The scope of the landscape: supergravity and string vacua in 10D, 8D, 6D, and 4D

Part III: Supergravity and string vacua in 6D

TASI, June 2010
Boulder, Colorado

June 3, 2010

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Supergravity in 6D

$\mathcal{N} = 1$ (1, 0) 6D SUGRA: 8 supercharges, gauge group + matter

6D (1, 0) multiplets:

Multiplet	Matter Content
SUGRA	$(g_{\mu\nu}, B^{+}_{\mu\nu}, \psi^{-}_{\mu})$
Tensor (T)	$(B^{-}_{\mu\nu}, \phi, \chi^{+})$
Vector (V)	(A_{μ}, λ^{-})
Hyper (H)	$(4\varphi, \psi^{+})$

T: B's transform under $SO(1, T)$, ϕ's $\rightarrow j \in SO(1, T)/SO(T)$

V: Semi-simple group $G = G_1 \times G_2 \times \cdots \times G_k$ $(/\Gamma)$ [ignore $U(1)$'s]

H: Matter $\mathcal{M}$ in (generally reducible) representation of G

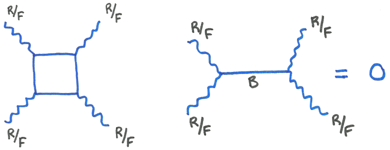
Plan:

- Constraints on $\mathcal{G}$
- Intersecting brane models (IBM's) in 6D
- Heterotic $\rightarrow$ 6D (instantons) [MBM]
- F-theory $\rightarrow$ 6D (B = cpx. surface, intersections $\rightarrow$ matter)

Anomaly cancellation in 6D

In 6D, chiral fields $B^{\pm}_{\mu\nu}, \psi^{-}_{\mu}, \lambda^{-}, \psi^{+}$ give anomalies

Anomalies characterized by 8−form $I_8 \sim R^4 + R^2F^2 + F^4$



can cancel by Green-Schwarz mechanism [G/S/West, Sagnotti, Sadov]

$$I_8 = \frac{1}{2}\Omega_{\alpha\beta}X_4^{\alpha}X_4^{\beta} \qquad X_4^{\alpha} = \frac{1}{2}a^{\alpha}\mathrm{tr}R^2 + \sum_i b_i^{\alpha}\left(\frac{2}{\lambda_i}\mathrm{tr}F_i^2\right)$$

$\mathrm{sign}(\Omega) = (+,-,-,-,\ldots)$, $a^{\alpha}, b_i^{\alpha} \in \mathbb{R}^{1,T}$, $\mathrm{tr} \to \lambda_{SU(N)} = 1, \lambda_{E_8} = 60, \ldots$

Constraints on 6D chiral theories

Claim: For $T < 9$, only a finite set of possible $G, \mathcal{M}$ [Kumar/Morrison/Taylor]

Primary constraint: anomaly cancellation

$$I_8 = \frac{1}{2}\Omega_{\alpha\beta}X_4^\alpha X_4^\beta \qquad X_4^\alpha = \frac{1}{2}a^\alpha \mathrm{tr}R^2 + \sum_i b_i^\alpha \left(\frac{2}{\lambda_i}\mathrm{tr}F_i^2\right)$$

Factorization simplifies for Lagrangian theories (one tensor)

$T = 1$:

$$\Omega_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad a = (-2, -2), \qquad b = \frac{1}{2}(\alpha, \tilde{\alpha})$$

$$I_8 = X^1 X^2 = (\mathrm{tr}R^2 - \sum_i \alpha_i \mathrm{tr}F_i^2)(\mathrm{tr}R^2 - \sum_i \tilde{\alpha}_i \mathrm{tr}F_i^2)$$

Further constraint: physical kinetic terms [Sagnotti]

SUSY $\rightarrow -j \cdot b\, \mathrm{tr}F^2 \rightarrow j \cdot b > 0$ $\qquad (j \cdot b = e^\phi \alpha + e^{-\phi}\tilde{\alpha}$ for $T = 1)$

