by $\sum_i |V_{e\nu_i}|^2 = 1$.)
From this formula we learn

- Heavier particles have *much* shorter lifetimes than particles of the same charge.
  - $m_\tau \sim 10 m_\mu \Rightarrow \Gamma_\tau \sim 10^5 \Gamma_\mu$
- Since $|V_{su}|^2 \sim .1$ and $|V_{cb}|^2 \sim 10^{-5}$, the lifetime of strange and bottom quarks are anomalously long compared to the charm quark, which has $|V_{cs}|^2 \sim .5$.

All the quarks except the top quark live so long that complicated QCD interactions are important. In particular these particles always form bound states.

The top quark has a very short lifetime, with a width well over 1 GeV. Its lifetime is so short that it decays before strong QCD physics has a chance to play any role; it never has time to form a bound state, and its decays can be described using perturbation theory.

Taus and Tau Identification

Taus decay 34% to e or μ, 50% to one charged hadron (plus neutral hadrons), and 15% to 3 charged hadrons (plus neutral hadrons) –

– **always** with a tau neutrino.
Therefore E_τ can never be measured!

- Leptonic taus look just like e, μ – cannot identify.
- Hadronic taus look like special jets –
  - 1 or 3 tracks (typical jets $\sim 5-20$ tracks)
  - invariant mass of decay products $< m_\tau \sim 1.8$ GeV (typical jet has $m_{inv} \sim 0.15\ p_T$)

  So look for ultra-narrow low-track-number jet. (Warning: high fake rate!)

Taus will be very important in LHC searches for Higgs, SUSY, etc.

Bottom Quarks and Heavy Flavor Tagging

Bottom quarks (and charm quarks) make jets much like other quarks and gluons, but with a single B (or D) meson [or baryon] which tends to carry a large fraction of the energy.

When a B meson decays, it will

- typically travel (E_B/m_B) 450 μm before decaying
- typically decay to $D\ell\nu$ or D plus other hadrons

A D meson has a 200 μm lifetime and decays to Kaons K plus leptons or hadrons.
Thus a b turns into a jet which

- typically has a displaced vertex (or two)
- $\Rightarrow$ often has ≥ 2 tracks missing collision point by 200-1000 μm
- has a μ embedded in it $\sim 15\%$ of time

For c, similar but less likely to have displaced tracks and vertex.

Typical rates for "heavy-flavor tagging" algorithms (numbers depend on technique used)

- 50–70% (15–20%)of b (c) jets tagged
- 1–2% of g jets tagged due to gluon splitting
  Unavoidable: if gluon splits $g \to c\bar{c}, b\bar{b}$, its jet contains real D, B mesons.
- 0.5–1% of other jets erroneously tagged

Comments

- Don't forget mistagging or gluon splitting. For some measurements, a large source of background.
- Hard to tell b from $\bar{b}$ but embedded $\mu^{\pm}$ charge is correlated with b charge.
- Distinguish μ's from heavy flavor (or π decay) from interesting muons by "isolation" from other particles.
- But b's *can* create jets where μ is only high-momentum particle: **Source of fake isolated-μs**

Top Quarks and their Role at LHC

LHC = Top Factory! Huge rate – 10^5–10^8 per year. Dominantly $gg \to t\bar{t}$ followed by

- $t \to bW^+$ (almost 100% of time)
- $W \to q\bar{q}$ (2/3), $\tau\nu$ (1/9), $\mu\nu$ (1/9), $e\nu$ (1/9)

so (ignoring q vs. $\bar{q}$, defining $\ell \equiv e, \mu$), we will have
$q\bar{q}$ or $gg \to t\bar{t} \to$

- $qqqqb\bar{b}$ 44% [fully hadronic, huge QCD background]
- $qq\ell^{\pm}\nu b\bar{b}$ 30% ["semi-leptonic", fully reconstructible]
- $\ell^+\nu\ell^-\bar{\nu}b\bar{b}$ 5% ["dilepton", low background;]

[Also 21% with τs – complicated.]

Understanding top physics in detail is a key goal for early LHC.

- Tops provide almost everything: jets, electrons, muons, taus and MET
- Production mechanism is simple (relatively!) QCD, masses, decays are essentially known
- Use the objects for calibration
- Must make sure it looks as expected; a huge background to many new physics signals

Easiest: semileptonic

- Trigger on lepton
- Use lepton, $\not{E}_T$ and b tags (0,1,2) to reduce QCD and W+jets background
- Find 2 untagged jets with $m_{jj} \sim m_W$
- Find another (possibly-)tagged jet with $m_{jjj} \sim m_t$.
- For remaining (possibly-)tagged jet, check $m_{j\ell} < 155$ GeV

EXERCISE – **IMPORTANT** – *Check that if we have two 2-body decays — $A \to BC$ and $B \to DE$ — then m_{CD} is bounded from above. Determine the upper bound in terms of m_A, m_B, m_E. This is especially useful when E is invisible. In our case, show that if $A = t$, $B = W$, $C = b$, $D = e$ and $E = \nu$, then $m_{be} < 155$ GeV. (The precise number depends on what you take for m_t – just try to get close.)*

If needed, can find the neutrino momentum assuming $m_{e\nu} = m_W$ and $\vec{p}_{\nu,T} = \vec{\not{p}}_T$.
(The required equation for $p_{\nu,z}$ is quadratic $\Rightarrow$ two solutions.)

