The scope of the landscape: supergravity and string vacua in 10D, 8D, 6D, and 4D

Part II: Supergravity and string vacua in 8D

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$\mathcal{N} = 1$ supergravity in 8 dimensions: 16 supercharges, gauge multiplets

Multiplets:	$\{g_{\mu\nu}, \psi_{\mu\alpha}, B_{\mu\nu}, 2A_\mu, \chi_\alpha, \sigma\}$	graviton multiplet
	$\{A_\mu, \lambda_\alpha, 2\phi\}$	gauge multiplet

Spinors pseudo-Majorana, not chiral, no (anti)-self-dual tensors
$\rightarrow$ no anomalies

So apparently $\mathcal{G} = \{$SUGRA + YM for any $G\}$; infinite set

What models can we get from string theory?

Plan:

- Compactification of SUGRA to lower D: general aspects
- Heterotic $\rightarrow$ 8D
- F-theory $\rightarrow$ 8D

Heterotic + F-theory $\rightarrow$ same answer for $\{\mathcal{N} = 1$ 8D theories$\}$ (duality)

Compactification to lower dimension

As for 11D on $S^1 \to$ IIA, generally cpt. on $T^d \to$ same # supercharges

What else can we do?

Many options:

- pure geometric (classic CY, $G_2, \ldots$)
- geometry + branes (IBM, F-theory, . . .)
- geometry + fluxes (IIB/IIA flux cpt., AdS/CFT, non-Kähler, . . .)

Start with pure geometry
(no H, RR flux, ϕ constant)

D-dim theory

$\downarrow X^k$

$(D - k)$-dim theory

To preserve SUSY need

$\delta\psi_\mu = D_\mu\eta = 0 \Rightarrow$ cov. constant spinor

$\exists$ for dim $X = 2N$ w/ $SU(N)$ holonomy

$\Leftrightarrow X$ cpx. Kähler, $c_1 = 0$ (Calabi-Yau)

Compactifications with covariantly constant spinor

dimension	mfd. type	holonomy	SUSY
D	T^k	$\{1\}$	1
4	K3 (CY2)	SU(2)	1/2
6	CY3	SU(3)	1/4
7	G_2	G_2	1/8
8	CY4	SU(4)	1/8

How can we get $\mathcal{N} = 1$ in 8D?

het = I (10D) [16 Q] → (T^2) → 8D [16 Q]

F-theory ("12D") [32 Q] → (K3) → 8D [16 Q]

Dimensional reduction of $\mathcal{N} = 1$ SUGRA ($G = SO(32)$ or $E_8 \times E_8$)

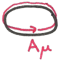
$g_{\mu c}, B_{\mu c}, A_\mu$ → $20\ A_\mu \quad [U(1)^4 \times G]$

$A_c, g_{cc'}, B_{cc'}$ → $36\ \phi$ (scalars/moduli)

Gives gravity multiplet + 18 vector multiplets

Simple reduction: $\rightarrow SO(32) \times U(1)^4$ or $E_8 \times E_8 \times U(1)^4$ (rank 20)

Can also turn on Wilson lines A_μ around circles

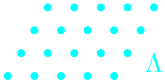


$E \sim F^2 \Rightarrow$ Wilson lines commute $[A_8, A_9] = 0$

So gauge group broken to $H \subset G, \quad [h, A_{c=8,9}] = 0 \ \ \forall h \in H$

But more — can enhance gauge symmetry at specific moduli

Interlude: basics of lattices



Why lattices? $T^k = \mathbb{R}^k/\Lambda^k$, Λ^k a lattice.