Anomaly conditions for factorization:

$$I_8 = \frac{1}{2}\Omega_{\alpha\beta}\left(\frac{1}{2}a^{\alpha}\mathrm{tr}R^2 + 2b_i^{\alpha}/\lambda_i\ \mathrm{tr}F_i^2\right)\left(\frac{1}{2}a^{\beta}\mathrm{tr}R^2 + 2b_i^{\beta}/\lambda_i\ \mathrm{tr}{F_i}^2\right)$$

give relations:

R^4:	$H - V = 273 - 29T$	$H - V \sim$ partition bound
F^4:	$0 = B^i_{Adj} - \sum_R x^i_R B^i_R$	
$(R^2)^2$:	$a \cdot a = 9 - T$	A_R, B_R, C_R from expanding :
F^2R^2:	$a \cdot b_i = \frac{1}{6}\lambda_i\left(A^i_{Adj} - \sum_R x^i_R A^i_R\right)$	$\mathrm{tr}_R F^2 = A_R \mathrm{tr} F^2$
$(F^2)^2$:	$b_i \cdot b_i = \frac{1}{3}\lambda_i^2\left(\sum_R x^i_R C^i_R - C^i_{Adj}\right)$	$\mathrm{tr}_R F^4 = B_R \mathrm{tr} F^4 + C_R(\mathrm{tr}F^2)^2$
$F_i^2 F_j^2$:	$b_i \cdot b_j = 2\sum_{R,S} x^{ij}_{RS} A^i_R A^j_S$	(x's: #'s of charged H's)

Remarkable fact: $a \cdot a,\ a \cdot b_i,\ b_i \cdot b_i,\ b_i \cdot b_j \in \mathbb{Z}$ if no local/global anomalies.

Defines integral lattice $\Lambda \subset \mathbb{R}^{1,T}$ [KMT]

Proof of finite possible $\mathcal{G}, \mathcal{M}$ in consistent 6D SUGRA's with $T < 9$

For fixed G, T, finite possible $\mathcal{M}$ from $H - V = 273 - 29T$.
So must have unbounded G, prove impossible by contradiction

Case 1: $\{G = G_1 \times \cdots \times G_k\}, \ |G_i| \leq D, \ k \to \infty$

Use $(1, T)$ geometry, bound on matter: $H \leq 273 - 29T + kD$;

$$\text{classify} \begin{cases} b^2 > 0 : \text{P} \\ b^2 = 0 : \text{Z} \\ b^2 < 0 : \text{N} \end{cases}$$

P: $b_i = (x_i, \vec{y}_i), |x_i| > |\vec{y}_i| \Rightarrow b_i \cdot b_j > 0 \ \Rightarrow$ at most $\mathcal{O}(\sqrt{k})$ type P's

N: At most T mutually orthogonal with $b_i^2 < 0, b_i \cdot b_j = 0$
Turán's theorem: graph on n nodes with $> (1 - 1/T)n^2/2$ edges $\supset T$-clique
$\Rightarrow$ at most $\mathcal{O}(\sqrt{k})$ type N's

Z: $b_i \cdot b_j > 0$ if not parallel, $\to \mathcal{O}(k)$ parallel (all but $\mathcal{O}(\ln k)$ Z's)
All Z's have positive $H - V$ (explicit check) $\Rightarrow$ exceed bound, contradiction $\forall T$.

Case 2: $|G_1| \to \infty$

Only limited possibilities for factors of unbounded dimension

- Schwarz: Two $T = 1$ families: $SU(N) \times SU(N), SO(2N + 8) \times Sp(N)$
- Also: $SU(N) \times SO(N + 8), SU(N) \times SU(N + 8), Sp(N) \times SU(2N + 8)$
- $T > 1$: Three 3-factor families $\sim SU(N - 8) \times SU(N) \times SU(N + 8)$

e.g. $SU(N) \times SU(N)$, matter = $2 \times (\square, \bar{\square})$

$$a \cdot b_1 = a \cdot b_2 = 0, \quad b_1^2 = b_2^2 = -2, \quad b_1 \cdot b_2 = 2$$

Families all have common problem for $a^2 > 0$ $(T < 9)$

$$a \cdot (b_1 + b_2) = 0 \;\; \& \;\; (b_1 + b_2)^2 = 0 \;\; \Rightarrow \;\; b_1 + b_2 = 0$$

$$\Rightarrow j \cdot b_1 = -j \cdot b_2 \;\; \Rightarrow \;\; \text{bad kinetic terms}$$

Proven finite models for $T < 9$; Proof fails at $T \geq 9$

What happens at $T = 9$?