Reconstruction of the hadronically-decaying W and t key tool for calibrating jet measurements!!

Next: dilepton.

Trigger easy; 2 ν's so hard to reconstruct. Low SM background.
But itself a large background to new $\ell^+\ell^-\not{E}_T$ signals!!

Finally: fully hadronic –
Large 6-jet QCD backgrounds, trigger challenge, combinatorics [choose correct jet triplets???]
Some progress possible if $p_{t,T} \gg m_t$, 3-jets quasi-collimated into one fat jet

Lecture 4

QCD: Evolution of pdfs and emergence of jets

Two crucially important effects we have not discussed:

- pdfs not $f(x)$ but $f(x, \mu^2)$; partonic initial states run — or "evolve" — with scale
- jets emerge because partonic final states themselves run, or evolve, in a similar way.
- Both of these effects are independent of confinement and arise from the quasi-conformal behavior of high-energy QCD interactions.
- Gluon emission dominates!
  - Large N_c plays a very important role in jet structure
  - Order-$N_f/N_c \sim 1$ effects from $q\bar{q}$ pairs plays limited role in perturbative jet structure, even though it protects that structure from hadronization.

Evolution of pdfs are easier so we'll do them first.

Consider a u quark in a proton.

As it travels it constantly emits and absorbs gluons:

- These virtual processes occur at all scales of quark "virtuality" q^2.
- Time/distance scale for virtuality q^2 of order $1/\sqrt{q^2}$.

A scattering process at scale Q^2, time scale $\frac{1}{\sqrt{Q^2}}$, interrupts absorption/emission for $q^2 < Q^2$.

- Some of the emitted partons will not be reabsorbed; become *real*
- The parton which scatters will have *non-negligible* p_T of order $\sqrt{q^2}$, virtuality of last emission.
- This emission, which carries off balancing p_T, becomes real and can turn into a soft jet.
- In general many gluons are emitted; only the highest-p_T gluons typically are observable experimentally as jets. This is called "initial state radiation", or "ISR".

Although initial parton carried $p_z = x_0 E_b$, ISR steals some of its p_z, leaving it $q_z = x E_b < x_0 E_b$.

Thus, the probability of finding a parton with momentum fraction x is changed!

$$\text{parton model} \Rightarrow \text{QCD}$$

$$f(x) \Rightarrow f(x, Q^2)$$

A parton which, averaging over times $\sim \Lambda^{-1}$, has $p = xP$ in the proton, will be at a lower x when probed on a shorter time $1/Q$ by the short-distance scattering process.

Slow, predictable "evolution" of pdfs through "parton splitting": a fundamental prediction of QCD, precisely tested in 1970s and 80s.

The emission, and therefore evolution, looks α_s-suppressed, but not quite so!.

Consider quark emitting a gluon: $p_\mu = k_\mu + q_\mu$; $p^2, k^2 \ll q^2 = -2p \cdot k < 0$.

Defining $E_0 = x_0 E_b$,

$$-\frac{1}{q^2} \approx \frac{1}{2p \cdot k} = \frac{1}{E_0 E_g (1 - \cos\theta_g)} \quad \text{which diverges when}$$

- $\cos\theta_g \to 0$ **collinear singularity**
- $E_g \to 0$ **soft singularity**

EXERCISE: Check the above formulas.

What is the amplitude $\mathcal{M}$ for a quark to emit a gluon before scattering?

Physics is gauge invariant, but intuitive picture agrees with diagrams only in light-cone gauge.

Simpler: do a scalar quark entering a subamplitude $\mathcal{A}$:

$$\mathcal{M} \sim \frac{\epsilon \cdot (p+q)}{q^2} \mathcal{A} = \frac{\epsilon \cdot p}{p \cdot k} \mathcal{A}$$

Here, used $\epsilon \cdot k = 0$ to write $\epsilon \cdot p = \epsilon \cdot q$. *Ignore color structure; simple to add.*

Square and sum over gluon helicities, and use

$$\sum_h \epsilon_\mu^{(h)} \epsilon_\nu^{(h)*} = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{n \cdot k}$$

where $n = (1, 0, 0, -1)$ defines the light-cone gauge $n^\mu \cdot \epsilon_\mu = 0$. Result proportional to

$$\frac{k \cdot p \; n \cdot p}{n \cdot k (p \cdot k)^2} \propto \frac{1}{p \cdot k}$$

and thus $\sum_h |\mathcal{M}^2|$ has same singularities as $\mathcal{M}$. *Also true for spin-$\frac{1}{2}$ quarks, spin-1 gluons.*

- Soft singularity (long-range field) not observable; $q^\mu \approx p^\mu$; pole cancels against virtual loop.
- But collinear singularity really reduces x of quark and is observable.

