The scope of the landscape: supergravity and string vacua in 10D, 8D, 6D, and 4D

Part I: Supergravity and strings in 10D

TASI, June 2010
Boulder, Colorado

June 1, 2010

Washington (Wati) Taylor, MIT

Introduction

QFT: powerful framework ⇒ Standard model, many possible extensions
But: many possible QFT's, few constraints

Question motivating these lectures:

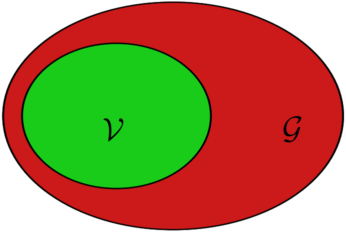
Does gravity + UV consistency
constrain low-E* physics (QFT)?
(* low-E = sub-Planck scale)

Study two aspects of global picture

1) Apparently consistent gravity + YM ($\mathcal{G}$)

2) Known string vacua ($\mathcal{V}$)

$\mathcal{G} \setminus \mathcal{V} =$ apparent "swampland" [Vafa]



Focus on supersymmetric theories. Why SUSY?

Phenomenological answers:

- Coupling constant unification
- Dark matter candidate
- Hierarchy problem

String theory answers:

- String theory needs SUSY at Planck scale
- Supersymmetry makes things simpler

Philosophy of these lectures: split by scale

$< \Lambda$: SUSY QFT
$> \Lambda$: SUGRA/strings

If SUSY breaks at Planck scale:
UV physics very hard to understand

Q. gravity [Planck]
↓
SUSY QFT (SUGRA) [$< \Lambda$]
↓
~~SUSY~~ QFT [$<$ TeV]

Plan of lectures:

Describe set of SUGRA + string theories with minimal SUSY in $D = 10, 8, 6, 4$

10D →	8D →	6D →	4D
gravity	gauge gp.	matter	Yukawas
simple	→	→	complicated

As we decrease dimensions:

- Theories richer, more structure
- Wider range of vacuum constructions
- Global description of landscape harder

10D: SUGRA, anomaly constraints, strings, branes

8D: Heterotic and F-theory compactifications

6D: Anomaly constraints, intersecting brane models, heterotic and F-theory cpt.

4D: IBM; IIB, IIA, NG flux vacua

I. Supergravity and string vacua in 10D

Supersymmetry:

- Symmetry relating bosons to fermions
- SUSY algebra: extends Poincare by fermionic generators

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\Gamma^\mu_{\alpha\beta} P_\mu$$

$$\text{basic action: } \delta\phi \sim \bar{\epsilon}\psi, \quad \delta\psi \sim \Gamma^\mu \epsilon \partial_\mu \phi$$

- In some cases, multiple SUSY's → generators $Q^A_\alpha, \quad A = 1, \ldots \mathcal{N}$

$$\{Q^A_\alpha, \bar{Q}^B_\beta\} = 2\delta^{AB} P_\mu \Gamma^\mu_{\alpha\beta}$$

- Can extend by central charges → identify topological charges

$$\{Q^A_\alpha, \bar{Q}^B_\beta\} = 2\delta^{AB} P_\mu \Gamma^\mu_{\alpha\beta} + Z^{AB} \delta_{\alpha\beta}$$

- SUSY + gravity ⇒ local SUSY = supergravity
- Details of SUSY + spinors in various dimensions: Polchinski App. B (v2)

Supergravity in maximum dimension: $D = 11$

In $D > 11$, any representation of Γ's (Clifford algebra) $\geq$ 64-dimensional
$\Rightarrow$ massless particles of spin > 2 – no known (interacting) realizations.

Maximum D for SUGRA: $D = 11, \mathcal{N} = 1$, 32 supercharges Q_α

Fields:

$g_{\mu\nu}$	graviton (metric)	$[\frac{9\times 10}{2} - 1 = 44$ DOF]
$C_{\mu\nu\lambda}$	antisymmetric 3-form	$[\frac{9\times 8\times 7}{6} = 84$ DOF]
$\psi_{\mu\alpha}$	graviton (metric)	[128 DOF]

Bosonic DOF = Fermionic DOF = 128

Action:

$$S = \frac{1}{2\kappa_{11}^2}\left[\int \sqrt{g}(R - \frac{1}{4}|F|^2) - \frac{1}{6}\int C \wedge F \wedge F\right]$$

$$F^{(4)} = dC^{(3)}$$

No gauge symmetries, matter. Global picture: $\mathcal{G}_{11} = \{M_{11}\}$ $(= \mathcal{V}_{11};$ M-theory)

10D supergravity

32 supercharges: IIA, IIB

$$\text{IIA}: \quad \begin{array}{c} \text{11D SUGRA} \\ \downarrow (S^1) \\ \mathbb{R}^{1,9} \end{array} \qquad \begin{array}{ccccc} g^{(11)}_{\mu\nu} & g^{(11)}_{\mu\ 11} & g^{(11)}_{11\ 11} & C^{(11)}_{\mu\nu\lambda} & C^{(11)}_{\mu\nu\ 11} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ g_{\mu\nu} & A_\mu & \phi & C_{\mu\nu\lambda} & B_{\mu\nu} \end{array}$$