Infinite families with anomaly cancellation, ok kinetic terms

e.g. for $SU(N) \times SU(N)$ family, $\Omega = \text{diag}\,(+1, -1, -1, \ldots), j = (1, 0, 0 \ldots)$

$$\begin{aligned} a &= (2, 1, 1, 1, 1, 0, 0, 0, 0, 0) \\ b_1 &= (1, 1, 1, 0, 0, 1, 0, 0, 0, 0) \\ b_2 &= (1, 0, 0, 1, 1, -1, 0, 0, 0, 0) \end{aligned}$$

Summary so far:

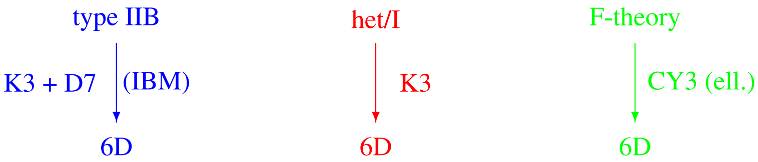
- Each consistent 6D SUGRA $\Rightarrow$ integral lattice Λ
- Finite gauge group, matter combinations for $T < 9$ (in principle denumerable)
- Infinite families at $T \geq 9$

Analysis so far independent of string theory.
Which models have string realizations?

6D $\mathcal{N} = 1$ string vacua

6D excellent playground for developing compactification methods
— Simpler than 4D, but rich group and matter structure.
— Anomalies provide strong checks on internal consistency

Consider several approaches:



Idea of Intersecting Brane Models:
Compactify on X, wrap branes $\Rightarrow$ gauge groups + matter

Problem: can't just have D-branes, since source RR fields

Need to cancel charge with SUSY *orientifolds*

Type I string theory: IIB/Ω

Ω reflects string	$x_L \leftrightarrow x_R$
oriented $\rightarrow$ unoriented	$\alpha \leftrightarrow \tilde{\alpha}$

Simplifies spectrum (*e.g.* removes $B_{\mu\nu}$; $[\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1} - \tilde{\alpha}^{\mu}_{-1}\alpha^{\nu}_{-1}]|0\rangle \rightarrow 0$)

Type I has D-strings + $\tilde{B}$, no B field

1-loop calculation $\rightarrow$ RR tadpole [Polchinski]

$\Rightarrow$ space-filling "orientifold" O9-plane

- O9 has negative tension, negative D-brane charge (-16 D9)
- Can combine Ω, reflection σ $\Rightarrow$ lower-D orientifold plane
- O-planes key in vacuum constructions; combine w/ branes for SUSY bg.

Type I: O9-plane + (16 D9-branes + images)/Ω $\rightarrow$ $SO(32)$ group.

Duality $\phi \rightarrow -\phi$ $(\sim \tau \rightarrow -1/\tau)$: I $\rightarrow$ het $SO(32)$

T-duality on D-branes and O-planes

Recall: for string compactified on circle S^1 of radius R,
given mass spectrum $M^2 = \frac{n^2}{R^2} + \frac{m^2 R^2}{\alpha'}$, T-duality: $R \leftrightarrow \alpha'/R, n \leftrightarrow m$

	T	
IIB on R	$\longleftrightarrow$	IIA on (α'/R)
x_R	$\longleftrightarrow$	$-x_R$
D_p	$\longleftrightarrow$	$D_{p\pm 1}$
A^μ	$\longleftrightarrow$	X_μ
O_p	$\longleftrightarrow$	$2^{\mp 1} O_{p\pm 1}$