Collinear: approximate $\theta \ll 1$, then

$$k_\mu = (1-z)E_0(1, \sin\theta, 0, \cos\theta) \approx (1-z)x_0 E_b (1, \theta, 0, 1) \; ; \; -q^2 = 2p \cdot k \approx E_0 E_g \theta^2 = (1-z)x_0^2 \theta^2$$

There is a phase space integral to do. For **fixed** z, we will have

$$\int \frac{d\cos\theta}{1 - \cos\theta} \approx \int \frac{d\theta^2}{\theta^2} = \int \frac{dq^2}{q^2}$$

So the emission is order $\alpha_s \ln Q_2^2/Q_1^2 \sim 1$ if $Q_2 \gg Q_1$. **Must** *resum* **numerous gluon emissions.**

In short, must integrate effect over Q^2.

$$f_u(x, Q_2^2) = f_u(x, Q_1^2) + \int_x^1 dx_0 \; f_u(x_0, Q_1^2) \times G[x, x_0; Q_1, Q_2]$$

where $G[x, x_0, Q_1, Q_2]$ = Probability for u of virtuality $q^2 = Q_1^2$ to cascade down from x_0 to x_1 by g emission, as $q^2 \to Q_2^2$.

$$G[x, x_0; Q_1, Q_2] = \int_{Q_1^2}^{Q_2^2} dQ^2 \int_0^1 dz \; \tilde{P}_{qg\leftarrow q}(x_0, x, z; Q^2) \; \delta(x - zx_0)$$

Here $\tilde{P}_{qg\leftarrow q}$ is the differential probability for a quark of virtuality Q^2 to emit a gluon such that its x decreases from x_0 to $x = zx_0$. [Relabel subscript $qg \leftarrow q$ by qq; note the order matters.]

- $\tilde{P}$ given by QCD tree graph, scale-invariant except for overall $\alpha_s(Q^2)$
- To make the rest of the formula scale invariant, $\tilde{P}(x, x_0, z, Q^2) = \frac{\alpha_s(Q^2)}{Q^2} P(x, x_0, z)$.
- Diagram describing splitting independent of E_b, so only depends on ratio $x/x_0 = z \Rightarrow P(x, x_0, z) = P(z)$

$$G[x, x_0; Q_1, Q_2] = \frac{\alpha_s}{2\pi} \int_{Q_1^2}^{Q_2^2} \frac{dQ^2}{Q^2} \int_0^1 dz \; P_{qq}(z) \; \delta(x - zx_0) = \frac{\alpha_s}{2\pi} \int_0^{\log Q_2^2/Q_1^2} d\log Q^2 \int_0^1 \frac{dz}{x_0} \; P_{qq}(z) \; \delta(z - x/x_0)$$

Note change in argument of delta function, and corresponding Jacobian. Now, we have

$$f_u(x, Q_2^2) = f_u(x, Q_1^2) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dx_0}{x_0} \; f_u(x_0, Q_1^2) \int_0^{\log Q_2^2/Q_1^2} d\log Q^2 \; P_{qq}(x/x_0)$$

and so

$$\frac{\partial f_u(x, Q_2^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dx_0}{x_0} P_{qq}(x/x_0) \; f_u(x_0, Q_1^2)$$

Not complete: can also get u starting from a g... so really

$$\frac{\partial f_u(x, Q_2^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dx_0}{x_0} \; \left[P_{qq}(x/x_0) \; f_u(x_0, Q_1^2) + P_{qg}(x/x_0) \; f_g(x_0, Q_1^2)\right]$$

where

$$P_{qq} = \frac{h_1(z)}{(1-z)} \; ; \; P_{gq} = \frac{h_2(z)}{z} \; ; \; P_{gg} = \frac{h_3(z)}{z} + \frac{h_4(z)}{1-z} \; ; \; P_{qg} = h_5(z)$$

All $h_i(z)$ calculable polynomials.

Note $P_{qq}(z) = P_{gq}(1-z) \propto \frac{1}{z}$; This soft singularity *observable*: gives large $f_g(x)$ for $x \ll 1$.

Summary

- Always collinear singularity.
- Gluon emission $\Leftrightarrow$ soft singularity
- Quark emission has no soft singularity.
- Gluons win big: $1/z$ emission enhancement.
- N_f enhancement of $g \to q\bar{q}$ ineffectual.

Evolution of the pdfs is slow at LHC scales, as shown in FIGURES.