Mathematically, lattice $\Gamma = \{n_i e_i, n_i \in \mathbb{Z}\}$, e_i basis vectors

Lattice: carries symmetric bilinear inner product $\cdot$, signature (p, q)

Integral lattice: $v \cdot w \in \mathbb{Z} \;\; \forall v, w \in \Gamma$

Even lattice: $v \cdot v \in 2\mathbb{Z} \;\; \forall v \in \Gamma$

Dual lattice: $\Gamma^* = \{v : v \cdot w \in \mathbb{Z} \;\; \forall w \in \Gamma\}$

Self-dual/unimodular lattice: $\Gamma = \Gamma^*$ ($\leftrightarrow$ $\det \Gamma = \pm 1 \leftrightarrow |\mathrm{Vol\ (unit\ cell)}| = 1$)
Unimodular signature $(p, q) \Rightarrow p \equiv q \pmod 8$ [Milnor]

Example:

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad e_1^2 = e_2^2 = 0, \quad e_1 \cdot e_2 = 1$$

$U = \Gamma^{1,1}$ is even, unimodular lattice with signature (1, 1)
$(e_1 + e_2)^2 = 2$, $(e_1 - e_2)^2 = -2$

Root lattices

Given simply laced Lie group $(SU(N), SO(2N), E_6, E_7, E_8)$

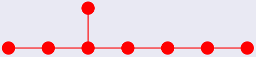
Simple roots $r_i \rightarrow$ lattice basis, Cartan matrix $a_{ij} = r_i \cdot r_j$ (norm $r_i^2 = 2$,)

Dynkin diagram: $\bullet = r_i$, connected by — if $a_{ij} = -1$

Example: SU(3)

Cartan matrix for SU(3): $(a_{ij}) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$; Dynkin diagram ($A_2$): 

Example: E_8

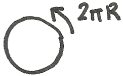


$$(a_{ij}) = \begin{pmatrix} 2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

Euclidean SD lattices $p \equiv 0 \pmod 8$: $E_8, E_8 \times E_8, \Lambda_{16}(SO(32)/\mathbb{Z}_2), \ldots$

Indefinite signature SD: all $= U \oplus \cdots \oplus U \oplus (\pm E_8) \oplus \cdots \oplus (\pm E_8)$

Toroidal compactification and enhanced symmetries



Flat space bosonic string: $X^\mu(\sigma, \tau) : \Sigma \to \mathbb{R}^{1,9}$
Compactify bosonic string on circle of radius R

$$X = x + w\sigma + p\tau + \text{oscillators}(\alpha_n, \tilde{\alpha}_n) \qquad [w = mR, p = n/R; T : w \leftrightarrow p]$$

$$= x + \frac{1}{\sqrt{2}} l_L(\tau + \sigma) + \frac{1}{\sqrt{2}} l_R(\tau - \sigma) + \cdots \qquad [l_L = \frac{p+w}{\sqrt{2}},\ l_R = \frac{p-w}{\sqrt{2}}]$$

Momenta $(k_L, k_R) = \sqrt{2}(l_L, l_R)$ on lattice U: $l \cdot l = l_L^2 - l_R^2 = 2pw = 2nm \in 2\mathbb{Z}$

Mass shell condition
for string states

$$M^2 = k_\mu k^\mu = 2l_L^2 + 4(N-1) = 2l_R^2 + 4(\tilde{N} - 1)$$

Massless vector fields from closed string states:

$$N = \tilde{N} = 1 : \qquad \alpha^\mu_{-1}\tilde{\alpha}^*_{-1}|l_L = 0; l_R = 0\rangle \qquad (g^{\mu *}, B^{\mu *} \to A^{1-4}_\mu)$$

$$N = 1, \tilde{N} = 0 \qquad \tilde{\alpha}^\mu_{-1}|l_L; l_R\rangle, \quad l_R = 0, \ l_L^2 = l \cdot l = 2 \Rightarrow R = 1 (= (\alpha')^{1/2})$$

Enhanced gauge symmetry if $l \cdot l = 2$ ($SU(2)^2$ at self-dual radius $R = \sqrt{\alpha'}$)

General compactification (Narain compactification)

Compactify on T^D $\rightarrow$ momenta in lattice $\Gamma^{D,D}$ with signature (D, D)

- $l \cdot l = 2 \Rightarrow$ enhanced (nonabelian) symmetry
- Modular invariance $\rightarrow \Gamma = \Gamma^*$ (Γ unimodular/self-dual)
- (Narain) moduli space of lattices:

$$G_D \backslash SO(D, D)/SO(D) \times SO(D), \quad [G_D \text{ discrete (T-duality) group}]$$

Heterotic string:

26 right-moving, 10 left-moving bosonic DOF [note convention]

Compactify in 10D: unimodular lattice $\Gamma^{0,16}$
$\Gamma^{0,16} = E_8 \oplus E_8$ or $\Lambda_{16} \Rightarrow G = E_8 \times E_8$ or $SO(32)/\mathbb{Z}_2$

Compactify in 9D: $\Gamma^{1,17} = U \oplus (-E_8) \oplus (-E_8)$
Same theory for $E_8 \times E_8, SO(32)$ on S^1 (T-duality)

Heterotic string in 8D

het $\rightarrow \Gamma^{2,18} = U \oplus U \oplus (-E_8) \oplus (-E_8)$

$\downarrow T^2$ 36D moduli space $SO(2,18;\mathbb{Z})\backslash SO(2,18;\mathbb{R})/SO(2)\times SO(18)$

$\mathbb{R}^{1,7}$

Can enhance symmetry to any G : $-\Lambda_G \hookrightarrow \Gamma^{2,18}$

Examples:

$SO(36)$	$(\Lambda = D_{18})$	[Polchinski]
$SU(18)/\mathbb{Z}_3$	$(\Lambda = A_{17})$	[Morrison/Seiberg]

Simplified embedding theorem (Nikulin)

For $S = \Gamma^{t_+,t_-}, T = \Gamma^{l_+,l_-}$ even, T unimodular, exists embedding $S \hookrightarrow T$ if the following conditions are satisfied:

1) $l_+ - l_- \equiv 0 \pmod 8$, 2) $l_- \geq t_-,\ l_+ \geq t_+,\ l_+ + l_- - t_+ - t_- > l(S^*/S)$
where $l(A)$ = minimum number of generators of finite group A

e.g. $G = SU(19)$ ok in 8D ($S^*/S = \mathbb{Z}_{19}$), but not $G = SU(20)$.
More precise necessary and sufficient conditions evolve number theory of S^*/S.

F-theory vacua in 8D

Brief introduction to F-theory [Vafa, Morrison/Vafa]

IIB: axiodilaton $\tau = \chi + ie^{-\phi}$

Classical $SL(2,\mathbb{R})$ duality: $B, \tilde{B}$ transform as doublet,

$$\tau \to \frac{a\tau + b}{c\tau + d}, \qquad ad - bc = 1$$

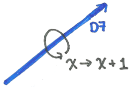
Quantum theory: strings quantized, (F1, D1) doublet

$$SL(2,\mathbb{R}) \;\to\; SL(2,\mathbb{Z})$$

Moduli space for τ : $\{\tau \in \mathbb{C}, \text{Im } \tau > 0\}/SL(2,\mathbb{Z})$ = moduli space for T^2

D7-brane: magnetically charged under χ

(p,q) 7-branes generalize to arbitrary $SL(2,\mathbb{Z})$ monodromy



Geometrize $\Rightarrow$ T^2 over space-time

F-theory $\sim$ 12D compactified on *elliptically fibered* CY (but no 12D SUGRA)

$$\begin{array}{ccc} T^2 & \longrightarrow & X \\ & & \downarrow \\ & & B \end{array}$$

Base B of complex dimension d

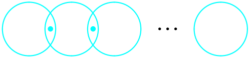
$\rightarrow$ compactify to $D = 10 - 2d$

To understand physics need geometry of elliptic fibrations

If $X \neq B \times T^2$, fibration singular (though total space may be smooth)

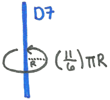
- Singularities classified by Kodaira for B complex 1D (2D real)
- A-D-E classification $\leftrightarrow$ nonabelian group symmetry

Simplest case: N D7-branes $\Rightarrow A_{N-1}$ singularity $\Rightarrow SU(N)$ gauge group

 represents shrunk 2-cycles 

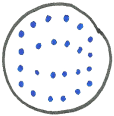
[Note: in M-theory dual massless fields from M2-brane wrapped on cycles]

F-theory in 8D



$\sim$ compactification on 2D space B with 7-branes

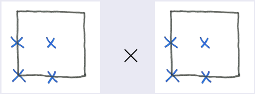
Elementary D7 solutions $\rightarrow$ deficit angle $\pi/6$
Positive curvature $\rightarrow$ B = sphere with 24 7-branes $(4\pi = 24(\pi/6))$