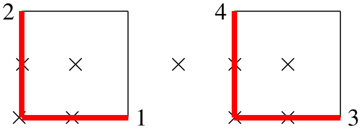
Upshot: I, with O9-plane + 16 D9-branes + images
T-dual on T^k to 2^k O$(9-k)$-planes + 16 D(9-k)-branes + images

Intersecting branes

D-branes intersecting in two planes SUSY if $\theta_1 = -\theta_2$ (same for D $\cap$ O)

Strings between branes $\rightarrow$ chiral matter in bifundamental

Orientifold on K3 (orbifold limit)



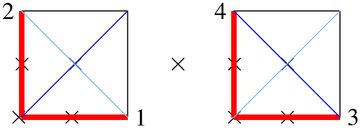
Orbifold action: $\rho : z_i \longrightarrow -z_i$

Orientifold action $\Omega\sigma$, $\sigma : z_i \rightarrow \bar{z}_i$ gives (13) O7-plane

Combined orientifold action: $\Omega\sigma\rho \rightarrow$ second O7-plane (-24)

Intersecting Brane Model

Given K3 orientifold, wrap D7-branes on cycles of two tori
$(n_1, m_1), (n_2, m_2)$ + images $(n_1, -m_1), (n_2, -m_2)$



$N = 8$ on (1, 1) $\times$ (1, -1)

SUSY condition: $m_1/n_1 = -\alpha m_2/n_2$ for all branes (α modulus)

Tadpole conditions: $\sum_a N_a n_1^a n_2^a = 8, \;\; \sum_a (-N_a m_1^a m_2^a) = 8$

Physical content of example model:

Gauge group: $G = SU(8)$ ($T = 1$ from reduction)

Matter: $D = 1$ Adjoint (4φ from 10D vector),
$F = 0$ fundamentals ($\square$), $A = 4$ antisymmetric ($\begin{smallmatrix}\square\\\square\end{smallmatrix}$)

Satisfies F^4 anomaly condition for $SU(N)$: $\boxed{F = 2N - 2ND - A(N - 8)}$

IBM systematics on K3 orbifold [a la Blumenhagen/Braun/Körs/Lüst]

D7-branes → "bulk" cycles from $H_2(T^4;\mathbb{Z}) = U \oplus U \oplus U; \pi_{ij} \cdot \pi_{kl} = 2\epsilon_{ijkl}$

$$
\begin{aligned}
\pi &= n_1 n_2 \pi_{13} + n_1 m_2 \pi_{14} + m_1 n_2 \pi_{23} + m_1 m_2 \pi_{24} \\
o_7 &= 2(\pi_{13} - \pi_{24})
\end{aligned}
$$

Multiplicity of matter reps.
(in each case $R + \bar{R}$)

representation	multiple
Adj	$\pi_a \cdot \pi_a + 1$
$(\square, \bar{\square})$	$\pi_a \cdot \pi_b$
$(\square, \square)$	$\pi_a \cdot \pi'_b$
$\begin{smallmatrix}\square\\\square\end{smallmatrix}_a$	$\frac{1}{2}(\pi_a \cdot \pi'_a + \pi_a \cdot o_7)$
$\square\!\square_a$	$\frac{1}{2}(\pi_a \cdot \pi'_a - \pi_a \cdot o_7)$

Combinatorial range of models limited; $\ll$ anomaly-free set

More sophisticated models include e cycles, fractional cycles
[BBKL ∼ Gimon/Polchinski]

Description away from orbifold K3 more difficult

6D heterotic vacua on K3

$$\begin{array}{c} \text{het} \\ \downarrow K3 \\ \mathbb{R}^{1,5} \end{array}$$

More complicated than 8D: instantons

Models all have $T = 1$ from 10D $B_{\mu\nu}$

Recall from 10D $H = dB - \omega_Y + \omega_R$

$\Rightarrow$ Bianchi identity $dH = \text{tr}R \wedge R - \text{Tr}F \wedge F$

on K3 $\rightarrow$ nonzero instanton number

$$\int_S c_2(F) = \frac{1}{8\pi^2} \int_s \text{Tr}F \wedge F = \frac{1}{16\pi^2} \int_S \text{Tr}R \wedge R = -\frac{1}{2} \int_S p_1(R) = 24$$

So need gauge field configuration with instanton number 24 (and $c_1 = 0$)

Special case: "magnetized brane models"

Have learned want $SO(32)$ or $E_8 \times E_8$ w/instanton # 24 on K3.