- In 10D: $\Gamma^{11} = \prod_{\mu=0}^{9} \Gamma^\mu$; Can have **16, 16'** Majorana-Weyl chiral spinors.
- In IIA: two SUSY's ($\mathcal{N} = 2$), $Q^1 \in \mathbf{16}, Q^2 \in \mathbf{16}'$
- Bosonic fields $g_{\mu\nu}, B_{\mu\nu}, \phi$ in $\mathcal{N} = 1$ multiplet
- $A_\mu, C_{\mu\nu\lambda}$ "R-R fields" (name from string construction)

IIB:

- Two M-W SUSY's with same chirality $Q^1, Q^2 \in \mathbf{16}$
- Fields: $g_{\mu\nu}, B_{\mu\nu}, \phi; \chi, \tilde{B}_{\mu\nu}, D^+_{\mu\nu\lambda\sigma}$

Type IIA, IIB supergravity theories uniquely determined by SUSY

10D $\mathcal{N} = 1$ SUGRA: 16 supercharges $Q_\alpha \in \mathbf{16}$

SUSY multiplets:

$$\begin{cases} \text{SUGRA} & g_{\mu\nu}, B_{\mu\nu}, \phi, \psi_{\mu\alpha}, \zeta_\alpha & [64 + 64 \text{ DOF}] \\ \text{vector} & A_\mu, \lambda_\alpha & [8 + 8 \text{ DOF}] \end{cases}$$

$$S \sim \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left[e^{-2\phi} \left(R + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}|H|^2 \right) - \frac{1}{g_{\text{YM}}^2} |F|^2 \right]$$

$$H = dB - \omega_Y \qquad d\omega_Y \sim \text{tr } F \wedge F$$

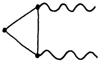
From classical supergravity point of view:

Appears that gauge group G can be anything

Does $\mathcal{G}_{10}$ contain infinitely many discrete components?

No: quantum constraints from **anomalies**

Anomalies

In 4D, chiral anomaly  $\partial_\mu j^5 \sim F \wedge F$ [see *e.g.* Peskin/Schroeder]

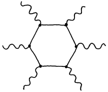
• Can be understood in terms of lack of invariance of measure in PI

$$\int d\psi d\bar{\psi} \neq \int d\psi' d\bar{\psi}'$$

• Related to index theorem

• Gives breakdown of gauge invariance w/chiral fields
anomalies in local symmetry $\Rightarrow$ quantum inconsistency

In dimensions $D = 4k + 2$, Weyl representations *self-conjugate*
$\Rightarrow$ particle + antiparticle have same chirality
$\Rightarrow$ gravitational (+ gauge, mixed) anomalies

10D anomalies:  $R^6, R^4F^2, R^2F^4, F^6 \rightarrow (D+2)$-form $\hat{I}$

Summary of 10D anomalies [Alvarez-Gaume/Witten]

Anomalies from n spinors (8), gravitino (56), and (anti-)self-dual $D^{\pm}_{\mu\nu\lambda\sigma}$ (70)

$$\begin{aligned}
\hat{I}_8 &= -\frac{\mathrm{Tr}(F^6)}{1440} + \frac{\mathrm{Tr}(F^4)\mathrm{tr}(R^2)}{2304} - \frac{\mathrm{Tr}(F^2)\mathrm{tr}(R^4)}{23040} - \frac{\mathrm{Tr}(F^2)[\mathrm{tr}(R^2)]^2}{18432} \\
&\quad + \frac{n\ \mathrm{tr}(R^6)}{725760} - \frac{n\ \mathrm{tr}(R^4)\mathrm{tr}(R^2)}{552960} + \frac{n\ [\mathrm{tr}(R^2)]^3}{1327104} \\
\hat{I}_{56} &= -495\frac{\mathrm{tr}(R^6)}{725760} - 225\frac{\mathrm{tr}(R^4)\mathrm{tr}(R^2)}{552960} - 63\frac{[\mathrm{tr}(R^2)]^3}{1327104} \\
\hat{I}_{70} &= 992\frac{\mathrm{tr}(R^6)}{725760} + 448\frac{\mathrm{tr}(R^4)\mathrm{tr}(R^2)}{552960} + 128\frac{[\mathrm{tr}(R^2)]^3}{1327104}
\end{aligned}$$

Type IIB: 2 ζ_α's (**8'**, $n = -2$), 2 $\psi_{\mu\alpha}$ (**56**), 1 $D^{+}_{\mu\nu\lambda\sigma}$ (**70**), $F = 0$

$$-2\hat{I}_8(F \to 0, n \to 1) + 2\hat{I}_{56} + \hat{I}_{70} = 0!!$$

$\mathcal{N} = 1$: no $\hat{I}_{70}$ $\rightarrow$ no cancellation?