Need Π ($SL(2, Z)$ monodromies) $= 1$

Elliptic fibration with 24 singularities over S^2 $\Rightarrow$ K3 surface (CY2)

K3: complex surface, $H_2(K3; \mathbb{Z}) = \Gamma^{3,19}$ lattice with intersection form product
Often considered in orbifold limit $T^4/\mathbb{Z}_2$



16 Z_2 singularities. Blow up $\rightarrow$ e_i (S^2's)
$U \oplus U \oplus U$ from T^4 (12, 34; 13, 24; 14, 23)
"bulk" cycles + e_i $\rightarrow$ Kummer $\subset \Gamma^{3,19}$
(fractional combinations complete)

Weierstrass form for singularities

Elliptic curve (torus) defined in $\mathbb{P}^{2,3,1}$ by

$$y^2 = x^3 + fxz^4 + gz^6$$

Singular when $\Delta = 4f^3 + 27g^2 = 0$

Local torus fibration over $\mathbb{C} = \{w\}$ near singular point $w = 0$

$$y^2 = x^3 + f(w)x + g(w)$$

Type of singularity depends on behavior of f, g, Δ near $w = 0$

Simplest example:

$$A_{n-1} \text{ singularity when } f(0), g(0) \neq 0, \;\; \Delta(w) \sim w^n + \mathcal{O}(w^{n+1})$$
$$\sim n \text{ D7's} \rightarrow SU(n) \text{ gauge group}$$

Kodaira singularity classification

Can identify type of singularity/enhanced gauge group from Weierstrass form

ord (f)	ord (g)	ord (Δ)	singularity	nonabelian symmetry
≥ 0	≥ 0	0	none	none
0	0	n	A_{n-1}	$SU(n)$
≥ 1	1	2	none	none
1	≥ 2	3	A_1	$SU(2)$
≥ 2	2	4	A_2	$SU(3)$
2	≥ 3	$n+6$	D_{n+4}	$SO(8+2n)$
≥ 2	3	$n+6$	D_{n+4}	$SO(8+2n)$
≥ 3	4	8	E_6	E_6
3	≥ 5	9	E_7	E_7
≥ 4	5	10	E_8	E_8

ord $(\Delta) > 10 \Rightarrow$ cannot be resolved to CY by blowing up fibers

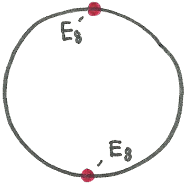
F-theory example: 8D with $G = E_8 \times E_8$ [Morrison/Vafa I]

(Heterotic: bundle with no Wilson lines $\Rightarrow$ 2 complex moduli $(\tau, \rho + iB)$)

On S^2: $\deg \Delta = 24 \Rightarrow \deg f = 8, \ \deg g = 12$

$E_8 : \text{ord}(f) \geq 4, \text{ord}(g) \geq 5, \text{ord}(\Delta) = 10$

Place singularities at $z = 0, \infty$ on S^2



$$f = \alpha z^4, \qquad g = z^5 + \beta z^6 + z^7, \qquad \Delta = -27 z^{10} + \cdots$$

Weierstrass: $\boxed{y^2 + \alpha z^4 x + z^5 + \beta z^6 + z^7 = 0}$

F-theory vacua in 8D: global picture

What nonabelian G can be realized in 8D?

Recall $H_2(K3;\mathbb{Z}) = \Gamma^{3,19}$. Elliptic fiber + section give U, reduce $\rightarrow \Gamma^{2,18}$.

In remaining $\Gamma^{2,19}$, can shrink any 2-cycles $\Rightarrow$ NA gauge bosons

$$\text{Possible } G: \quad -\Lambda_G \hookrightarrow \Gamma^{2,18}$$

Agrees with heterotic. (Dual descriptions of same physics)

Moduli space: $D\backslash SO(2,18;\mathbb{R})/SO(2)\times SO(18)$, D discrete.

8D summary: (Challenge: find new consistency conditions, constrain G)