SO(32) with $U(1)$ fluxes F on 2-cycles
= "Magnetized branes" in type I language
"T-dual" to IBM $(F = \partial A \leftrightarrow \partial X$ under T-duality)

In type I language:

F^2 on D9-brane carries D5-brane charge from $\int F \wedge F \wedge \hat{C}_6 \quad (d\hat{C}_6 = *d\tilde{B}_2)$
SUSY $\rightarrow$ analogous coupling to R^2 for D-branes, O-planes

O9 $\rightarrow$ 16 D9's + images
each D9 on K3 $\rightarrow$ -1 D5-brane charge
O9 on K3 $\rightarrow$ -8 D5-brane charges

$\Rightarrow$ need 24 D5-branes = instantons $\int F \wedge F$

- Can study on general K3
- Was essentially first class of $\mathcal{N} = 1$ 6D theories studied [Green/Schwarz/West]

Complex structure and Kähler moduli of K3

Complex structure $\Omega \in H^{2,0}(S) \subset H^2(S)$ fixed up to scale on cpx. surface S

satisfies $\int \Omega \wedge \Omega = 0, \quad \int \Omega \wedge \bar{\Omega} \propto \text{Vol}(S) > 0$

Writing $\Omega = x + iy$, $\boxed{x \cdot y = 0,\ x \cdot x = y \cdot y > 0}$.

Kähler form $J \in H^{1,1}(S)$

$$\begin{aligned} \int J \wedge \Omega &= 0 \Rightarrow \boxed{J \cdot x = J \cdot y = 0} \\ \int J \wedge J &\propto \text{Vol}(S) > 0 \Rightarrow \boxed{J \cdot J > 0} \end{aligned}$$

Thus, (x, y, J) define a positive-definite 3-plane in $H^2(S; \mathbb{R}) = \mathbb{R}^{3,19}$

Constraints on fluxes

- Fluxes quantized, normalize $F = 2\pi i f \rightarrow f \cdot f \in 2\mathbb{Z},\ f \in H^2(K3, \mathbb{Z}) = \Gamma^{3,19}$
- Consider N_a fluxes f_a; on 16 U(1)'s in $SO(32)$, $\sim$ 16 branes + $(-f)$ on images

$$F = \begin{pmatrix} \left.\begin{matrix} f_1 & & \\ & \ddots & \\ & & f_1 \end{matrix}\right\} N_1 & & 0 \\ & \left.\begin{matrix} f_2 & & \\ & \ddots & \\ & & f_2 \end{matrix}\right\} N_2 & \\ 0 & & \ddots \end{pmatrix}$$

Tadpole constraint : $\displaystyle \frac{1}{8\pi^2}\int_s \mathrm{Tr} F \wedge F = 24 \ \Rightarrow \sum_a N_a f_a \cdot f_a = -24$

SUSY constraints : $\displaystyle \int f^a \wedge \Omega = 0 \quad \int f^a \wedge \bar{\Omega} = 0 \quad \int f^a \wedge J = 0$

Λ negative definite $\Rightarrow \exists\ \Omega, J$ giving SUSY (pos. def. 3-plane in $\Gamma^{3,19}$)

Fluxes $\{f_a\}$ generate even lattice $\Lambda \subset \Gamma^{3,19}$ characterizing theory

Gauge group:

$$G = U(N_1) \times U(N_2) \times \cdots \times U(N_K) \times SO(32 - 2\sum_a N_a)$$

[Technical points: 1) Some U(1)'s anomalous $\rightarrow$ massive,
2) G may be enhanced when $J \cdot f = 0, f^2 = -2$]