Green-Schwarz anomaly cancellation

Problem: $\mathcal{N} = 1$ SUGRA + YM group G has hexagon anomalies $\sim (R, F)^6$.

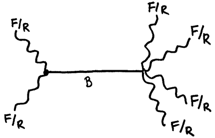
Solution: careful treatment of B couplings, new B couplings at higher order

$$S_B \sim (dB - \omega)^2 + B \wedge d\tilde{\omega}$$

$$\begin{aligned} d\omega &= c\, \mathrm{tr}F^2 + c' \mathrm{Tr}R^2 = Y_4(F, R) \\ d\tilde{\omega} &= X_8(F, R) \end{aligned}$$

Variation of $\delta B = c\ \mathrm{Tr}(\Lambda F) + c'\mathrm{tr}(\Theta R)$ where $\delta A = d\Lambda, \delta\omega_1 = d\Theta$
$\delta(B \wedge X_8) \neq 0$ can cancel anomalous variation of PI

Can cancel anomaly if $\hat{I}_{12} \sim Y_4 X_8$ factorizes

Tree diagrams: 

$Y_4(F, R)X_8(F, R)$

Anomaly cancellation for $\mathcal{N} = 1$ SUGRA, gauge group G

Fields: gravitino (**56**), neutral fermion (**8'**), $n (= \dim G)$ gauginos (**8**)
Want

$$\begin{aligned} \hat{I}_{12} &= (n - 496)\frac{\mathrm{tr}(R^6)}{725760} - \frac{\mathrm{Tr}(F^6)}{1440} + \cdots \\ &= Y_4(F,R) X_8(F,R) \end{aligned}$$

For factorization: R^6 term must cancel $\Rightarrow$ $n = 496$

F^6 cancellation + factorized form only if

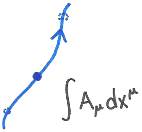
$$\mathrm{Tr}(F^6) = \frac{1}{48}\mathrm{Tr}(F^2)\mathrm{Tr}(F^4) - \frac{1}{14400}[\mathrm{Tr}(F^2)]^3$$

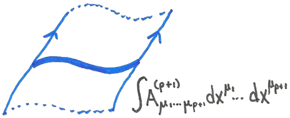
Only satisfied for 4 groups: $SO(32)$, $E_8 \times E_8$, $U(1)^{496}$, $E_8 \times U(1)^{248}$

So, including IIA, IIB only 6 candidates for UV-consistent SUGRA in 10D.

UV completions of 11D + 10D SUGRA

Rely on extended objects

Just as 

similarly 

pointlike particle couples to A_μ

p-brane couples to $(p+1)$-form $A^{(p+1)}_{\mu_1 \cdots \mu_{p+1}}$

Theory	Field	Brane
11D	$C_{\mu\nu\lambda}$	M2-brane (+ dual M5)
$\mathcal{N} = 1, 2$ 10D	$B_{\mu\nu}$	(F) string (+ NS5-brane)
IIA IIB	$A_\mu, C_{\mu\nu\lambda}$ $\tilde{B}_{\mu\nu}, D^+_{\mu\nu\lambda\sigma}$ [RR-fields]	(D)-branes

Brane democracy

Quantize any brane ⇒ Quantum gravity (but usually hard except in limits)

11D supergravity (M-theory):

M2-brane: In light cone ⇒ M(atrix) theory (also from D0)

10D supergravity:

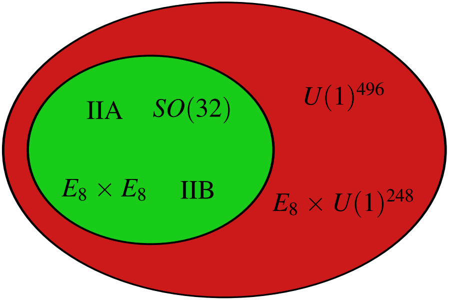
Dp-branes: Near horizon limit ⇒ AdS/CFT

- A_μ, X_μ fields *on* D-brane $\rightarrow$ SYM

Strings (F1-branes): perturbative string theory

- Best understood in g, B, ϕ background; RR bg tricky [see Berkovits]
- D-branes: loci for open string endpoints
- Strings between branes $\rightarrow$ nonabelian G  $\rightarrow SU(N)$
- $\mathcal{N} = 2$: IIA, IIB; $\mathcal{N} = 1$: heterotic: $E_8 \times E_8, SO(32)/\mathbb{Z}_2$ = type I

Summary of global situation in 10D



6 distinct theories w/o known inconsistencies; 4 realized in string theory
2 with no known string realization–apparent swampland

Remaining challenge:
Find string realization or UV inconsistency
for $U(1)^{496}$ and $E_8 \times U(1)^{248}$ theories

[Update: inconsistency may be proven, stay tuned [Adams/de Wolfe/WT]]

Henceforth, focus on nonabelian part of gauge groups