Matter content: depends only on $N_a, m_{ab} = f_a \cdot f_b$

Rep. (+ c.c.)	# hypermultiplets
Adjoint $U(N_a)$	1
$(N_a, \bar{N}_b)$	$(-2 - (f_a - f_b)^2)$
(N_a, N_b)	$(-2 - (f_a + f_b)^2)$
Antisym. $U(N_a)$	$(-2 - 4f_a^2)$
$(N_a, 2M)$	$(-2 - f_a^2)$
Neutral	20

Anomalies: F^4, R^4 cancel (*e.g.* $n_H - n_V + 29n_T = 273$) [Green/Schwarz/West]

Classification of K3 magnetized brane models

Criterion for existence of model, given spectrum $\Leftrightarrow N_a, m_{ab}$:

Must exist lattice embedding $\Lambda(m) \rightarrow \Gamma^{3,19}$

Using Nikulin embedding theorems:

Embedding always possible, generally unique [Kumar/Taylor]

Gives (almost) all anomaly-free models w/ $SO(2M) \times SU(N_1) \times \cdots \times SU(N_K)$

- One class of exceptions: $SO(8) \times SU(24)$ with $3 \times \square_{24}$ + relatives
- Can generalize approach to $E_8 \times E_8$
- More general models from nonabelian bundles [Friedman/Morgan/Witten, ...]
- Models connected by moduli, including different G through Higgsing
- Still limited: Fixed at $T = 1$ for smooth solutions, limited matter reps.
  Note: misses $SU(8)$ IBM despite heuristic "T-duality"
  (include singularities, singular instantons = 5-branes $\rightarrow$ more general)

F-theory in 6D

Compactify on:
elliptic CY3
with section

$$\begin{array}{ccc} T^2 & \longrightarrow & X \\ & & \downarrow \\ & & B \end{array}$$

$X =$ CY 3-fold

$B =$ complex surface

Canonical class of B: $K = T^* \wedge T^* \in H^2(B)$ $(\sim dz_1 \wedge dz_2)$

Canonical class $\sim$ measure of total curvature

For CY: $K_{\text{CY}} = 0$. (Kähler + $K = 0 \leftrightarrow$ CY)

Divisor: $\mathbb{Z}$-linear combination of irreducible algebraic hypersurfaces $\sum_i n_i H_i$

Discriminant locus $\Delta =$ vanishing locus of $4f^3 + 27g^2$ is divisor

Kodaira condition for $X =$ CY $(K_X = 0)$

$$-12K = \Delta$$

(Example: $\mathbb{P}^1$, $K_{\mathbb{P}^1} = c_1(T^*) = -2H$; $-12K_{\mathbb{P}^1} = \Delta = 24H$ $[H = \text{pt.}]$)

Codimension 1 singularities and gauge symmetries

Effective divisor: divisor $\sum_i n_i H_i, n_i \geq 0$

Irreducible effective divisor:
associated with irreducible hypersurface

Discriminant locus Δ = effective divisor

Irreducible components $\rightarrow$ gauge group factors

$$\Delta = \sum_i \nu_i \xi_i + Y$$

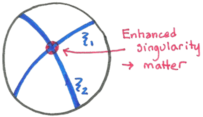
$\xi_i \rightarrow G_i$ through Kodaira/Tate classification

ν_i = multiplicity (*e.g.* $\nu = N$ for $SU(N)$, $\nu = 10$ for E_8, etc.)

Y = residual divisor locus (no nonabelian structure)

Codimension 2 singularities and matter representations

At codimension 2 loci, singularities worsen $\rightarrow$ matter

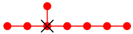


Example: $A_{n-1}\ (\xi_1 \sim z^n) \cap A_{m-1}(\xi_2 \sim w^m)$ at A_{n+m-1} (ord $(\Delta) = n + m$)

$\Rightarrow$ bifundamental $(\square_n, \bar{\square}_m)$

$\sim$ decomposition of adjoint of $SU(n+m)$ (but no actual enhanced symmetry)

- Like IBM but more general,

e.g.  $\Rightarrow (\square_3, \square_2, \square_4)$.

- Some matter not from localized singularities (# Adj's $=$ genus g)
- Possible codimension 2 singularities not completely classified

Bases for elliptic fibrations of CY 3-folds [Morrison/Vafa]

If B smooth, $K \neq 0$, then $B = \mathbb{P}^2, \mathbb{F}_m$ or blow-up thereof

- Follows from minimal surface theory
- For $B = K3$, Enriques surface, $K = 0$ so $\Delta = 0$, higher SUSY.
- $\mathbb{P}^2$: divisors nH, $H^2 = 1$; canonical class $-K = 3H$; $K^2 = 9$
- No systematic classification of B's after many blow-ups

Hirzebruch surfaces $\mathbb{F}_m$:

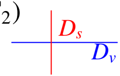
F_m = $\mathbb{P}^1$ bundle over $\mathbb{P}^1$, like compactifying line bundle w/ $c_1 = -m$

Examples: $\mathbb{F}_0 = \mathbb{P}^1 \times \mathbb{P}^1$, $\quad \mathbb{F}_1 =$ blow-up of $\mathbb{P}^2$, $\quad \mathbb{F}_2 = \text{cpt}(T^*\mathbb{P}^1)$

Basis for divisors:

D_v = section, $D_v^2 = -m \quad$ (*e.g.* $D_v^2 = -2$ for $\mathbb{F}_2$)

D_s = fiber, $D_s^2 = 0, D_s \cdot D_v = 1$



Irreducible effective divisors: $aD_v + bD_s$, $b \geq ma$

Canonical class: $-K = 2D_v + (2+m)D_s \quad (K^2 = 8)$; (each blow-up $\rightarrow$ -1)

6D F-theory compactifications: Examples

- $SU(N)$ on $\mathbb{F}_2$ $\quad (-K = 2D_v + 4D_s, \quad \Delta = 24D_v + 48D_s)$

  Wrap on $N \times \xi, \xi = D_v$ $\quad (\Rightarrow Y = (24 - N)D_v + 48D_s)$
  Matter: # fundamental's $\xi \cdot Y = 2N$ anomalies ok
  (no adjoints: $2g - 2 = \xi \cdot \xi + K \cdot \xi = D_v \cdot (-4D_s - D_v) = 2$)

- $E_7 \times E_8$ on $\mathbb{F}_{12}$

  E_8 on D_v, E_7 on $D_u = D_v + 12D_s$
  No bifundamental matter since $D_v \cdot D_u = 0$.

  More generally: heterotic $E_8 \times E_8$, $(12 + n, 12 - n)$ instantons $\rightarrow \mathbb{F}_n$

Big groups [Aspinwall/Morrison]

$G = E_8^{17} \times F_4^{16} \times G_2^{32} \times SU(2)^{32} \quad (T = 192)$
(heterotic dual: all instantons on a singularity)

How to organize the large space of possibilities?
Can systematically understand from constrained set of low-energy theories

Identifying topological F-theory data from SUGRA structure

$T = h_{1,1}(B) - 1; \quad K^2 = 9 - T.$ Each factor G_i maps to singular divisor ξ_i

Matching intersections [Sadov, Grassi-Morrison]

$$\begin{aligned} -a \cdot b &= -K \cdot \xi_i \\ b_i \cdot b_j &= \xi_i \cdot \xi_j \end{aligned}$$

motivates Mapping from low-energy data to F-theory [KMT]

$$\begin{aligned} a &\rightarrow K & \qquad \Lambda &\hookrightarrow H_2(B; \mathbb{Z}) \\ b_i &\rightarrow \xi_i & \qquad j &\rightarrow J \end{aligned}$$

Explicit mapping for $T = 1$:

$$\boxed{b = \frac{1}{2}(\alpha, \tilde{\alpha}) \;\rightarrow\; \xi = \frac{\alpha}{2}(D_v + \frac{m}{2} D_s) + \frac{\tilde{\alpha}}{2} D_s\,.}$$

Agrees with $-a = (2, 2) \rightarrow -K = 2D_v + (m+2)D_s$.

Examples: 6D supergravity and F-theory images

Map $\Lambda \to H_2(B; \mathbb{Z})$ determines topological F-theory data (not nec. unique)
Must check data is topologically ok, Weierstrass.

- $T = 1$: $SU(N)$ with $2N$ fundamentals

  $b = \frac{1}{2}(\alpha, \tilde{\alpha}) = (1, -1) \to D_v + (m/2 - 1)D_s$
  only effective divisor if $m = 2$, where $b \to \xi = D_v$.

- There are 68,997 $T = 1$ models with $G = \prod_i SU(N_i > 2)$, matter $\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}, \square \times \bar{\square}$

–all seem topologically consistent with F-theory on $\mathbb{F}_{0,1,2}$

–identified some Weierstrass models

–# DOF $= 273 - 29 = 244$; imposing $\mathcal{G}, \mathcal{M} \Rightarrow H - V$ constraints
seems plausible all admit Weierstrass models

- Exotic matter representations $\to$ unknown F-theory singularities $(T = 1)$

– *e.g.* $\square + 3\,\square\square + 2\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} + \begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}$, $\quad H - V = 243$

Some $\mathcal{G}, \mathcal{M}$ topologically inconsistent w/F-theory (rare at $T = 1$)

– *e.g.* $SU(4)$ with 1 adjoint, $10 \times \square\square + 40 \times \square$:

$$\Lambda = \begin{pmatrix} 8 & 10 \\ 10 & 10 \end{pmatrix}$$ admits no unimodular embedding

– *e.g.* $SO(8) \times SU(24)$, embedding ok but divisors not both effective

Infinite families revisited

Recall infinite $SU(N) \times SU(N)$ family at $T = 9$

$$a \cdot (b_1 + b_2) = 0 \ \ \& \ \ (b_1 + b_2)^2 = 0$$

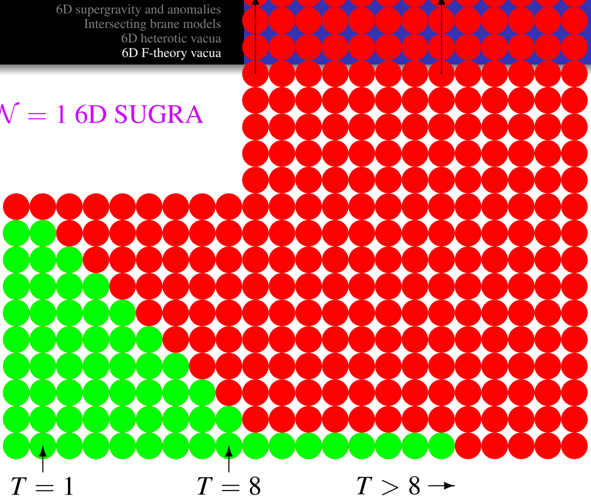
When $a^2 = 0$, this implies $b_1 + b_2 = na$. a primitive $\rightarrow n \in \mathbb{Z}$.

Kodaira constraint $\rightarrow j \cdot Y = j \cdot (12a - N(b_1 + b_2)) > 0$

limits $N \leq 12$ in F-theory [$N = 8$: Dabholkar/Park].

Global picture of $\mathcal{N} = 1$ 6D SUGRA

(cartoon)



- Each $\mathcal{G}, \mathcal{M} \rightarrow$ continuous (quaternionic) moduli space
- Connected by Higgs/small instanton/massless string transitions
  [Witten, Duff/Minasian/Witten, Seiberg/Witten, Morrison/Vafa, KMT III]
- Can we find new consistency conditions, perhaps from F-theory?
- Can we chart regions, connect $\rightarrow$ one theory; string universality?

Summary + Conclusions

- 11D, 10D: Consistent SUGRA ($\mathcal{G}$) = SUSY string vacua ($\mathcal{V}$)
- 8D: $\mathcal{V}$ from het = F-theory $\ll \mathcal{G}$: new constraints?
- 6D: Have general sense of $\mathcal{G}, \mathcal{V}$: new constraints/vacua?
- 4D: Understanding only in local patches

- String theory strongly constrains low-energy physics
- Some (all?) of these constraints are visible in low-energy theory
- Vacuum constructions more complicated in lower dimension
- Fitting into global picture in 4D a challenge
- 6D a good place to learn about constructions, global picture